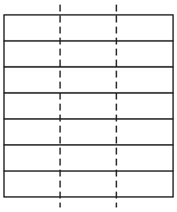
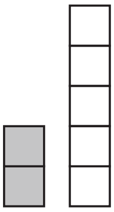
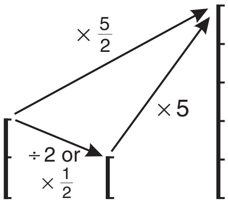
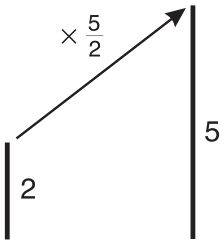
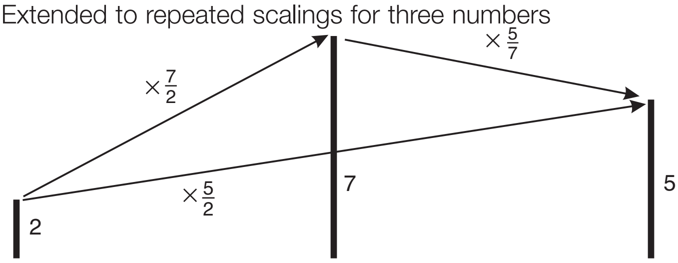
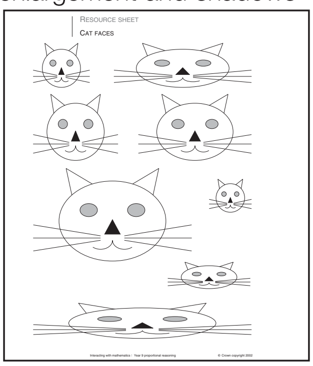
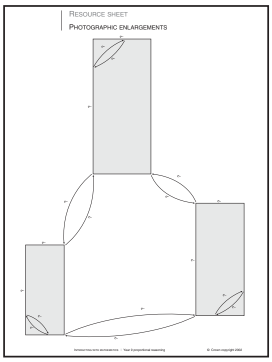
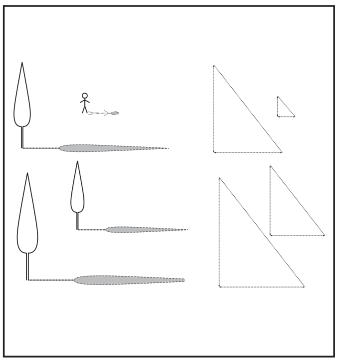
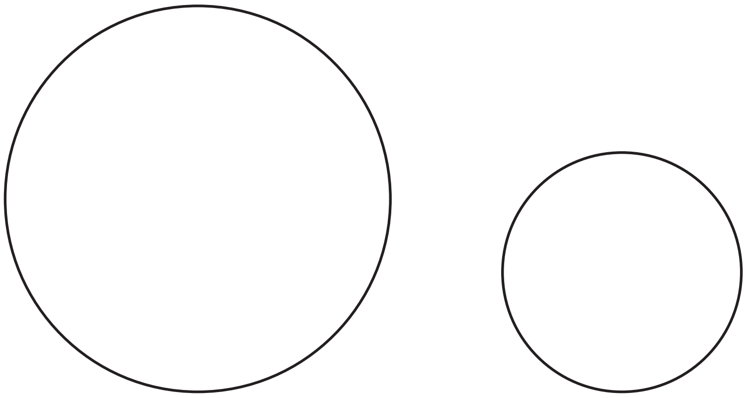
Images for fractions, scaling and proportion — Handout PR1

Year	Image used ...	To develop ...
7	Single strips stacked to represent a number; stacks sectioned using vertical lines 	Fractions as numbers, multiples of unit fractions Unit, proper and improper fractions as operators
	Linking blocks forming **strips** to illustrate ratios 	Ratio as a comparison of two quantities Ratio, fractions, decimals and percentages as equivalent ways of comparing
8	Sets of three parallel, **graduated line segments**  Scaling represented as a two-stage operation perhaps through 1, e.g. from 2 to 5 in two steps by $\div$ 2 (or $\times\frac{1}{2}$) then $\times$ 5	Establishing a single multiplier between any two numbers Middle line segment used to show interim step of multiplication or division (as used in unitary method)
9	Sketches of pairs of **line segments** used to illustrate a single multiplier from one number to another  Extended to repeated scalings for three numbers 	Identifying a single multiplier between any two numbers without recording an interim step
	Links to enlargement shown using paper folding, cat faces, photographic enlargement and shadows 	Identifying the 'within' and 'between' aspect of dimensions of similar shapes

Chain wheel problems

A and B are two chain wheels.



Year 7

For every 2 complete turns that wheel A makes, B makes 5 complete turns.

If wheel B completes 30 turns how many turns does wheel A complete?

Year 8

For every 3 complete turns that wheel A makes, B makes 5 complete turns.

If wheel B completes 84 turns how many complete turns does wheel A make?

Year 9

For every 3 complete turns that wheel A makes, B makes 7 complete turns.

If wheel B completes 83 turns how many complete turns does wheel A make?

How far through the next full turn is wheel A?

A lesson plan from phase 2 of Year 8 multiplicative relationships

Objectives

- Consolidate understanding of the relationship between ratio and proportion.
- **Identify the necessary information to solve a problem** (by recognising problems involving direct proportion).

Starter

Vocabulary
corresponding pairs
direct proportion
relationship

Resources
Data sets selected from resource sheet PR1
Calculators

Proportion or not?

On the OHP display a pair of data sets which are in direct proportion. (See the resource sheet PR1, 'Proportion or not? Data sets'.)

Q Are the two sets in direct proportion?

Q How do you know?

Explore a range of different strategies to draw out different relationships between the numbers.

Repeat using different pairs of sets, some in direct proportion and some not.

When the pairs are in direct proportion:

Q Can you name another pair of numbers we could add to the set (which share the same relationship)?

Q How could you describe the relationship between the sets? (corresponding pairs)

Q If x is added as member of the first set, which y goes with it in the second? For example:

2	7
8	28
20	70
10	?

Draw out $(2 + 8) : (7 + 28)$. Emphasise the greater flexibility of the multiplicative relationship $(20 \div 2) : (70 \div 2)$.

Main activity

Vocabulary
As starter

Resources
Collection of real data sets from resource sheet PR2
Collection of problems from resource sheet PR3

Solving problems

Describe/illustrate a proportion problem. For example:

Q A bottle of diet cola indicates that 100 ml contains 0.4 kcal of energy. How much energy would 200 ml contain?

Construct a table of values:

Cola (ml)	Energy content (kcal)
100	0.4
200	0.8

Add other 'easy' figures (e.g. 150 ml of cola and 1 litre of cola) then less obvious figures (e.g. a typical glass of 180 ml).

Q How can we work out the amount of energy?

Q Are the two sets of values in direct proportion?

Q Why/how do we know?

Relate the answers to the table of numbers and also to the situation (i.e. the uniform nature of cola); draw out the use of 'for every'. Ensure this is well established with the class.

If time permits, use this question to confirm pupils' understanding of the situation:

Q Can you calculate or estimate how much cola would provide 5 kcal?

Point out that the volume of cola and the amount of energy are two values which can vary and are connected in some way. This is typical of many mathematical problems. To solve them, it is important to know how the variables are connected.

Distribute a selection of data sets from resource sheet PR2, 'Proportion or not? Contextualised data sets' (or similar). Say that you want pupils to work in pairs to identify quickly which sets of variables are in direct proportion. They should record their reasons briefly.

Mini-plenary

After a few minutes, select two or three of the examples and ask selected pairs to share their thinking with the class. Address any misconceptions revealed.

Now distribute a selection of problems from resource sheet PR3, 'Proportion or not? Problems' (or similar). Ask pupils to classify them according to whether the variables are in direct proportion or not and to solve those they can.

Simplification

Initially, restrict situations and problems to those with more obvious relationships.

Extension

Think about a length of elastic tied to a fixed hook with a light pan suspended on the other end. Ask pupils to consider the length of the elastic as different weights are added to the pan. Will the length be proportional to the mass? Will the extension be proportional to the mass? Ask them to design an experiment to test their theories.

Plenary

Resources

Collection of situations from resource sheet PR4

Address any issues which you have identified while pupils were working on the problems.

Select two or three situations from resource sheet PR4, 'Proportion or not? Situations' (or elsewhere), and present them on an OHT.

Q What are the variables?

Q Are they in direct proportion?

Q How can you justify your answer?

Remember

- You might know that two sets of numbers are in direct proportion because you are familiar with the context and know how one variable relates to the other.
- You might observe that two sets of numbers are in proportion by looking at a table of values and noting the pattern of entries.
- Both of these points can be checked by ensuring that a constant multiplier connects every pair of values.

Main teaching activity (14 minutes)

The problem discussed in depth in the video extract is:

> *A telephone call lasting 45 seconds costs 13p. How much will a call lasting 17 minutes cost? If a call costs 28p. how long did it last?*

First, set out how you would solve the two parts of the problem.

As you watch the video extract look out for:

- the deliberate way in which Ali manages the discussion:
  - telling the class what they will be doing and what they will not be doing;
  - keeping their attention on one phase of the process until they have talked it through;
  - getting them to think about or comment on the problem-solving processes in which they are engaged;
- how the class use tabulation and scaling methods and the confidence this gives them in tackling problems with difficult numbers.

Extracts from the plenary (6 minutes)

Pupils worked in pairs on two or three other problems during this lesson. In the plenary Ali invites pupils to reflect on the process they have been going through. As you watch the extract look out for:

- the careful way in which Ali captures the experiences and thought processes of the class in the visual image of the problem-solving cycle on the board;
- the quality of pupils' responses to her questions;
- at the very end, how Ali invites the class to generalise their learning by stepping back from the lesson to consider how problem solving strategies may be useful in other work.

Note

Video sequence 3, 'Plenary' (12 minutes), shows a fuller version in which Ali takes pupils through all the stages of the problem-solving cycle. It is preceded by a short sequence of extracts from the many occasions during the lesson when Ali draws pupils' attention to the processes in which they are engaged. Although too long to include in the training session, this sequence gives a stronger flavour of the pacing and sequencing of the discussion and would make good viewing on another occasion.

Kelsey’s solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

$$9\overline{)5.0^50}\quad 0.55$$

1 CD $\simeq$ 0.55cm wide

$\times 14$
0.55
7.7cm

14 CDs side by side would measure
7.7cm

David’s solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?

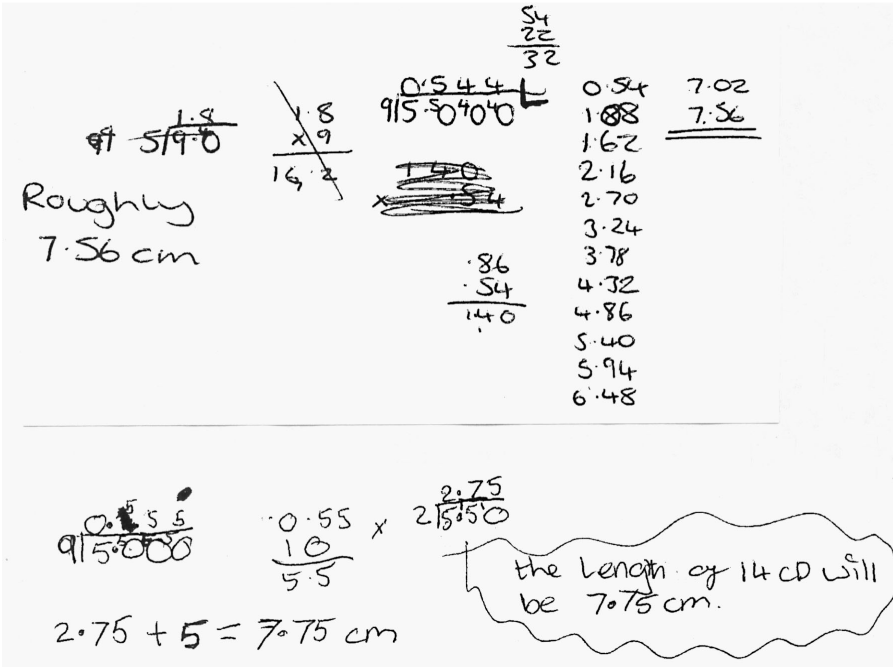
$\frac{1}{2}$ of 9 = $4\frac{1}{2}$ cd's

$\frac{1}{2}$ of 5cm = $2\frac{1}{2}$ cm

9 cds + $4\frac{1}{2}$ cd's about 14 cd's
5 cm + $2\frac{1}{2}$ cm about $7\frac{1}{2}$ cm

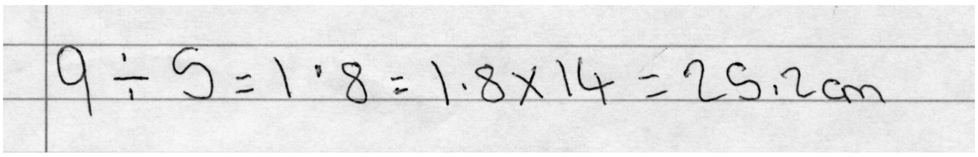
Anna's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?



Sam's solution

9 CDs put side by side on a shelf measure 5 cm. How many centimetres would 14 CDs placed side by side measure?



Selecting scripts before the lesson

- Choose a small selection of pupils' solutions, based on one or two problems – perhaps no more than four altogether for the group task.
- Ensure pupils are familiar with and reasonably successful at problems of the chosen kind, not just meeting them for the first time.

Modelling the task

Plan to model what will be a new task for the class, in a clear structured sequence, thinking aloud about what to do, so that pupils can imitate it.

- Choose one or two solutions to a problem to discuss as a class. Question pupils in order to elicit evaluative comments, refined to written statements.
- Ensure the class understands that they are commenting on the solutions and giving advice which could help the pupils improve their work, not just putting ticks, crosses and corrections.

Paired or small-group work

Organise well-focused small-group work, to give all pupils an opportunity to interpret the scripts, share thinking and refine ideas together.

- Ask pupils to begin by classifying the solutions as correct or incorrect.
- Pupils might write their comments on sticky notes.
  - Correct solutions should be checked for efficiency and a clear target for improvement noted, if appropriate.
  - Incorrect solutions should have the errors identified and a clear strategy for correction and improvement noted.

Plenary

Plan some key questions for the plenary, particularly to elicit:

- comments on the approaches used in the written solutions;
- views on what kinds of feedback or written comments would help the pupils to improve their work;
- reflections on what the class themselves can learn from the process of discussing other pupils' work.

