Algebraic notation

How to use these resources

In a departmental meeting:

- consider the pupil's responses to the questions set (resource D1) and discuss the misconceptions that are evident;

- compare your response with the findings from the researchers (resource D2);

- predict how the pupil might answer an associated test question (resource D3);

- explore approaches that target the misconceptions (resource D4) and do the card sort activity (resource D5);

- discuss likely outcomes from pupils' discussions when they use the card sort;

- consider how to use pupils' responses to create and resolve the cognitive conflict by encouraging them to discuss their imagery and reasoning.

In teaching:

- consolidate skills by setting questions that focus on the application of the newly acquired concept;

- adjust your schemes of work to incorporate the activities designed to counter misconceptions.

Samples of a pupil's work

6   A piece of rope 5 metres long is cut into two pieces.
One piece is x metres long.
How long is the other piece?   ......2.5 metres.....................

There are 24 hours in one day. $y = 3$ = 72 hours.
How many hours are there in y days?   ....y x 24 = 24 hours.........

It costs £140 to hire a coach.
This cost is shared equally among n people.   14
How much does each person pay?   ......$n \cdot \cancel{14}$ = £10 each...

A plumber charges £30 to come to your house plus an extra £20 for each
hour that the job takes.
A job takes x hours.
How much does the plumber charge? $x = 3$ £90.

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7.   What can you say about x if $x + y = 10$ and x is less than y ?

     $x = 4$ $y = 6$ $4 + 6 = 10$ . ...

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9   A piece of rope 60 metres long is cut into two pieces.
One piece is x metres long and the other is y metres long.

Write down two equations.
Each equation should use x, y and 60.

     .....$x + y$............ = 60

     x = ...30.....................

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13.   If $y = 1 + 4x$ and $x = 3$
then y = ...$1 + 4 = 5 \times 3 = 15$..............................

If $A = 3r^2$ and r = 4
then A = ...$\cancel{3 \times 3 \times 3}$ $4 + 4 + 4 = 12^2 = 144$.................

Commentary on the pupil's work

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The pupil's answers to questions 6, 7 and 9 show that she does not recognise that letters represent variables. In every case, she substitutes particular values for the letter. In questions 6, 7 and 9, she substitutes 3, 4 and 30 for x, so there is some realisation that x can take different values in different questions, but she does not allow for this within a single question.

Notice also how she has let $n = 14$ in question 6. This is presumably because n is the 14th letter in the alphabet. This is reminiscent of children's secret codes, where $a = 1$, $b = 2$ and so on.

Her responses reveal a general reluctance to leave operations in answers. She appears to think that if an operation is present then something still needs to be done.

Her answer to question 13 shows that she does not recognise the conventions of algebra: that multiplication precedes addition and that squaring should precede multiplication.

Her use of the equals sign is idiosyncratic. As with many pupils, she writes such things as $1 + 4 = 5 \times 3 = 15$ while evaluating an expression. This tendency is consistent with an interpretation of the '=' symbol as meaning 'makes' – a signal to evaluate what has gone before. This is the same meaning as the button with this label on a calculator.

Key Stage 3 test questions

1 Simplify these expressions.

$5k + 7 + 3k =$ _____

$k + 1 + k + 4 =$ _____

2 Solve these equations.

$8k - 1 = 15$ $\qquad$ $k =$ _____

$2m + 5 = 10$ $\qquad$ $m =$ _____

3 You can work out the cost of an advert in a newspaper by using this formula:

$C = 15n + 75$

C is the cost in pounds.
n is the number of words in the advert.

(a) An advert has 18 words.
Work out the cost of the advert.

Show your working.

£ _____

(b) The cost of an advert is £615.
How many words are in the advert?

Show your working.

_____

Always, sometimes or never true?

Answer these questions **on your own**, without talking to your neighbour.
If you get stuck, don't worry. You can come back to them later.
In each question, n stands for any number.

1 When are the following true – always, never or sometimes?
If you choose 'sometimes true' then say when it is true.

	Always true	**Sometimes true when …**	**Never true**
$n + 2 = 3$	No	Yes, when $n = 1$	No
$2n - 3 = 3 - 2n$			
$n + 12 = m + 12$			
$3(n + 3) = 3n + 3$			
n^2 is greater than n			
$4n$ is greater than $4 + n$			

Now try the following activity **in a small group**.
You will need the cards for sorting from resource D5.

2 Divide a sheet of paper into three columns. Label them 'Always true', 'Sometimes true', 'Never true'.

Decide whether each of the statements on the cards is

- Always true   (it is true for all possible values)
- Sometimes true   (it is true for just some values)
- Never true   (no values make the statement true)

Take it in turns in your group to explain your thinking.

Explain using examples how you made your decision.
If you think a statement is sometimes true, say which numbers make it true. If you think a statement is never true, then say how you can be sure.

Statements which are always true are called **identities**.

When you reach agreement, stick the statement into the correct column.

On your own, go back and revise your answers to question 1.
Make notes on any mistakes you made and the reasons for them.
Make notes on new things you have learned.

Equations for sorting

D1 $a + 5 = 12$	**D2** $b + 12 = b + 16$
D3 $2c + 3 = 3 + 2c$	**D4** $2d - 5 = 5 - 2d$
D5 $4 + 2e = 6e$	**D6** $f + 12 = g + 12$
D7 $4h > 4 + h$	**D8** $k + 5 < 20$
D9 $3(m + 3) = 3m + 3$	**D10** $4(3 + n) = 12 + 4n$
D11 $p^2 > 4$	**D12** $q^2 = 10q$
D13 $r^2 > r$	**D14** $16s^2 = (4s)^2$