Stacks

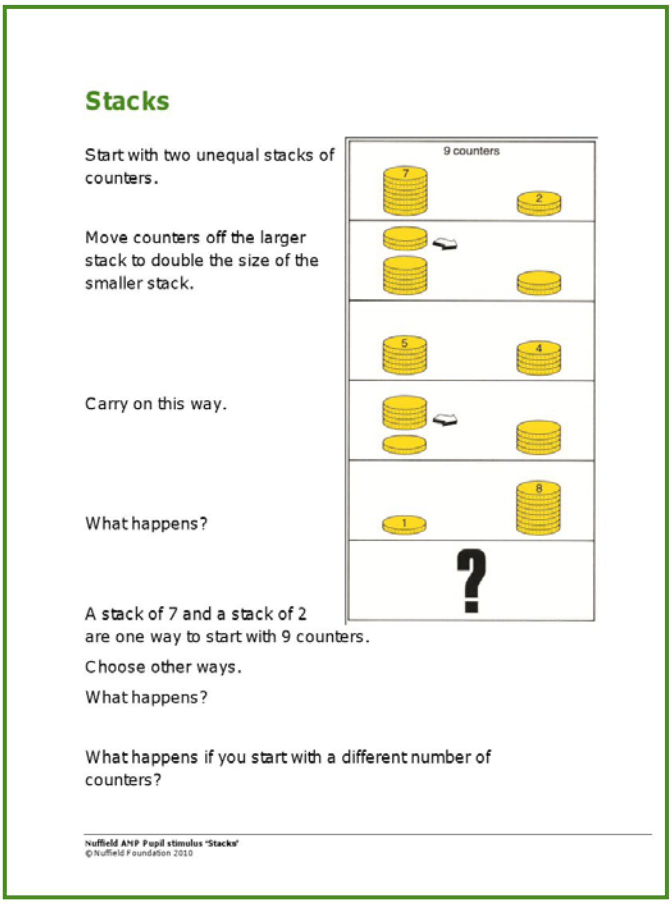
Applying Mathematical Processes

In this **investigation** pupils explore, analyse and describe the patterns generated by moving counters between two stacks according to a fixed rule, always doubling the size of the smaller stack.

Suitability Pupils working at all levels; individuals or groups

Time 2 hours

Equipment
Counters or multilink cubes
Interactive software
Spreadsheet



Stacks

Start with two unequal stacks of counters.

Move counters off the larger stack to double the size of the smaller stack.

Carry on this way.

What happens?

A stack of 7 and a stack of 2 are one way to start with 9 counters.

Choose other ways.

What happens?

What happens if you start with a different number of counters?

Nuffield AMP Pupil stimulus 'Stacks'

Resources

- PUPIL STIMULUS
- FLASH INTERACTIVE
- TEACHER SUMMARY
- TEACHER GUIDANCE
- PROGRESSION TABLE
- SAMPLE RESPONSES

The Clothworkers' Foundation

Nuffield Foundation

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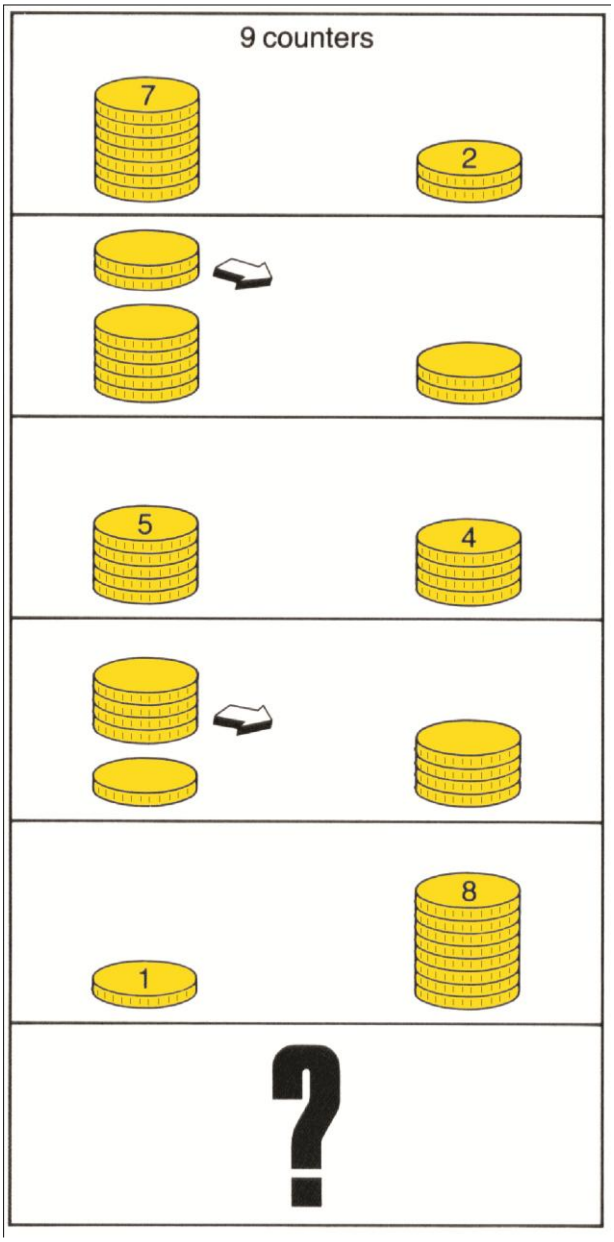

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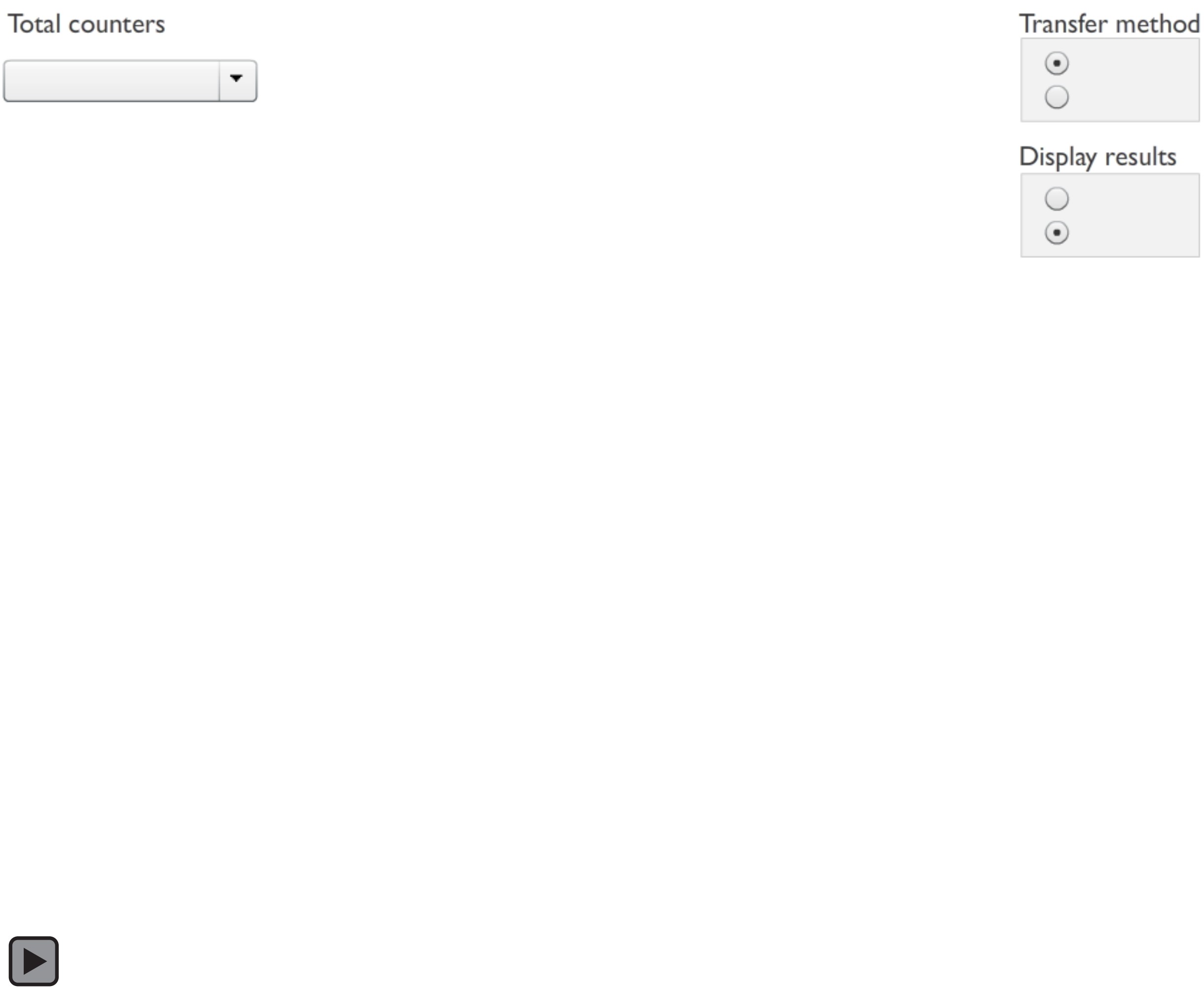
Stacks
Flash interactive



Teacher notes Stacks

Activity description

(interactive not shown on this sheet)

Pupils start by exploring the patterns generated by moving counters between two stacks according to a fixed rule, doubling the size of the smaller stack. They are then asked to explore and describe the patterns arising from using different numbers of counters.

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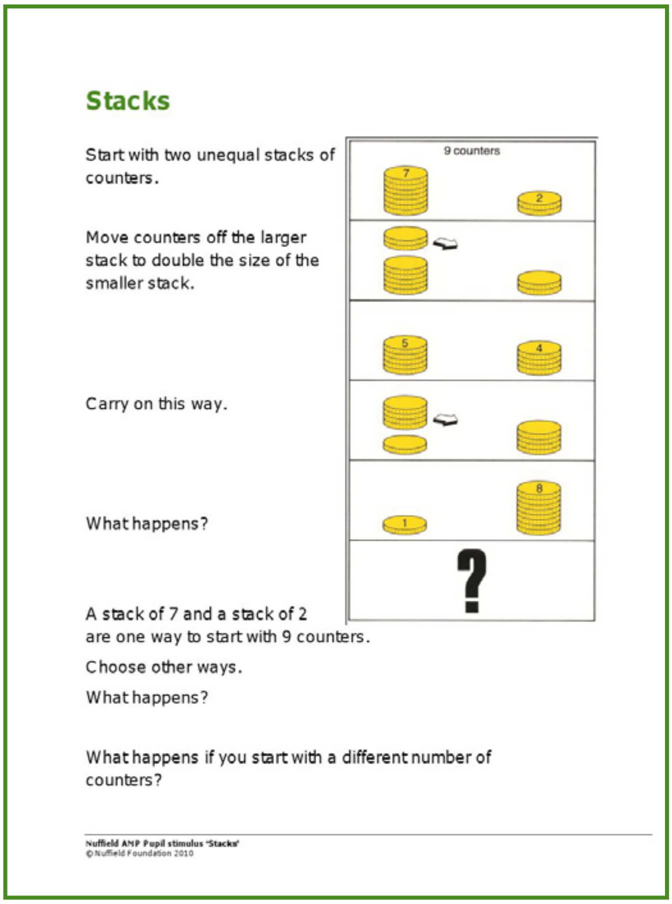
Time 2 hours

AMP resources

Pupil stimulus, Flash interactive

Equipment

Counters or multilink cubes
Spreadsheet



Stacks

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Key mathematical language

cycle, repeat, sequence, reflection, predict, conjecture, proof

Key processes

Representing Identifying which variables and other mathematical aspects to focus on; devising appropriate forms of representation.

Analysing Working systematically; forming conjectures about relationships.

Interpreting and evaluating Exploring, verifying and justifying patterns and generalisations.

Communicating and reflecting Describing decisions, conclusions and reasoning clearly.

Teacher guidance

Ensure that the pupils understand the rule for creating the sequences of stacks and that they have equipment to experiment with. The rule can be demonstrated using large discs, an overhead projector, the interactive program, or using pupils in two rows to enact the movements between them.

Explore with pupils at least two different starting stacks for a chosen total of counters. Pupils should determine how they will gather and record results.

During the activity

Pupils who stop when the second stack is bigger than the first should be reminded that the rule is to double the smaller stack.

Make sure that pupils are clear about why they are stopping when they finish exploring a particular starting point.

A potential stumbling block with this activity is that, given the wealth of results, it can be difficult to manage all the data and to sort meaningful connections from red herrings. Pupils will need time, and may need some help, in establishing a systematic approach and a clear recording system.

As presented, this is a 2-variable problem – for instance the total number of counters and the number in one of the stacks to start with. Encourage pupils to be clear about any labels they choose for different numbers (variables) to lessen the chances of confusion.

If necessary, help pupils see that a useful strategy can be to explore all possibilities with a given total number of counters before moving on to a different total number.

There is potential in this activity to extend even the most capable pupils, but to do this they may need to be encouraged to continue when they have found generalisations for a limited set of numbers of counters.

As pupils become familiar with the practical process of moving the counters, they may naturally move to more abstract representations. After this has happened, some may prefer to record their results in a spreadsheet.

Encourage sharing between pupils of areas they have explored. These might include:

- exploring which starting numbers produce a complete set of possible column heights;
- exploring which starting numbers produce 'reversing' chains, such as (2,3) > (4,1) > (3,2) > (1,4) > (2,3) > ...;
- exploring which starting numbers produce different types of 'sub-cycles';
- discovering results about specific families of numbers, such as prime numbers or powers of two.

Probing questions and feedback

- How do you know when you have got all possible configurations for a given number of counters?
- Can you see patterns that are in common? See if you can group your results to highlight any patterns.
- Have you made any predictions? How will you decide if you are correct?
- Can you conjecture any general rules? If so, what are they?
- Explain or prove why your rules must be true.

Progression table

Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
Choice(s) about mathematical features to investigate; choice of variables	*Working systematically; forming conjectures about relationships*	*Exploring, verifying and justifying patterns and generalisations*	*Describing decisions, conclusions and reasoning clearly*
Applies the rule and keeps a record of results Pupils A, B	Accurately generates sequences for one or more total numbers of counters Pupils A, B	Understands that the defining rule can be applied to any number of counters	Communicates a finding sufficiently clearly for someone else to understand Pupils C, E
Presents results clearly and consistently, e.g. in a table, as number pairs or a mapping diagram Pupil C	Makes some attempt to select and control variables	Demonstrates, possibly by stopping, that a stacking sequence is determined when the original pair of numbers is reached, or when a number pair is repeated Pupils A, B	Describes a stacking sequence they have found, and describes their approach in a way that is fairly easy to follow Pupil B
Recognises the need to collect all possible results for a given total number of counters, and uses a systematic approach to do this Pupil C	Seeks a relationship, e.g. 'Is there a connection between the total number of counters and the number of steps?' Pupil D	Makes a simple observation, e.g. 'some chains have all possible numbers in them; others don't' Pupils C, D, E	Communicates patterns in some detail Pupil D
Uses algebraic terms in attempting to generalise stacking sequences Pupil E	Uses an effective method to work towards a solution, including developing conjectures and considering counter-examples	Develops a coherent picture by collating and building on their findings	Communicates findings clearly and shows some evidence of reflecting on their approach
By working through different examples, searches for a general classification of stacking sequences	Systematically explores relationships between the nature of the stacking sequences and the starting stack sizes	Justifies accurate generalisations for relationships between stacking sequences and the starting stack sizes	Describes decisions, conclusions and reasoning clearly and reflects on their approach

PROGRESSION

Download a Word version of this Progression Table from
www.nuffieldfoundation.org/AMP

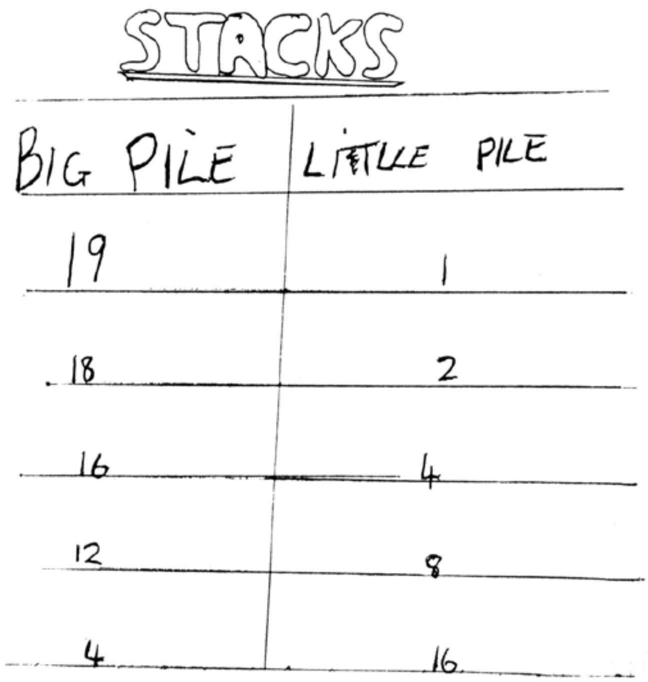
Sample responses

Pupil A

Pupil A understands and applies the rule and records results systematically. The columns are labelled in a way that initially represents the variable number of counters in each column, but this breaks down.

Probing questions

- Are your column headings consistent with your work? If not, how could you address this?
- What happens next if you apply the rule to the last step?



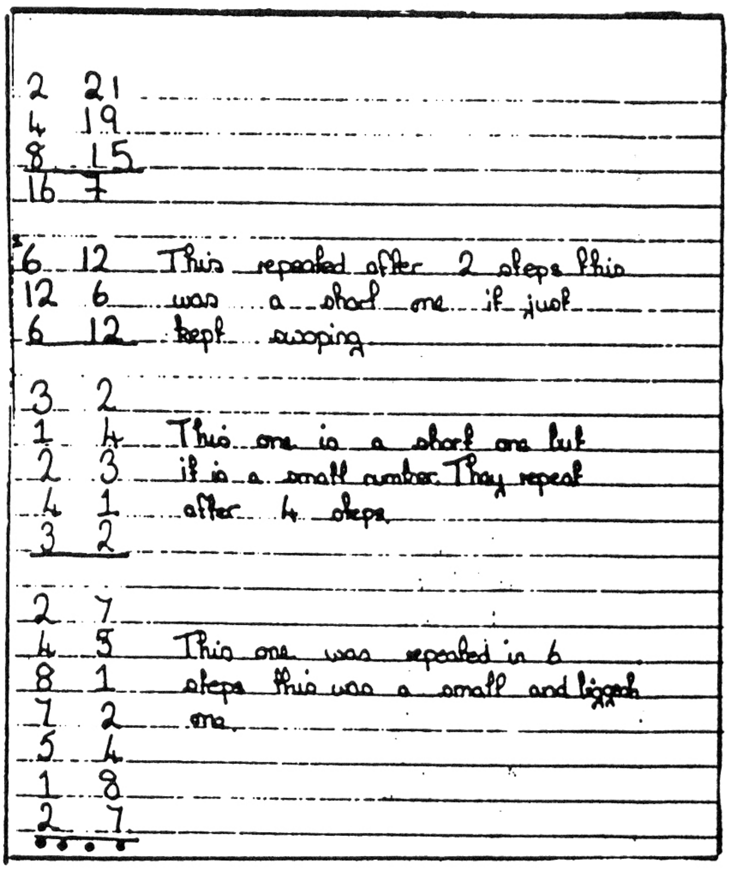
Pupil B

Pupil B has presented results clearly and consistently, and has demonstrated that a stacking sequence ends when the original pair of numbers is reached, or when a number pair is repeated.

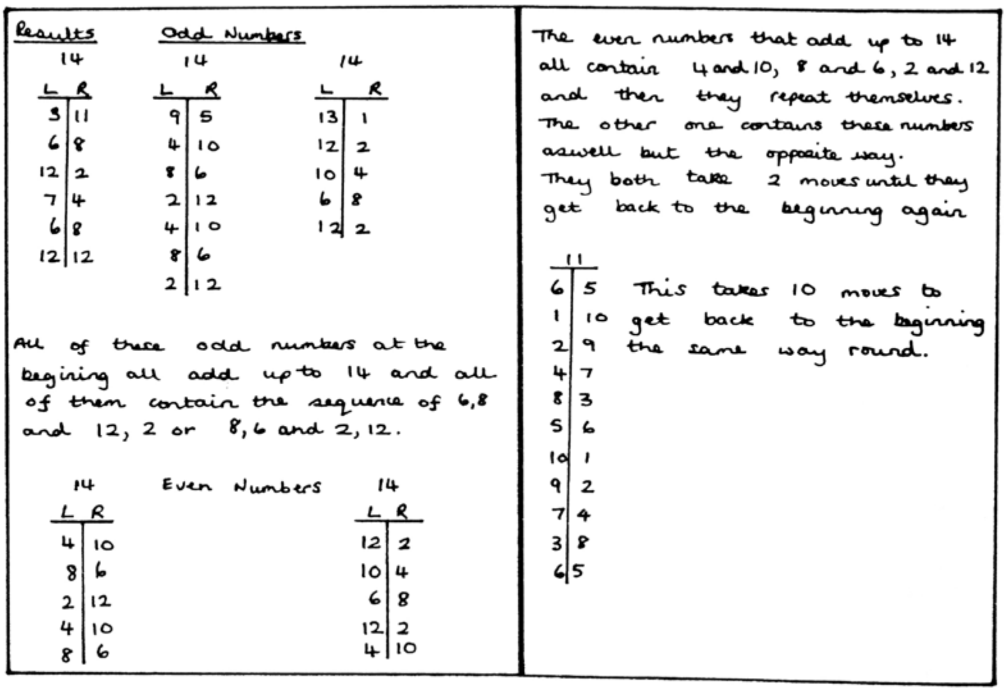
The selection of starting numbers appears random, suggesting that the value of focused work has not yet been recognised – for example by collecting all possible results for a given total number of counters.

Probing questions

- How have you selected your starting numbers?
- Could you make other patterns starting with 9 counters?



Pupil C



Results

Odd Numbers

14 L	14 R
3	11
6	8
12	2
7	4
6	8
12	12

14 L	14 R
9	5
4	10
8	6
2	12
4	10
8	6
2	12

14 L	14 R
13	1
12	2
10	4
6	8
12	2

All of these odd numbers at the begining all add up to 14 and all of them contain the sequence of 6,8 and 12, 2 or 8,6 and 2,12.

Even Numbers

14 L	14 R
4	10
8	6
2	12
4	10
8	6

14 L	14 R
12	2
10	4
6	8
12	2
4	10

The even numbers that add up to 14 all contain 4 and 10, 8 and 6, 2 and 12 and then they repeat themselves. The other one contains these numbers aswell but the opposite way. They both take 2 moves until they get back to the beginning again

11	
6	5
1	10
2	9
4	7
8	3
5	6
10	1
9	2
7	4
3	8
6	5

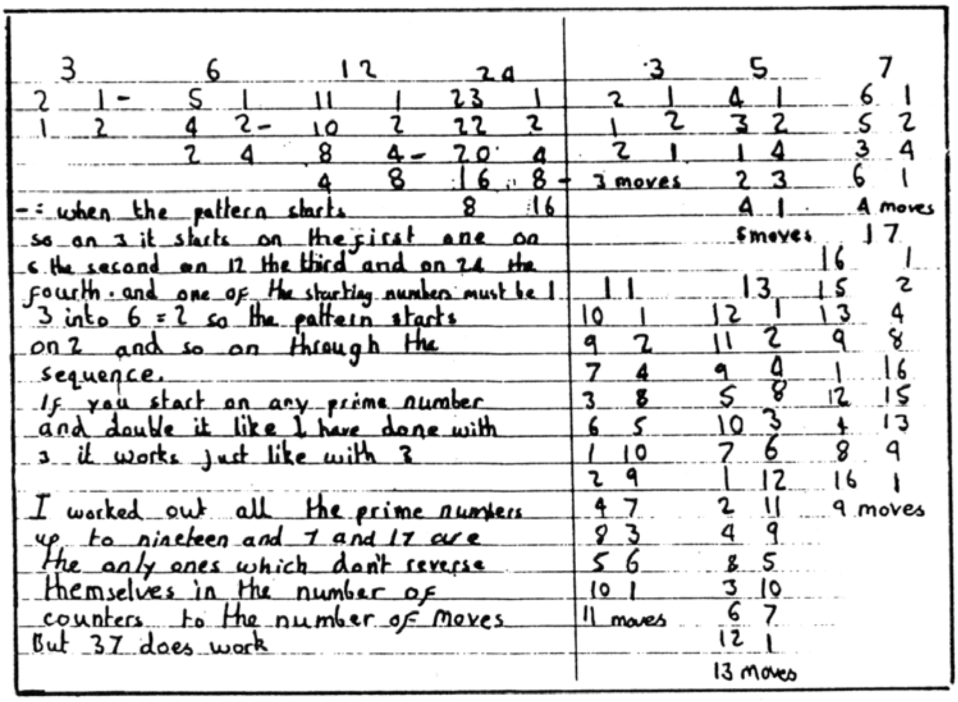
This takes 10 moves to get back to the beginning the same way round.

Pupil C uses an effective recording system, improving as the work progresses in accuracy and, mostly, recognising when to stop.

Some attempt to control variables is shown by considering odd then even starting numbers, and simple statements are made for each, but without connecting the two. The result for 11 counters is correct, and it is recognised that getting back to the beginning ‘the same way round’ is important.

Probing questions

- What can you tell me if I start with 14 counters altogether?
- What are you going to look at next?



3 6 12 24
2 1 – 5 1 11 1 23 1
1 2 4 2 – 10 2 22 2
2 4 8 4 – 20 4
4 8 16 8 –
8 16

– = when the pattern starts
so on 3 it starts on the first one on 6 the second on 12 the third and on 24 the fourth. and one of the starting numbers must be 1
3 into 6 = 2 so the pattern starts on 2 and so on through the sequence.
If you start on any prime number and double it like I have done with 3 it works just like with 3

I worked out all the prime numbers up to nineteen and 7 and 17 are the only ones which don't reverse themselves in the number of counters to the number of moves
But 37 does work

3
2 1
1 2
2 1
3 moves

5
4 1
3 2
1 4
2 3
4 1
5 moves

7
6 1
5 2
3 4
6 1
4 moves

11
10 1
9 2
7 4
3 8
6 5
1 10
2 9
4 7
8 3
5 6
10 1
11 moves

13
12 1
11 2
9 4
5 8
10 3
7 6
1 12
2 11
4 9
8 5
3 10
6 7
12 1
13 moves

17
16 1
15 2
13 4
9 8
1 16
12 15
4 13
8 9
16 1
9 moves

Pupil D

Pupil D has made some significant progress in managing the range of possible variables. This work is close to a generalisation where the smaller stack starts at 1, but the explanation could be clearer. With prime numbers, there is a description of emerging patterns, but with missing elements.

Probing questions and feedback

- See if you can clarify your statement about prime numbers working ‘just like with 3’.
- Make a general statement which you think would be true for *any* prime number of starting counters.
- Explore patterns obtained when you start with a smaller stack of size other than 1.

Pupil E

For 2, 4, ~~6~~, 8, 16, 32 etc
stopping stack is $\frac{2^n}{2}$

So for 64 counters
stopping stack will be 32

Pupil E has not summarised or otherwise presented how the work was tackled, but a systematic approach can be inferred since a relevant sub-set has been identified. (A spreadsheet to generate and record data may have been used.)

A general formula is found for pairs of stacks containing 2^n counters but no explanation given for the variables. There is no attempt made to justify generalisation or explain the significance of dividing by 2, and the conclusion is so brief that no other mathematical insight is shown.

Probing questions

- Explain how you have arrived at your conclusion.
- Why do you think this rule works?