

# 7 Number Patterns and Sequences

In this unit we consider how number patterns arise, how to find particular patterns and finding the formula for a general term in a sequence. Again, this topic is an important building block in mathematical understanding.

## 7.1 Multiples

We start by looking at a sequence formed by taking multiples of a particular number. For example,

$$3, 6, 9, 12, 15, \dots, \dots$$

which are the multiples of 3.



### Example

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

This square shows the multiples of a number. What is this number?

Write down the numbers that should go in each of these boxes. The number square will help you with some of them.

(a) The 5th multiple of  is .

(b) The th multiple of  is 36.

- (c) The 12th multiple of  is .
- (d) The 20th multiple of  is .
- (e) The th multiple of  is 96.
- (f) The 100th multiple of  is .



### Solution

The number is 4, and

- (a) the 5th multiple of 4 is 20,  
 (b) the 9th multiple of 4 is 36,  
 (c) the 12th multiple of 4 is 48,  
 (d) the 20th multiple of 4 is 80,  
 (e) the 24th multiple of 4 is 96,  
 (f) the 100th multiple of 4 is 400.



### Exercises

1. On a number square like this one, shade all the multiples of 6. Then answer the questions.

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

- (a) What is the 4th multiple of 6?  
 (b) What is the 10th multiple of 6?  
 (c) What is the 12th multiple of 6?  
 (d) What is the 100th multiple of 6?
2. The multiples of a number have been shaded on this number square. What is the number?

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Copy each statement about these multiples and write down the numbers that should go in the boxes.

- (a) The 3rd multiple of  is .
- (b) The 9th multiple of  is .
- (c) The 200th multiple of  is .
- (d) The th multiple of  is 66.
- (e) The th multiple of  is 330.

3. (a) Write down the first 8 multiples of 8.  
 (b) Write down the first 8 multiples of 6.  
 (c) What is the smallest number that is a multiple of both 6 and 8?  
 (d) What are the next two numbers that are multiples of both 6 and 8?
4. (a) Write down the first 6 multiples of 12.  
 (b) What is the 10th multiple of 12?  
 (c) What is the 100th multiple of 12?  
 (d) What is the 500th multiple of 12?  
 (e) If 48 is the  $n$ th multiple of 12, what is  $n$ ?  
 (f) If 96 is the  $n$ th multiple of 12, what is  $n$ ?
5. (a) What multiples have been shaded in this number square?

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

- (b) What is the first multiple *not* shown in the number square?
6. (a) Explain why 12 is a multiple of 6 and 4.  
 (b) Is 12 a multiple of any other numbers?
7. The number 24 is a multiple of 2 and a multiple of 3. What other numbers is it a multiple of?

8. Two multiples of a number have been shaded on this number square. What is the number?

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

9. Two multiples of a number have been shaded on this number square.

|    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10  |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20  |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30  |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40  |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50  |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60  |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70  |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80  |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90  |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

- (a) What is the number?  
 (b) What is the 19th multiple of this number?

10. Three multiples of a number are 34, 170 and 255. What is the number.
11. Three multiples of a number are 38, 95 and 133. What is the number?
12. Four multiples of a number are 49, 77, 133 and 203. What is the number?

## 7.2 Finding the Next Term

Here we use the given numbers of the sequence to deduce the pattern and hence find the next term



### Example

What are the next 3 numbers in the sequences:

- (a) 12, 17, 22, ...
- (b) 50, 47, 44, 41, 38, ...



### Solution

- (a) To spot the pattern, it is usually helpful to first find the differences between each term; i.e.

$$\begin{array}{ccc} 12 & 17 & 22 \\ \swarrow & \searrow & \\ & 5 & 5 \end{array}$$

So the next term is found by adding 5 to the previous term; this gives 27, 32, 37.

- (b) Again we find the difference:

$$\begin{array}{cccccc} 50 & 47 & 44 & 41 & 38 \\ \swarrow & \searrow & \swarrow & \searrow & \\ & -3 & -3 & -3 & -3 \end{array}$$

So the next term is found by taking away 3 from the previous term, giving 35, 32, 29.



## Exercises

1. Copy each of the sequences below and write in the next 3 numbers in each sequence. Complete the working that is shown.

(a) 1, 4, 7, 10, 13, ...  


(b) 3, 5, 7, 9, 11, ...  


(c) 5, 8, 11, 14, 17, ...  


(d) 6, 8, 10, 12, 14, ...

(e) 20, 19, 18, 17, 16, ...

(f) 6, 9, 12, 15, 18, ...

(g) 22, 20, 18, 16, 14, ...

2. Copy each sequence and fill in the missing number.

(a) 4, 7, , 13, 16, ...

(b) 7, , 15, 19, 23, ...

(c) 8, 14, 20, , 32, ...

(d) 3, 11, , 27, 35, ...

(e) 15, , 27, 33, 39, ...

3. Copy and continue each sequence, giving the next three numbers.

(a) 18, 30, 42, 54, 66, ...

(b) 4.1, 4.7, 5.3, 5.9, 6.5, ...

(c) 14, 31, 48, 65, 82, ...

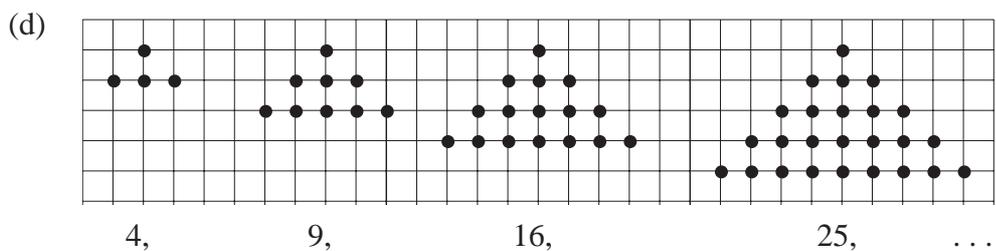
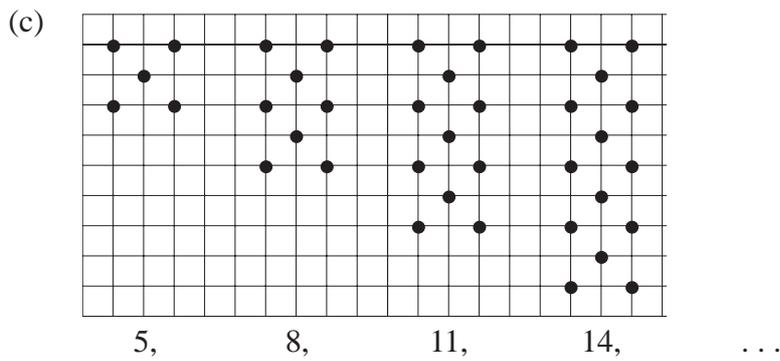
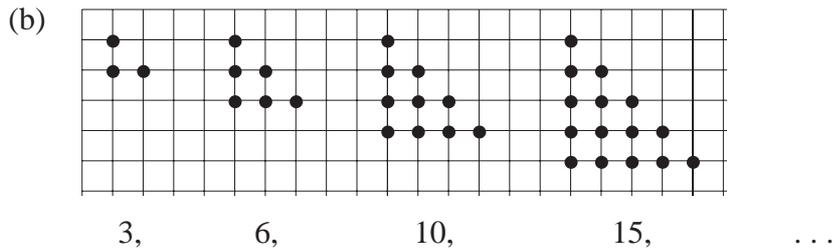
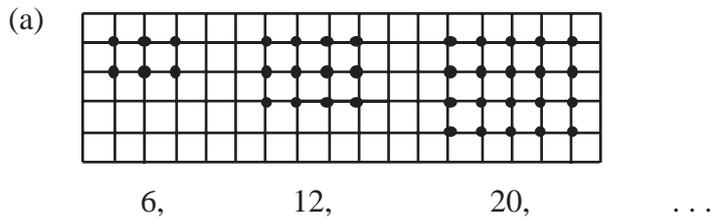
(d) 101, 119, 137, 155, 173, ...

(e) 3.42, 3.56, 3.70, 3.84, 3.98, ...

(f) 10, 9.5, 9, 8.5, 8, 7.5, ...

(g)  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$ , 1,  $1\frac{1}{4}$ ,  $1\frac{1}{2}$ , ...

4. For each sequence of patterns, draw the next two shapes and find the next 3 numbers in the sequence.



5. Find the first number in each of the sequences.

(a) , 6, 11, 16, 21, ...

(b) , 7, 9, 11, 13, ...

(c) , 6, 5, 4, 3, ...

(d) , 19, 28, 37, 46, ...

(e) , 12, 9, 6, 3, ...

6. Copy each sequence and write in the next three terms.

- (a) 1, 4, 9, 16, 25, ...
- (b) 2, 5, 10, 17, 26, ...
- (c) 0, 3, 7, 12, 18, ...
- (d) 6, 12, 20, 30, 42, ...
- (e) 0.5, 2.0, 4.5, 8.0, 12.5, ...

7. Copy each sequence and fill in the missing numbers.

- (a) 2, 4, , 16, 32, ...
- (b) 100, 81, 64, , 36, ...
- (c) 6, 9, , 21, 30, ...
- (d) 0, 1.5, 4, , 12, ...
- (e) 1, 7, 17, , 49, ...

8. Write down the next two terms in each sequence.

- (a)  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$
- (b)  $\frac{9}{11}, \frac{8}{12}, \frac{7}{13}, \frac{6}{14}, \dots$
- (c)  $\frac{3}{6}, \frac{5}{7}, \frac{7}{8}, \frac{9}{9}, \dots$
- (d)  $\frac{2}{1}, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \dots$
- (e)  $\frac{0}{2}, \frac{3}{5}, \frac{8}{10}, \frac{15}{17}, \dots$

## 7.3 Generating Number Sequences

Here we introduce the concept of the general terms of a sequence. For example, the formula  $5n$ , with  $n = 1, 2, 3, 4, \dots$ , generates the sequence

$$5 \times 1, 5 \times 2, 5 \times 3, 5 \times 4, \dots$$

that is  $5, 10, 15, 20, \dots$

Similarly,  $5n + 1$  gives, in the same way,

$$(5 \times 1) + 1, (5 \times 2) + 1, (5 \times 3) + 1, (5 \times 4) + 1, \dots$$

that is 6, 11, 16, 21, ...



### Example 1

What sequence is generated by the formulae

(a)  $5n - 1$                       (b)  $6n + 2$ ?



### Solution

(a) Putting  $n = 1, 2, 3, 4, \dots$  gives

$$4, 9, 14, 19, \dots$$

(b) Putting  $n = 1, 2, 3, 4, \dots$  gives

$$8, 14, 20, 26, \dots$$



### Example 2

What is the formula for this sequence

$$11, 21, 31, 41, 51, 61 ?$$



### Solution

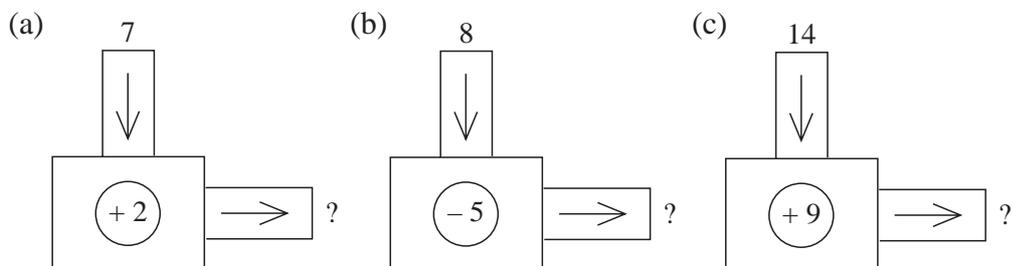
As you are starting with 11, and  $11 = 10 + 1$ , and you continue to add 10 each time, the formula will be

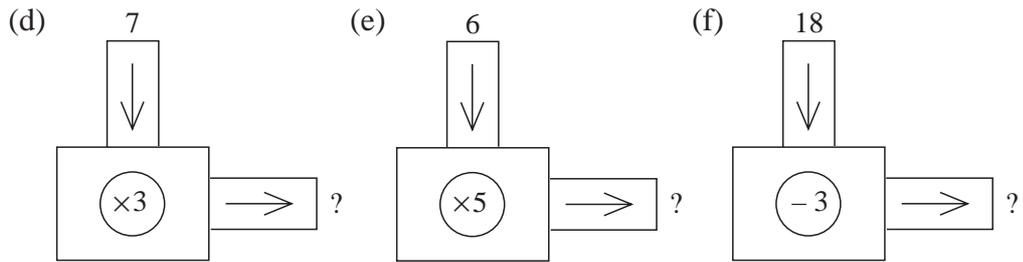
$$10n + 1$$



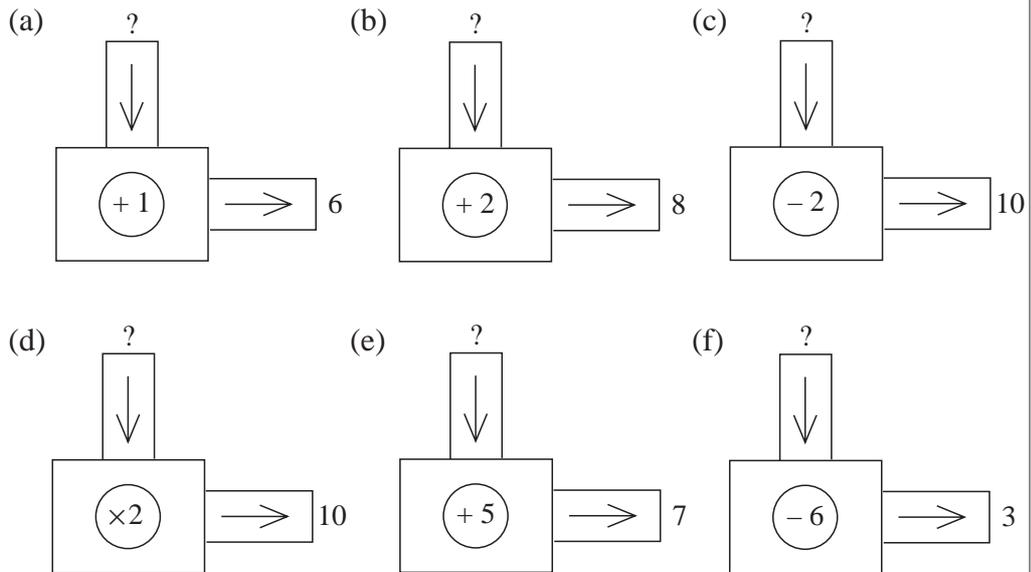
### Exercises

1. What number comes out of each of these number machines?

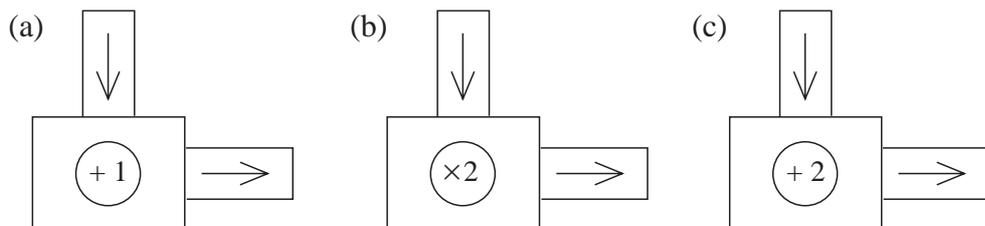




2. What number was put into each of these number machines?

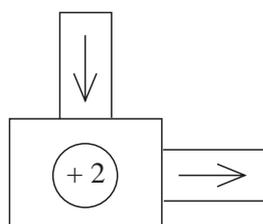


3. The sequence 1, 2, 3, 4, 5, ... is put into each of these machines. Write down the first 5 terms of the sequence that comes out of each machine.

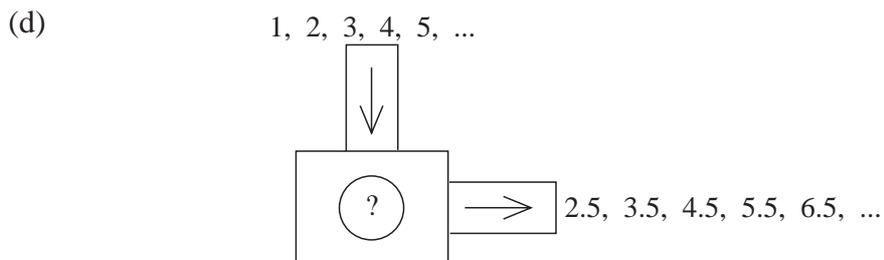
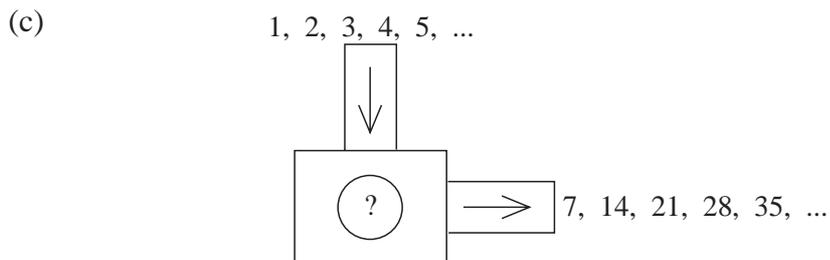
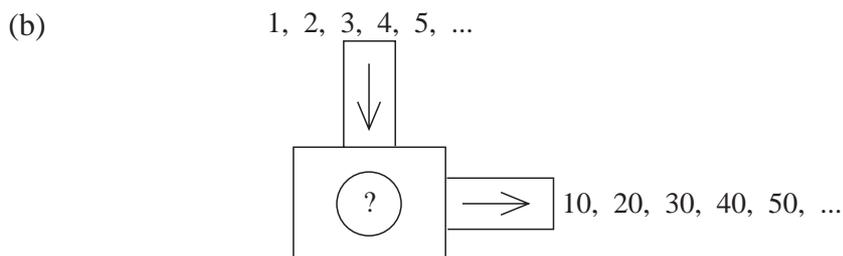
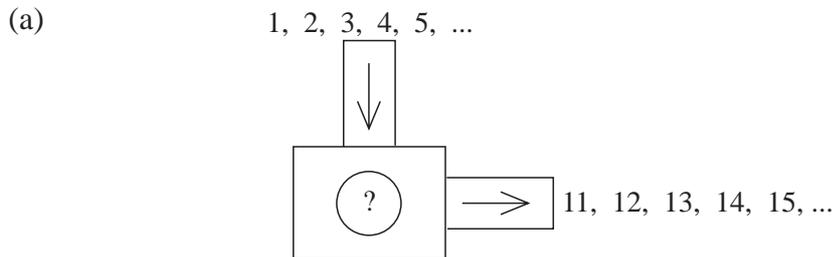


Which machine gives the *even* numbers?

4. (a) Write down the first 5 multiples of 2.  
 (b) What happens if you put these multiples of 2 into this machine?



5. The sequence 1, 2, 3, 4, 5, ... is put into each number machine. What does each machine do?



6. Write down the first 5 terms of the sequence given by each of these formulae:

(a)  $3n$                       (b)  $3n + 1$                       (c)  $3n + 4$

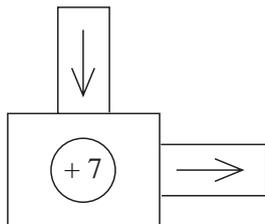
(d)  $3n - 1$                       (e)  $3n - 2$

7. Write down the first 5 terms of the sequence given by each of these formulae:

(a)  $9n$                       (b)  $12n$                       (c)  $2n + 1$

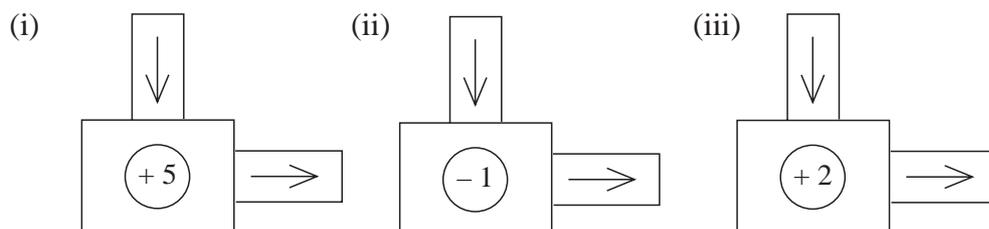
(d)  $3n + 2$                       (e)  $5n - 2$                       (f)  $7n - 1$

8. (a) What is the 10th term of the sequence  $2n + 1$  ?  
 (b) What is the 8th term of the sequence  $3n + 6$  ?  
 (c) What is the 5th term of the sequence  $4n + 1$  ?  
 (d) What is the 7th term of the sequence  $5n - 1$  ?
9. When the sequence  $1, 2, 3, 4, 5, \dots$  is put into the machine:

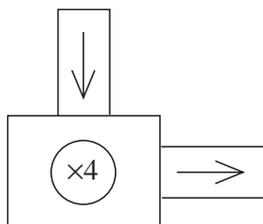


it creates the sequence with formula  $n + 7$ .

- (a) Write down the first 6 terms of the sequence with formula  $n + 7$ .  
 (b) What happens if the sequence  $1, 2, 3, 4, 5, \dots$  is put into these machines? Write down the formula for the sequence you get.

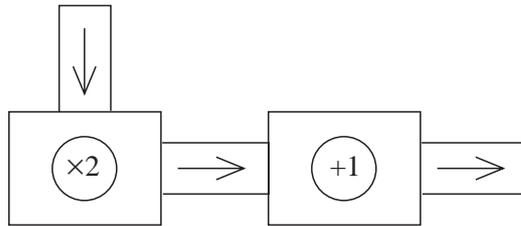


10. You need this machine to get the sequence with formula  $4n$  from the sequence  $1, 2, 3, 4, 5, \dots$



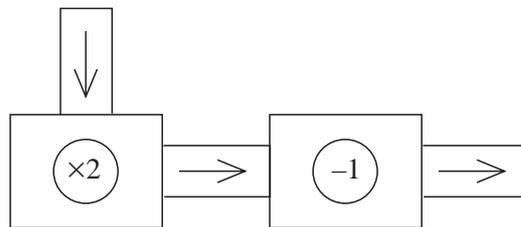
- (a) Write down the first 5 terms of this sequence.  
 (b) Draw the machine you would need to get  $6, 12, 18, 24, 30$  from  $1, 2, 3, 4, 5, \dots$   
 (c) Draw the machine you would need to get the sequence with formula  $7n$  from  $1, 2, 3, 4, 5, \dots$

11. Two machines can be put together like this, to make a double machine.

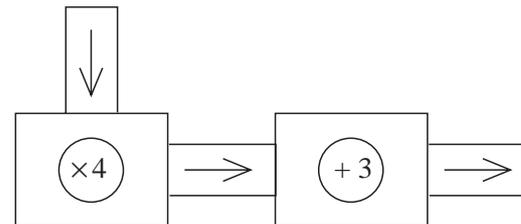


- (a) What do you get if you put 1, 2, 3, 4, 5, ... into this double machine? What is the formula for the sequence you get?
- (b) Repeat part (a) for each of these double machines.

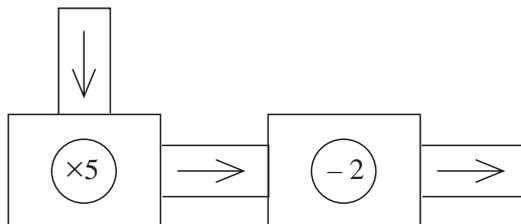
(i)



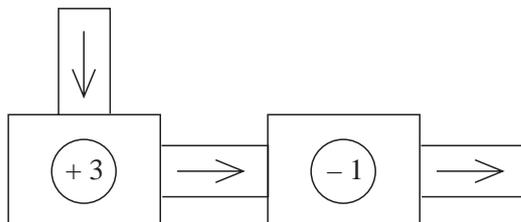
(ii)



(iii)



(iv)



- (c) What single machine has the same effect as the double machine in part (b)(iv)? What is the formula for the single machine?

12. Draw double machines that could be used to get each of these sequences from 1, 2, 3, 4, 5, ...

Also write down the formula for each sequence.

- (a) 5, 9, 13, 17, 21, ...  
 (b) 2, 5, 8, 11, 14, ...  
 (c) 6, 11, 16, 21, 26, ...  
 (d) 4, 9, 14, 19, 24, ...  
 (e) 102, 202, 302, 402, 502, ...

## 7.4 Formulae for General Terms

It is very helpful not only to be able to write down the next few terms in a sequence, but also to be able to write down, for example, the 100th or even the 1000th term!



### Example

For the sequence

$$3, 7, 11, 15, \dots, \dots$$

- find (a) the next *three* terms,  
 (b) the 100th term,  
 (c) the 1000th term.



### Solution

- (a) Looking at the differences,

$$\begin{array}{cccc} 3, & 7, & 11, & 15 \\ \swarrow & \searrow & \swarrow & \searrow \\ & 4 & 4 & 4 \end{array}$$

we can see that 4 is added each time to get the next term.

So we obtain 19, 23, 27.

- (b) To find the 100th term, starting at 3 (the first term), you add on 4, ninety-nine times, giving

$$\begin{aligned} 3 + 4 \times 99 &= 3 + 396 \\ &= 399 \end{aligned}$$

- (c) Similarly, the 1000th term is

$$\begin{aligned} 3 + 4 \times 999 &= 3 + 3996 \\ &= 3999 \end{aligned}$$



## Note

You can go one stage further and write down the formula for a general term, i.e. the  $n$ th term.

This is

$$\begin{aligned} 3 + 4 \times (n - 1) &= 3 + 4n - 4 \\ &= 4n - 1 \end{aligned} \quad \text{(Check the previous answers.)}$$



## Exercises

1. For the sequence:

$$2, 5, 8, 11, 14, \dots$$

- (a) What is the difference between each term?  
 (b) Explain why the formula for the  $n$ th term is  $3n - 1$ .

2. For the sequence

$$6, 8, 10, 12, 14, \dots$$

- (a) find the difference between each term,  
 (b) explain why the formula for the  $n$ th term is  $2n + 4$ .

3. For each sequence, write down the difference between each term and the formula for the  $n$ th term.

- (a) 3, 5, 7, 9, 11, ...  
 (b) 5, 11, 17, 23, 29, ...  
 (c) 4, 7, 10, 13, 16, ...  
 (d) 2, 5, 8, 11, 14, ...  
 (e) 6, 10, 14, 18, 22, ...

4. (a) What formula gives the sequence

$$4, 8, 12, 16, 20, \dots$$

- (b) What formula gives the sequence that is the multiples of 5?

5. (a) What is the formula for the  $n$ th term of this sequence?

$$7, 14, 21, 28, 35, \dots$$

- (b) How can you get this sequence from the sequence in (a)?

$$8, 15, 22, 29, 36, \dots$$

- (c) What is the formula for the  $n$ th term of the sequence in (b)?

6. (a) Write down the first 6 multiples of 11.  
 (b) What is the formula for the  $n$ th term of the sequence of the multiples of 11?  
 (c) What is the formula for the  $n$ th term of this sequence?

$$10, 21, 32, 43, 54, \dots$$

7. Write down the formula for the  $n$ th term of each of these sequences.

- (a) 3, 6, 9, 12, 15, ...  
 (b) 5, 12, 19, 26, 33, ...  
 (c) 21, 29, 37, 45, 53, ...  
 (d) 8, 11, 14, 17, 20, ...  
 (e) 1, 4, 7, 10, 13, ...  
 (f) 103, 106, 109, 112, 115, ...

8. (a) Explain why the formula for the  $n$ th term of this sequence,

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \dots$$

is  $\frac{1}{2n}$ .

- (b) What is the formula for the  $n$ th term of this sequence?

$$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \dots$$

9. Find formulae for the  $n$ th term of each of these sequences.

- (a)  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots$   
 (b)  $\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \frac{5}{8}, \dots$   
 (c)  $\frac{1}{10}, \frac{2}{11}, \frac{3}{12}, \frac{4}{13}, \frac{5}{14}, \dots$   
 (d)  $\frac{2}{8}, \frac{4}{9}, \frac{6}{10}, \frac{8}{11}, \frac{10}{12}, \dots$   
 (e)  $\frac{3}{5}, \frac{6}{6}, \frac{9}{7}, \frac{12}{8}, \frac{15}{9}, \dots$

10. The formula for the  $n$ th term of this sequence is  $n^2$ .

1, 4, 9, 16, 25, ...

What is the formula for the  $n$ th term of the following sequences?

- (a) 0, 3, 8, 15, 24, ...
- (b) 10, 13, 18, 25, 34, ...
- (c) 2, 8, 18, 32, 50, ...
- (d) 1, 8, 27, 64, 125, ...