Interacting with mathematics in Key Stage 3

Years 8 and 9 enhancement materials: notes for departmental discussion

Introduction

The following notes are presented in an ordered sequence, with suggested timings. Use them flexibly to meet the needs of your department when discussing the materials provided to enhance the teaching of proportional reasoning.

The notes for each session are organised as follows:

• background notes and key points (outlined in a box);

• resources needed;

• suggested activities.

Resources

For each session, teachers will need:

Year 8 multiplicative relationships: mini-pack

Year 9 proportional reasoning: mini-pack

Resources needed for a particular session are listed after the key points.

Areas for discussion

Images for scaling and proportion	20 minutes
Proportion or not?	25 minutes
Using a small set of problems	40 minutes
Using pupil scripts	30 minutes
The big picture	20 minutes

Images for scaling and proportion **20 minutes**

Background

Three mini-packs for proportional reasoning have now been published: Year 7 fractions and ratio, Year 8 multiplicative relationships and Year 9 proportional reasoning. Each mini-pack uses images to scaffold pupils' understanding of scaling and proportionality. This session explores the development of these images from Year 7 to Year 9.

Key points

A major emphasis has been on using a line segment image to illustrate the concepts of scaling and ratio: from ratio strips in Year 7 to three graduated line segments in Year 8 and sketches of line segments in Year 9.

Line segments help pupils develop a language to describe a problem and the calculation needed to help with the solution.

The image can be used flexibly to represent quantities of all kinds.

Resources: copies of handout PR1, 'Images for fractions, scaling and proportion'; (optional) copies of handout PR2, 'Chain wheel problems'; computer and ITP
Photographic enlargements

Briefly talk through the **background** notes and then use the **key points** to outline the use of images in the three proportional reasoning mini-packs.

Activity 1 (12 minutes)

Give everyone a **few minutes** to study **handout PR1**, then use the notes below to talk through the progression:

- from Year 7 to Year 8;

- from Year 8 to Year 9.

From Year 7 ...	**... to Year 8**
In the early stages, most problems are best dealt with using informal mental strategies. The image maintains the essence of discrete chunks of data and provides a simple scaffold for thinking through problems.	With problems that require more formal methods, the unitary method is a useful strategy. The image shown draws out clearly the operation needed, for example, to scale the two-block strip down to one.
Focusing on one of the strips a decision is made about the labelling of each block. Focusing on a two-block strip requires dividing by 2 or finding half of the strip in order to label each block.	Linking with the approach in Year 7 this can be referred to as dividing by 2 or halving. To make progress with the notion of inverses it is more helpful for pupils to see this operation as multiplying by $\frac{1}{2}$.

From Year 8 ...	... to Year 9
The two stages of calculation are clearly shown before a single step is recorded. This may be considered as: $\div 2 \times 5$ becoming a single scaling of $\times \frac{5}{2}$ or $\times \frac{1}{2} \times 5$ becoming a single scaling of $\times \frac{5}{2}$ Both of these are useful. The first helps pupils to see the actual steps required for the calculation to be performed on a calculator. The second shows clearly that they are dealing with 'five lots of one half of' and links more easily with inverses.	The interim stage is omitted with an expectation that pupils can quickly see a single multiplier (in this case a choice between $\times \frac{2}{5}$ and $\times \frac{5}{2}$). The accuracy of their choice should be underpinned by the development from Year 8 images and by a secure understanding of whether they expect an increase or decrease in the result. The complexity of calculation is stripped away allowing pupils to concentrate on other aspects of the problem solving process.

Discuss the progression in terms of developments in your school and how you can secure a consistent approach with all classes.

Activity 2 (8 minutes)

At this point you could **either** (a) use **handout PR2** to talk through the use of images to support solving problems **or** (b) demonstrate the ITP provided to support links with enlargement in Year 9.

Summarise by returning to the **key points**.

Proportion or not?

Proportion or not?   **25 minutes**

> **Background**
>
> Phase 2 of the Year 8 multiplicative relationships unit is described on page 9 of that mini-pack. It suggests a lesson on determining whether particular problems or situations are examples of proportions or not.
>
> **Key point**
>
> Determining the type of problem you are dealing with is the first crucial step towards solving it. In teaching, this stage is often rushed or overlooked.

Resources: copies of handout PR3, 'Lesson plan: proportion or not?'; resource sheets PR1 to PR4.

Refer everyone to phase 2 of the Year 8 multiplicative relationships unit (Year 8 mini-pack page 9). Introduce the session using the **background** note and the **key point**. Explain that **handout PR3** is a possible plan for the lesson, with four sets of resource sheets PR1–PR4 to draw on.

Activity 1 (20 minutes)

Ask colleagues to study the plan in pairs and to identify examples from the resource sheets which would be suitable for one of the classes they teach.

After about **15 minutes**, ask for feedback on:

- why they made the choices they did;
- the adaptations they might make to the plan to suit their own needs.

Emphasise that it is important to get pupils to justify the reasons why two variables are in proportion or not.

Activity 2 (5 minutes)

Note that these ideas should be revisited on a number of occasions, not just in a single lesson. Discuss and identify other opportunities in your scheme of work.

Background

Phase 3 of Year 8 multiplicative relationships and Year 9 proportional reasoning are each devoted to solving proportion problems. Page 9 of each mini-pack sets out four suggested activities. This discussion session focuses on a video of the third suggestion, taking a small set of problems and working through the initial stages only.

Key points

The unit also identifies four problem-solving strategies.

- These strategies need to be taught explicitly, not just practised.
- Research shows that teaching approaches such as that adopted in the video are more effective than just working through exercises.

The stages in solving a problem could be classified as follows:

- understanding and representing the problem – recognising problem types and organising relevant information;
- selecting or planning the solution – breaking the problem into steps and reviewing alternative strategies;
- executing the plan – carrying out routines, such as calculations;
- evaluating results – reviewing and making sense of answers.

The first two stages are critical, but pupils often rush to the third stage. Effective problem-solving habits need to be taught, not left to chance.

Resources: video clip 1, 'Enhancing proportional reasoning'; copies of handout PR4, 'Notes for viewing "Enhancing proportional reasoning"'

Use the **background** notes to introduce the session.

Ask everyone to study the notes for main activities on page 9 of either the Year 8 or the Year 9 mini-pack. Clarify the points using the **key points** and briefly discuss the stages in problem solving (perhaps set out on a flipchart or board).

Activity 1 (25 minutes)

Say that you will show a video of Ali teaching her Year 8 class. The pupils cover a wide range of attainment and are expected to achieve results from level 5 to level 7 in the Key Stage 3 tests. The lesson is from phase 3 of the Year 8 multiplicative relationships unit.

Refer everyone to **handout PR4**. Give them a few minutes to examine and set out the calculations they would use to solve each part of the telephone problem.

Explain that the extract from the main activity lasts for about **14 minutes**. Ask teachers to consider the points listed on the handout as they view the video.

After viewing, allow a couple of minutes in pairs or small groups to discuss the points raised on the handout and any other observations. Allow about **5 minutes** for feedback and further discussion.

Emphasise that the video illustrates a teaching approach which helps pupils develop critical problem solving skills.

Activity 2 (15 minutes)

Say that the extract from the plenary lasts for about **6 minutes**. Again, ask teachers to consider the points on handout PR4 as they view the video.

After viewing, allow a couple of minutes in pairs or small groups to discuss the points raised on the handout and any other observations. Allow about **5 minutes** for feedback and further discussion.

Point out that the plenary illustrates how it is possible to support pupils in the process of reflecting on their learning. Through such reflection pupils come to understand the processes of problem solving.

Background

Another teaching approach suggested in phase 3 of the Year 8 and Year 9 units is to present some solutions to a problem and ask pupils to evaluate the efficiency of the strategies chosen, identify errors, and make suggestions for improvement.

The CD-ROM contains about six pupil solutions for each of four proportion problems. None of the pupils had encountered either of the Year 8 or Year 9 mini-packs. Handout PR5 is a sample of the pupils' solutions on the CD-ROM.

Key points

Analysing sets of solutions can help pupils develop their problem-solving strategies:

- by having to explain solutions, discuss and evaluate their effectiveness;
- by addressing misconceptions and inappropriate methods, which they might recognise as their own.

Discussing the solutions of other pupils is an effective strategy for developing problem-solving skills.

There would be value in the department collecting some scripts to use as a shared resource.

More informal approaches, such as asking one pupil to present their solution to the class, can be a regular feature of teaching. Pupils learn that they can overcome difficulties through open discussion.

Resources: copies of handout PR5, 'Four solutions to "Stacking CDs" problem' and handout PR6, 'Planning a lesson using pupils' scripts'

Activity 1 (15 minutes)

Refer everyone to the notes for main activities on page 9 of one of the mini-packs. Use the **background** to explain the source for **handout PR5** and allow **5 minutes** for everyone to work through the examples. Ask them to note particularly:

- how pupils organise the data;
- whether they are clear about the value they are calculating and the operation to use;
- whether they avoid strategies inappropriate to the problem, such as approximating or using informal mental methods.

Spend **a minute or two** discussing colleagues' comments. Then say that you want to concentrate on the benefits to pupils of this kind of activity. Refer to the first paragraph of the **key points** above, adding any other points from colleagues.

Activity 2 (15 minutes)

Refer everyone to **handout PR6** and allow them **a few minutes** to read and absorb the points. Spend about **5 minutes** working through the sections on the handout together, thinking about how the lesson could be structured.

Now ask everyone, in pairs, to discuss the key questions they would want to ask in the plenary. It may be helpful to refer to the prompts on the handout. Allow about **5 minutes**.

Take feedback, noting the critical importance of the plenary to securing learning in this kind of lesson. Allow **a minute** for everyone to record their choice of key questions.

Briefly discuss the last two paragraphs of the **key points**.

Background

You now have a folder containing three mini-packs and there are also three pairs of transition lessons available on the web. This provides a substantial set of resources for developing the teaching of proportional reasoning in Key Stage 3.

Key points

The development of proportional thinking is important because:

- the concept of proportionality pervades all strands of mathematics;
- the ability to use proportional thinking is central to people's daily and working lives and also gives access to much higher mathematics.

The materials in the mini-packs have explored a range of strategies to make teaching and learning more effective. Many of these strategies are illustrated in the accompanying videos of lessons:

- Raj (Multiplicative relationships – in the Year 9 pack) interconnects the mathematics which underpins multiplicative processes by linking language, symbols and images in ways that change pupils' thinking about numbers.
- Ali (Enhancing proportional reasoning) is rigorous in engaging pupils in problem-solving processes and reflecting on those processes, using pupils' confidence in handling more difficult numbers to raise their level of thinking.

Activity 1 (10 minutes)

Use the **background** notes to outline the materials available from the Key Stage 3 Strategy and the **key points** to emphasise the importance of developing proportional thinking (first two bullets). Then discuss how as a department you can:

- make full use of the materials and take account of implications for other number units as well as units in other strands;
- use these resources to develop a 'big idea', helping to reduce the time needed to teach discrete objectives and simplify your scheme of work.

Activity 2 (10 minutes)

Use the second part of the **key points** to summarise the focus of the two videos on teaching proportional reasoning. Then discuss as a department how you can:

- learn from the teaching approaches in the videos, perhaps supporting one another through observation or team-teaching;
- adapt the materials in the packs to challenge your classes;
- focus your attention on in-depth development of key aspects of the scheme of work and benefit from the spin-off in other lessons.