

Multiplication and division

How to use these resources

In a departmental meeting:

- consider the pupil's responses to the questions set (resource B1) and discuss the misconceptions that are evident;
- compare your response with the findings from the researchers (resource B2);
- predict how the pupil might answer an associated test question (resource B3);
- explore approaches that target the misconceptions (resource B4) and do the card sort activity (resource B5);
- discuss likely outcomes from pupils' discussions when they use the card sort;
- consider how to use pupils' responses to create and resolve the cognitive conflict by encouraging them to discuss their imagery and reasoning.

In teaching:

- consolidate skills by setting questions that focus on the application of the newly acquired concept;
- adjust your schemes of work to incorporate the activities designed to counter misconceptions.

Samples of a pupil's work

1. Do these in your head and write down your answers as decimals:

- (a) $4 \div 20 =$ 5.0
 (b) $6 \times 0.5 =$ 3.0
 (c) $10 \div 0.5 =$ 2
 (d) $0.7 \div 0.7 =$ 0.7
 (e) $0.2 \times 0.4 =$ 0.8
 (f) $60 \div 0.3 =$ 20
 (g) $60 \times 0.3 =$ 18.0
 (h) $16 \div 20 =$

5. The answer to $26.12 \div 0.286$ will be....

Ring two correct statements

Bigger than 26

Smaller than 26

Bigger than 13

Smaller than 13

Give a rough estimate of the answer: 12.00

7.

(a) The boxes contain **six** statements.

Tick every statement that means the same as $85 \div 17$

How many 17's go into 85 ?

What fraction of 85 is 17 ?

$85 \overline{)17}$ ✓

$17 \overline{)85}$

$\frac{17}{85}$ ✓

$\frac{85}{17}$

(b) Tick every statement that means the same as $19 \div 76$

How many 19's go into 76? ✓

What fraction of 76 is 19 ?

$76 \overline{)19}$

$19 \overline{)76}$ ✓

$\frac{19}{76}$ ✓

$\frac{76}{19}$

Commentary on the pupil's work

The pupil's answer of 5.0 to $4 \div 20$ suggests that he recognises the concept of division, but that he may read $4 \div 20$ as: 'How many 4s are there in 20?'. This conjecture is not supported by later answers, however. His answers to question 7 show that he reads $85 \div 17$ as: 'How many 17s go into 85?' and $19 \div 76$ as: 'How many 19s go into 76?'. He appears to reverse his reading of the symbol \div to accommodate his feeling that one must always divide the larger number by the smaller.

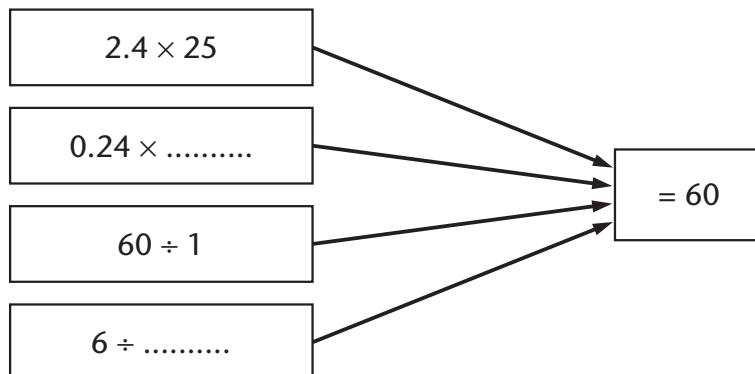
Notice how question 7 reveals that the pupil also has difficulty with other symbols for division. He appears to view $a \overline{)b}$ and $a \div b$ as identical in meaning. He may also have a resistance to 'top heavy' fractions.

Questions 1 and 5 show that the pupil has further difficulties when estimating the result of division by a decimal less than 1; he produces an answer that suggests he believes that division 'makes numbers smaller'. The pupil appears to ignore the decimal point in the questions $10 \div 0.5$ and $60 \div 0.3$. His rough estimate (17.00) to the question $26.12 \div 0.286$ further suggests that he may think that division of a number by a small quantity reduces that number by a small quantity.

Key Stage 3 test questions

- 1 Each of these calculations has the same answer, 60.

Fill in each gap with a number.

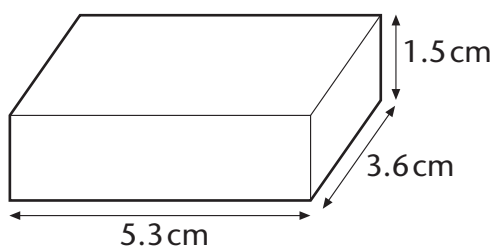


- 2 The diagram shows a matchbox.

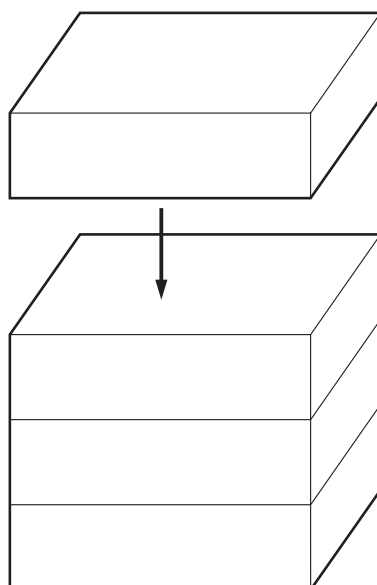
Its length is 5.3cm.

Its width is 3.6 cm.

Its height is 1.5 cm.



I start building matchboxes like this.



How many matchboxes will be in the pile when its height is 12 cm?

Sorting representations

Do these three questions **on your own**, without talking to your neighbour. Do **not** use a calculator.

1 Tick (✓) every statement below that means the same as $85 \div 17$.

☐ How many 17s go into 85?

☐ What fraction of 85 is 17?

☐ $85 \overline{)17}$

☐ $17 \overline{)85}$

☐ $\frac{17}{85}$

☐ $\frac{85}{17}$

2 Tick (✓) every statement below that means the same as $19 \div 76$.

☐ How many 19s go into 76?

☐ What fraction of 76 is 19?

☐ $76 \overline{)19}$

☐ $19 \overline{)76}$

☐ $\frac{19}{76}$

☐ $\frac{76}{19}$

3 Write answers to the following.

$$4 \times 0.5 = \underline{\hspace{2cm}} \quad 0.5 \times 4 = \underline{\hspace{2cm}}$$

$$2 \div 0.5 = \underline{\hspace{2cm}} \quad 0.5 \div 2 = \underline{\hspace{2cm}}$$

Try the following activity **in a small group**. You will need the cards for sorting from resource B5. You can use a calculator for this part.

4 Sort the cards into groups so that each group contains cards that mean the same thing.

Take it in turns to do this. Challenge answers you do not agree with and explain your reasons.

Make sure everyone in your group agrees that your final answers are correct.

5 Make a poster to show your group's answer by gluing the cards onto paper.

On your own, go back and do questions 1 to 3 again.

Write about what you have learned about multiplication and division through these activities.

Cards for sorting



B1 I share 5 litres of lemonade equally among 20 people. How much lemonade does each person get?	B13 I share 20 litres of lemonade equally among 5 people. How much lemonade does each person get?
B2 I pour 20 litres of water into buckets. Each bucket holds 5 litres. How many buckets do I fill?	B14 I pour 5 litres of water into a tank which holds 20 litres. What fraction of the tank does it fill?
B3 I cut a 20 metre rope into 0.5 metre lengths. How many pieces do I get?	B15 I cut a 0.5 metre wire into 20 equal pieces. How long is each piece?
B4 A box holds 20 jars of jam. Each jar weighs 0.5 kg. What weight is in the box?	B16 A full box holds 20 kg of potatoes. What weight does a half full box contain?
B5 $20 \div 5$	B17 $5 \div 20$
B6 $20 \div 0.5$	B18 $0.5 \div 20$
B7 20×0.5	B19 0.5×20
B8 $5 \overline{)20}$	B20 $20 \overline{)5}$
B9 $0.5 \overline{)20}$	B21 $20 \overline{)0.5}$
B10 40	B22 10
B11 $\frac{20}{5}$	B23 $\frac{5}{20}$
B12 4	B24 $\frac{1}{4}$