MODULE

9

Using and applying mathematics

OBJECTIVES

This module is for study by an individual teacher or group of teachers. It:

- considers the nature of problem solving in Key Stage 3 and the implications for planning and teaching;
- discusses examples on using and applying mathematics in the *Framework for teaching mathematics: Years 7, 8 and 9*;
- considers types of questions that will engage pupils in problem solving and probe their understanding.

CONTENT

The module is in six parts.

1 Introduction

2 Devising questions to probe pupils' understanding

3 Exploring the supplement of examples

4 Two problem-solving activities

5 Focusing teaching on objectives

6 Summary

RESOURCES

Essential

- Your personal file for inserting resource sheets and making notes as you work through the activities in this module
- The *Framework for teaching mathematics: Years 7, 8 and 9*
- A calculator
- A highlighter pen
- The resource sheets at the end of this module:
  - 9a Examples of probing questions
  - 9b Questioning pupils about percentages
  - 9c Everything 15% off
  - 9d Classifying quadrilaterals
  - 9e Lesson 1: Year 8
  - 9f Starter and plenary for Lesson 1
  - 9g Lesson 2: Year 7
  - 9h Starter and plenary for Lesson 2
  - 9i Summary and further action on Module 9

Desirable

Assess and review lessons, which you can download from:
http://www.standards.dfes.gov.uk/primary/publications/mathematics/assess_review/

STUDY TIME

Allow approximately 2 to 2.5 hours

Part 1 Introduction

1 From the summer of 2003, Key Stage 3 mathematics test papers have included a greater emphasis on using and applying mathematics. The questions assess pupils' skills in communicating mathematically, reasoning and solving problems.

Module 9 looks at the teaching of problem solving and draws on examples for using and applying mathematics in the *Framework for teaching mathematics: Years 7, 8 and 9,* section 4.

Find page 20 of the Framework section 1. Read the first three paragraphs, including the four bullet points.

Consider how closely this description matches your approach to teaching pupils to:

- use and apply their mathematics in the context of problem solving;
- develop their general thinking skills.

In your personal file, comment on how you can ensure that your teaching incorporates the elements of a good 'diet' listed in the third paragraph.

Part 2 Devising questions to probe pupils' understanding

1 All teaching of mathematics is dependent on good questioning strategies. This is especially so in the teaching of problem solving.

The National Numeracy Strategy produced some guidance materials on probing questions. Read **Resource 9a, Examples of probing questions**, which relate to the key objectives for Year 6. As you study the questions, use a highlighter pen to indicate those that you could use with any of the Key Stage 3 classes that you teach.

2 Now try drafting some probing questions. Try the activity on **Resource 9b, Questioning pupils about percentages**.

As you develop and refine each question, think about these factors.

- How much will this question reveal about the depth of pupils' understanding and thinking?
- As pupils consider this question, will it move their thinking forward?
- Is this question sufficiently open to be asked of a large group or whole class, or would it be better addressed to an individual or small group of pupils?

3 Use the list of questions below to refine and add to your list of sample questions on Resource 9b.

- What percentages can you easily work out in your head? Talk me through a couple of examples.
- When calculating percentages of quantities, what percentage do you usually start from? How do you use this percentage to work out others?
- Are there any percentages that you cannot work out?
- 50% of the numbers on this 1–100 grid are even. How would you check?
- Give me a question with an answer of 20%.

- 'To calculate 10% of a quantity you divide it by 10. So to find 20% you must divide by 20.' What is wrong with this statement?

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4 It is not easy to devise probing questions. Certain starting phrases can be helpful. For example:

- How do you know …? What do you look for?
  e.g. How do you know you have found the simplest form of a fraction? What do you look for?
- How do you go about …?
  e.g. How do you go about ordering a set of decimal numbers? What do you look for first? Which kinds of numbers are particularly difficult to order?
- Make up some questions …
  e.g. Make up some questions that have the answer 'two-fifths'
  … that have the answer 'a parallelogram'.
- Give me a —— that is the same as ——. How did you do it?
  e.g. Give me an equation that has the same solution as 6 = 2p – 8. How did you do it?
- Is … always true, sometimes true, or never true? How do you know?
  e.g. Consider the statement $(3x)^2 = 3x^2$. Is this always true, sometimes true or never true? Explain your reasoning.
- What clues do you use when …?
  e.g. What clues do you use when you are trying to find the size of an unknown angle in a geometric diagram?

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5 Carefully targeted probing questions are a vital assessment tool for any teacher. An important reason for using them regularly is that it helps pupils to see their power and to begin to formulate similar questions for themselves.

Pupils should also learn the habit of regularly asking themselves questions such as:

- What do I know that will help me here?
- Is that the best way of tackling this?
- How can I check that this is a reasonable answer?

These are high-level thinking skills that pupils need to develop as part of their problem-solving repertoire.

Part 3 Exploring the supplement of examples

1 We will now consider some examples of problems suitable for Key Stage 3 pupils.

In your *Framework for teaching mathematics: Years 7, 8 and 9,* turn to the Year 8 yearly teaching programme, section 3, page 8. Study the objectives for 'using and applying mathematics to solve problems'. Compare the Year 8 objectives with those for Year 7, page 6, and Year 9, page 10. Do you get a sense of the expected progression across Years 7, 8 and 9? Pay special attention to the key objectives, to which special emphasis should be given.

In your personal file, jot down all the verbs associated with using and applying mathematics (for example, *solve* …, *identify* …, *justify* …, *explain* …).

Think about how and when Key Stage 3 objectives for using and applying mathematics are taught in your school. Do they get enough emphasis?

If there are any points that you would like to discuss later with your head of department, allocate a page of your personal file for them and jot them down.

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2 The supplement of examples, Framework section 4, starts by illustrating 'using and applying mathematics to solve problems'. The first part of the section (pages 2–25) contains examples for solving word problems. These examples cover all strands of mathematics: number, algebra, shape, space and measures, and handling data. After the examples for word problems, other objectives for using and applying mathematics are exemplified on pages 26–35.

Turn to the supplement of examples, pages 16–17. Spend 10 minutes or so solving a small selection of the problems on these pages in your personal file.

If you have studied Modules 5 and 6, how do these problems and solutions link to the work you did on geometrical reasoning? Next to each of your solutions, make a note of the possible links with geometrical reasoning and how they could be made explicit to pupils.

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3 Now turn to the supplement of examples, pages 26–27. The problems on these pages relate to the two objectives:

- Identify the information necessary to solve a problem.
- Represent problems mathematically in a variety of forms.

Choose one or two of the problems. Spend 10 minutes or so exploring the problems and considering possible solutions.

In your personal file, make a note of any of the problems that you have tried that you would like to give to pupils. Note also any implications for your planning and teaching, such as appropriate points of your school's scheme of work when the problems could be introduced, or the prerequisite skills that pupils would need.

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4 Now go back and look again at each of the problems that you have considered above. For each problem, decide whether you would use it to follow up some previous teaching, so that pupils can use and apply their skills, or whether you would use it to introduce some new mathematics.

Once again, if there are any points that you would like to discuss later with your head of department, add them to the relevant page of your personal file. For example, you may be unsure how a problem could be used to launch a new mathematical topic – if so, your head of department will be able to advise you.

Part 4 Two problem-solving activities

1 There are two different types of problem solving:

- using and applying in a problem-solving context the mathematics that has already been taught;
- problems or starting points that enable pupils to explore and investigate new mathematics.

In thinking about these two types of problem solving, there are several points to note.

- Some problems require pupils to draw on objectives that they have already learned and to use them in a problem-solving context. These problems are often used in the middle or towards the end of a unit of work.
- Some problems are designed to motivate pupils to explore and develop understanding of new objectives. These problems can be used at any stage of a unit.
- Problems may be of short duration and completed in one lesson; others may require more sustained work and span two or more lessons.

You will now work on two activities taken from the Key Stage 3 Framework, section 4, the supplement of examples. The activities have been chosen to illustrate the two main types of problem solving and to link back to Modules 5 to 8, which focused on proportional and geometrical reasoning.

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2 First, explore the problem on **Resource 9c, Everything 15% off**.

Then try the problem on **Resource 9d, Classifying quadrilaterals**.

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3 Possible lines of questioning that you could use with these two problems could include:

- **Everything 15% off**

  What could you change in the original problem in order to explore further?
  For example:
  - exploring for any percentage reduction;
  - exploring for any percentage increase;
  - exploring what happens when an amount is increased by a percentage and then increased again, or vice versa.

  Which is better: a 10% reduction before VAT is added, or a 10% reduction after VAT is added? Justify your answer.

- **Classifying quadrilaterals**

  Why is it not possible to put a quadrilateral in this cell?

  What other pair of categories could you use to classify quadrilaterals?

  What about classifying other shapes?

Part 5 Focusing teaching on objectives

1 Effective teaching is based on clear objectives. Look again at the objectives for the two activities that you have been working on, shown on Resources 9c and 9d. You are now going to plan two lessons, each incorporating one of these activities. This will involve thinking about and making notes on *how to teach* the objectives.

Before you begin to plan the lessons, read 'The focus on direct teaching', in the Framework section 1, pages 26–27.

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2 Direct teaching does not necessarily have to happen before pupils start to work on the activity. You may feel it is appropriate for pupils to begin to explore the activity before teaching the whole class.

While you plan the direct teaching for the main part of your lessons, you may need to:

- focus on particular elements of direct teaching, such as demonstrating and explaining, questioning and discussing;
- think about how you will ensure that all pupils gain from the teaching and are able to tackle the problem;
- think about the specific questions that you will use;
- think about the kinds of resources that would be appropriate.

Start by planning the direct teaching for the main part of the lesson. Begin with the Year 8 lesson, and the problem of 15% off everything. Complete the relevant sections on **Resource 9e, Lesson 1: Year 8**, or, if you prefer, design your own format in your personal file.

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3 Now plan an oral and mental starter and plenary for your lesson. Use **Resource 9f, Starter and plenary for Lesson 1**, or design your own format in your personal file. Refer back to your notes on probing questions as you do this, particularly those on percentages.

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4 Next, repeat what you have done, this time focusing on the classifying quadrilaterals problem for Year 7. Use **Resource 9g, Lesson 2: Year 7**, and **Resource 9h, Starter and plenary for Lesson 2**, or design you own format in your personal file.

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5 If possible, discuss your plans with your head of department, a Key Stage 3 consultant, an advisory teacher or an experienced colleague. Ask them for feedback on:

- whether the plans would be effective in teaching pupils to use and apply mathematics (with specific reference to the particular objectives of the lesson);
- whether the plans cater sufficiently well for the different needs and abilities of the pupils in the class that you have in mind;
- whether the questions that you will use will support pupils, probe their thinking and help to extend their problem-solving skills.

Part 6 Summary

1 You can introduce problem solving and opportunities to use and apply mathematics at many points in a unit of work. A problem can serve as an introduction, to assess pupils' prior knowledge or to set a context for the work; it can be used to provide motivation for acquiring a skill; or it can be set as a class activity or as homework towards the end of a topic, so that pupils use and apply the mathematics they have been taught.

Aim to provide a range of opportunities that include:

- problems and applications that extend content beyond what has just been taught;
- familiar and unfamiliar problems in a range of numerical, algebraic and graphical contexts, some with a single solution and some with several possible solutions;
- activities that develop short chains of deductive reasoning and concepts of proof in algebra and geometry;
- occasional opportunities to sustain thinking by tackling more complex problems.

Wherever possible, use these opportunities to help pupils to appreciate the connections between different aspects of mathematics.

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2 A teacher's questions are central to the development of pupils' reasoning. Good questions prompt pupils to analyse, justify and evaluate their problem-solving strategies.

Several different prompts can be useful in probing pupils' thinking.

- How do you go about …?
- What do you look for when …?
- How do you know that …?
- Why do you think that …?
- How did you reach that conclusion?
- Can you explain why that is right?
- What might explain that?
- How is that possible?
- Is there another way?
- What explanation do you think is best?
- Does it always work? Why?
- Is it always true, sometimes true, or never true? Why?

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3 Look back over the notes you have made during this module. Have you identified all the factors that you want to consider and adopt when you are planning and teaching pupils to use and apply their mathematics and to solve problems?

Use **Resource 9i, Summary and further action on Module 9**, to list key points you have learned, points to follow up in further study, modifications you will make to your planning or teaching, and any points to discuss with your head of department.

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4 If you are interested in looking at more examples of the National Numeracy Strategy's suggestions for probing questions, download *Assess and review lessons* from http://www.standards.dfes.gov.uk/primary/publications/mathematics/assess_review/.

You might also wish to read the leaflet *Changes to assessment 2003: guidance for teachers of KS3 mathematics* (ref: QCA/02/980).

Resource 9a Examples of probing questions

<table>
<tr><th>Objective</th><th>Sample probing questions</th></tr>
<tr><td>Multiply and divide decimals mentally by 10 or 100, and integers by 1000, and explain the effect.</td><td>
• Why do 5 ÷ 10 and 50 ÷ 100 give the same answer?
• This calculator display shows 0.1. Tell me what will happen when I multiply by 100. What will the display show?
• What number is ten times as big as 0.01? How do you know that it is ten times 0.01?
• I divide a number by 10, and then again by 10. The answer is 0.3. What number did I start with? How do you know?
• How would you explain to someone how to multiply a decimal by 10?
</td></tr>
<tr><td>Order a mixed set of numbers with up to three decimal places.</td><td>
• What did you look for first?
• Which part of each number did you look at to help you?
• Which numbers did you think were the hardest to put in order? Why?
• What do you do when numbers have the same digit in the same place?
• Can you explain this to me using a number line?
• Give me a number somewhere between 3.12 and 3.17. Which of the two numbers is it closer to? How do you know?
</td></tr>
<tr><td>Reduce a fraction to its simplest form by cancelling common factors.</td><td>
• What clues did you look for to cancel these fractions to their simplest form?
• How do you know when you have the simplest form of a fraction?
• Give me a fraction that is equivalent to 2/3, but has a denominator of 18. How did you do it?
</td></tr>
<tr><td>Use a fraction as an operator to find fractions of numbers or quantities (e.g. 5/8 of 32, 7/10 of 40, 9/100 of 400 cm).</td><td>
• 2/5 of a total is 32. What other fractions of the total can you calculate?
• Using a set of fraction cards (e.g. 3/5, 7/8, 5/6, 3/4, 7/10, etc.) and a set of two-digit number cards, ask how the fractions and numbers might be paired to form a question with a whole-number answer. Ask: What clues did you use?
</td></tr>
<tr><td>Solve simple problems involving ratio and proportion.</td><td>
• There are 20 boys and 10 girls in Class 6. Give me a sentence using the word ‘ratio’ (or ‘proportion’). What other possibilities are there?
• Look at this Carroll diagram.
<table>
<tr><th></th><th>Boys</th><th>Girls</th></tr>
<tr><td>Wear glasses</td><td>1</td><td>3</td></tr>
<tr><td>Don’t wear glasses</td><td>2</td><td>4</td></tr>
</table>
Give me a question that has the answer:
3 : 7
40%
2 : 1
2/3
</td></tr>
</table>

Objective	Sample probing questions
Carry out column addition and subtraction of numbers involving decimals.	• Make up an example of an addition/subtraction involving decimals that you would do in your head and one you would do on paper. Explain why. • Give pupils some completed questions to mark. Questions need to be written horizontally as well as in column form. Include incorrect answers such as 12.3 + 9.8 = 21.11; 4.07 – 1.5 = 3.92; 3.2 – 1.18 = 2.18. Ask: Which are correct/incorrect? How do you know? Explain what has been done wrong and correct the answers.
Derive quickly division facts corresponding to multiplication tables up to 10 × 10.	• Start from a two-digit number with at least 6 factors, e.g. 56. How many different multiplication and division facts can you make using what you know about 56? How have you identified the different divisions? What if you started with 5.6? What about 11.2? Or 1120?
Carry out 'short' multiplication and division of numbers involving decimals.	• The answer is 12.6. Make up some questions using multiplication and division with decimal numbers.
Carry out long multiplication of a three-digit by a two-digit integer.	• Give pupils three or four long multiplications with mistakes in them. Ask them to identify the mistakes and talk through what is wrong and how they should be corrected. • Give pupils a multiplication question (for example, 147 × 32) calculated by both the grid method and long multiplication. Ask questions such as: What two numbers multiplied together give 4410? Or 294?
Use a protractor to measure acute and obtuse angles to the nearest degree.	• Ask pupils to estimate and measure a range of acute and obtuse angles using a transparency of a protractor with the numbers removed. • As above, but with the two corners broken off. • What important tips would you give to a person about using a protractor?
Calculate the perimeter and area of simple compound shapes that can be split into rectangles.	• Why is it a good idea to split this shape into rectangles to find the area? • How do you go about calculating the dimensions of the rectangles? … the compound shape? • Form a compound shape by pushing together two rectangles. Compare the area and perimeter of the rectangles with the compound shape. What has changed and why? What happens if you join the rectangles in a different way? Why?
Read and plot coordinates in all four quadrants.	• A square has vertices at (0, 0), (3, 0) and (3, 3). What are the coordinates of the fourth vertex? • A square has vertices at (3, 0), (0, 3) and ($^-$3, 0). What are the coordinates of the fourth vertex? • A square has vertices at (0, 0) and (2, 2). Give two possible answers for the positions of the other two vertices. • A square has vertices at ($^-$1, 1) and ($^-$2, $^-$3). Give two possible answers for the positions of the other two vertices.

Objective	Sample probing questions
Identify and use the appropriate operations (including combinations of operations) to solve word problems involving numbers and quantities, and explain methods and reasoning.	• What clues do you look for in the wording of questions? What words mean you need to add, subtract, multiply or divide? • Make up two different word problems for each of these calculations. Try to use a variety of words. $(17 + 5) \times 6$ $12.5 \div 5 - 0.25$
Solve a problem by extracting and interpreting information presented in tables, graphs and charts.	• From a given graph/table/chart, make up three questions that can be answered using the graph/table/chart. • What makes the information easy or difficult to interpret?

Resource 9b Questioning pupils about percentages

Imagine that you are planning a lesson on percentages for a mixed-ability Year 7 class. The objective of the lesson is:

- **Understand percentage as 'the number of parts per 100'; recognise the equivalence of percentages, fractions and decimals; calculate simple percentages and use percentages to compare simple proportions.**

In the space below, make a note of half a dozen questions that you could use in the final plenary of such a lesson, working with the whole class.

The questions should probe pupils' thinking and help you to assess how well they have absorbed the lesson.

Sample probing questions

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Resource 9c Everything 15% off

Problem (see the Framework section 4, page 29, Year 8)

In a gift shop sale everything is reduced by 15%.
A quick way of calculating the sale price is to multiply the original price by a number.
What is the number?
Give a mathematical reason to justify your answer.

After two weeks, the sale price is reduced by a further 15%.
Show that this means the original price has been reduced by 27.75%.

Main objective

- Solve more complex problems by breaking them into smaller steps, choosing efficient techniques for calculation, algebraic manipulation and graphical representation, and choosing suitable resources, including ICT.

Related objectives

- Use logical argument to establish the truth of a statement; give solutions to an appropriate degree of accuracy in the context of the problem.

Links to other strands/objectives

- Express a given number as a percentage of another; use the equivalence of fractions, decimals and percentages to compare proportions; calculate percentages and find the outcome of a given percentage increase or decrease (Framework section 4, pages 70–77).

Possible approaches or methods to use for this problem and potential for exploring it further:

[continued on the next page]

Possible questions that you could use to support, extend or challenge pupils' thinking:

Resources or special preparation that would be needed:

Other notes:

Resource 9d Classifying quadrilaterals

Problem (see the Framework, section 4, page 34, Year 7)

Copy the table below onto a large piece of paper.
Draw and name quadrilaterals in the appropriate spaces.

		Number of pairs of parallel sides		
		0	1	2
Number of pairs of equal sides	0			
	1			
	2			

Will any of the spaces remain empty? If so, explain why.

Main objective

- Suggest extensions to problems by asking 'What if …?'; begin to generalise and to understand the significance of a counter-example.

Related objectives

- Identify the necessary information.
- Represent problems mathematically, making correct use of symbols, words, diagrams, tables or graphs.

Links to other strands/objectives

- Identify and use angle, side and symmetry properties of triangles and quadrilaterals (Framework section 4, pages 184–189).

Possible approaches or methods to use for this problem:

[continued on the next page]

Possible questions that you could use to support, extend or challenge pupils' thinking:

Resources or special preparation that would be needed:

Other notes:

Main activity

Objectives

- Solve more complex problems by breaking them into smaller steps, choosing efficient techniques for calculation, algebraic manipulation and graphical representation, and choosing suitable resources, including ICT.
- Use logical argument to establish the truth of a statement; give solutions to an appropriate degree of accuracy in the context of the problem.
- Express a given number as a percentage of another; use the equivalence of fractions, decimals and percentages to compare proportions; calculate percentages and find the outcome of a given percentage increase or decrease.

Key vocabulary

Activity

Problem

In a gift shop sale everything is reduced by 15%.
A quick way of calculating the sale price is to multiply the original price by a number.
What is the number?
Give a mathematical reason to justify your answer.

After two weeks, the sale price is reduced by a further 15%.
Show that this means the original price has been reduced by 27.75%.

Notes on direct teaching for main part of lesson

[continued on the next page]

Notes on direct teaching for main part of lesson (continued)

Resource 9f Starter and plenary for Lesson 1

Plan an oral and mental starter and a plenary for Lesson 1.

Starter for Lesson 1

Teaching group and objective(s)

Key vocabulary

Notes on organisation, activity, key questions

[continued on the next page]

Plenary for Lesson 1

Plenary organisation and activity

Examples of probing questions to use in the plenary

Key points for pupils to remember to be drawn out at end of plenary

Main activity

Objectives

- Suggest extensions to problems by asking 'What if …?'; begin to generalise and to understand the significance of a counter-example.
- Identify the necessary information.
- Represent problems mathematically, making correct use of symbols, words, diagrams, tables or graphs.
- Identify and use angle, side and symmetry properties of triangles and quadrilaterals.

Key vocabulary

Activity

Problem

Copy the table below onto a large piece of paper.
Draw and name quadrilaterals in the appropriate spaces.

Will any of the spaces remain empty? If so, explain why.

		Number of pairs of parallel sides		
		0	1	2
Number of pairs of equal sides	0			
	1			
	2			

Notes on direct teaching for main part of lesson

[continued on the next page]

Notes on direct teaching for main part of lesson (continued)

Resource 9h Starter and plenary for Lesson 2

Plan an oral and mental starter and a plenary for Lesson 2.

Starter for Lesson 2

Teaching group and objective(s)

Key vocabulary

Notes on organisation, activity, key questions

[continued on the next page]

Plenary for Lesson 2

Plenary organisation and activity

Examples of probing questions to use in the plenary

Key points for pupils to remember to be drawn out at end of plenary

Resource 9i Summary and further action on Module 9

Look back over the notes you have made during this module. Identify the most important things to consider and modify in your planning and teaching of using and applying mathematics.

List two or three key points that you have learned.

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List two or three points to follow up in further study.

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List two or three modifications that you will make to your planning or teaching of using and applying mathematics.

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List the most important points that you want to discuss with your head of department, or any further actions you will take as a result of completing this module.

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