Fractions and decimals

How to use these resources

In a departmental meeting:

- consider the pupil's responses to the questions set (resource A1) and discuss the misconceptions that are evident;
- compare your response with the findings from the researchers (resource A2);
- predict how the pupil might answer an associated test question (resource A3);
- explore approaches that target the misconceptions (resource A4) and do the card sort activity (resource A5);
- discuss likely outcomes from pupils' discussions when they use the card sort;
- consider how to use pupils' responses to create and resolve the cognitive conflict by encouraging them to discuss their imagery and reasoning.

In teaching:

- consolidate skills by setting questions that focus on the application of the newly acquired concept;
- adjust your schemes of work to incorporate the activities designed to counter misconceptions.

Samples of a pupil's work

4. Write the following fractions as **decimals**:

a) six tenths 06.10

b) four fifths 04.50

c) sixteen tenths 16.10

d) three hundredths 30.10

11. Write these numbers in order of size, from **smallest** to **largest**:

0.625 0.25 0.3753 0.125 0.5

0.3753, 0.625, 0.125, 0.25, 0.5

Smallest **Largest**

Explain how you chose the smallest.

I chose 0.3753 as the smallest as it is the one which is furthest away from 0.5 which is 0.5 away from being a whole number

12. Write these fractions in order of size, from smallest to largest:

$\frac{1}{2}$ $\frac{3}{8}$ $\frac{5}{16}$ $\frac{5}{8}$ $\frac{1}{4}$

5/16, 3/8, 5/8, 1/4, 1/2

Smallest **Largest**

Explain how you chose the smallest.

because 5/16 is much smaller than 1/2 if you cut the fraction out of a cake there would be a lot of 16 small pieces instead of 2 large pieces

Commentary on the pupil's work

This pupil has a misconception that often goes unrecognised. Most teachers are aware of the tendency to ignore decimal points and treat decimals as if they were whole numbers: many pupils obtain such answers as $0.75 > 0.8$. Here is evidence of the reverse tendency to say that numbers with more decimal places are smaller in value.

There are two common reasons why pupils might believe this. First, they feel that, say, 0.45 goes into hundredths while 0.7 only goes into tenths. Thus $0.45 < 0.7$ because 'tenths are bigger than hundredths'. Secondly (and this is the reason that is suggested here), they believe that 0.45 is analogous or equivalent to $\frac{1}{45}$.

This pupil shows in his answers that he does understand one meaning of the denominator in a fraction. He sees $\frac{3}{8}$ as involving the cutting of a cake into 8 parts. He seems, however, to ignore the value of the numerator when comparing fractions.

Key Stage 3 test questions

1 Add $\frac{6}{10}$ and $\frac{6}{5}$.

_____

2 Fill in the missing numbers:

$\frac{1}{2}$ of 20 = $\frac{1}{4}$ of _____

$\frac{3}{4}$ of 100 = $\frac{1}{2}$ of _____

$\frac{1}{3}$ of 60 = $\frac{2}{3}$ of _____

3 A bag has 20 cubes in it. 6 of the cubes are green.
You take one cube out of the bag at random.

Which values below show the probability that you take out a cube that is green?

Circle the correct four values.

$\frac{6}{14}$	30%	0.6	$\frac{3}{10}$
6%	$\frac{3}{5}$	$\frac{6}{20}$	0.03
0.3	$\frac{6}{10}$	60%	$\frac{6}{26}$

4 Peter's height is 0.9 m.
Lucy is 0.3 m taller than Peter
What is Lucy's height?

_____ m

Lee's height is 1.45 m.
Misha is 0.3 m shorter than Lee
What is Misha's height?

_____ m

Sorting representations

Answer these questions **on your own**, without talking to your neighbour.
You may **not** use a calculator for these questions.

1 Write these decimals in order of size, from smallest to largest:

0.4 0.8 0.75 0.3753 0.25 0.125 0.04

_____

2 Write these fractions in order of size, from smallest to largest:

$\frac{3}{4}$ $\frac{3}{8}$ $\frac{2}{5}$ $\frac{8}{10}$ $\frac{1}{4}$ $\frac{1}{25}$ $\frac{1}{8}$

_____

Now try the following activity **in a small group**.
You will need the cards for sorting from resource A5.

3 Sort the cards into groups so that each group contains cards that mean the same thing.

Take it in turns to do this. Challenge answers you do not agree with and explain why.

4 Now arrange each group of cards in order of size.

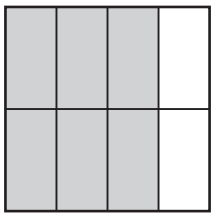
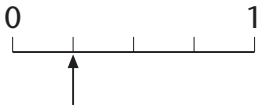
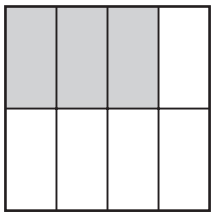
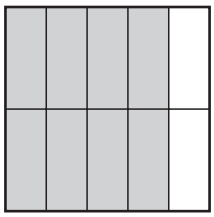
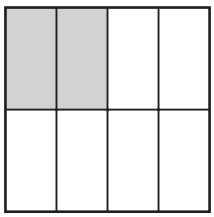
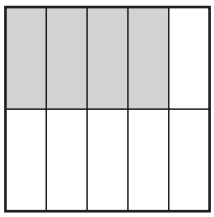
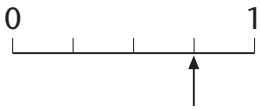
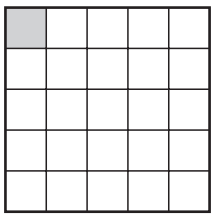
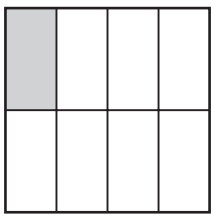
5 Check your work using a number line and a calculator.
Make sure everyone in your group agrees that your final answers are correct.

6 Make a poster to show your group's answers by gluing the cards onto paper.

On your own, compare your answers from the card sorting with your answers to the first two questions. Write about what you have learned. Make notes on any mistakes you made.

Source: This activity is adapted from Swan, M. (2001) 'Dealing with misconceptions in mathematics', pp. 161–162, in Gates, P. (ed.) *Issues in mathematics teaching*, London: Routledge Falmer.

Cards for sorting

A1 	A8 0.8	A15 $\frac{3}{8}$	A22 
A2 	A9 0.25	A16 $\frac{2}{5}$	A23 
A3 	A10 0.4	A17 $\frac{1}{4}$	A24 
A4 	A11 0.125	A18 $\frac{3}{4}$	A25 
A5 	A12 0.75	A19 $\frac{8}{10}$	A26 
A6 	A13 0.04	A20 $\frac{1}{25}$	A27 
A7 	A14 0.375	A21 $\frac{1}{8}$	A28