



Guidance

Curriculum and  
Standards

# Key Stage 3

## National Strategy

Interacting with mathematics in Key Stage 3

# Constructing and solving linear equations

Year 9 booklet

Teachers of mathematics

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# Introduction

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This booklet is to be used with the *Framework for teaching mathematics: Years 7, 8 and 9*. It provides additional guidance on developing progression in the teaching of *constructing and solving linear equations*. Although specific in this focus, it illustrates an approach that is designed to serve the broader purpose of developing the teaching of all aspects of algebra. The booklet:

- supports the training session 'Constructing and solving linear equations in Year 9';
- provides a resource for mathematics departments to use in collaborative planning for the teaching of algebra.

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## Key teaching principles

### Principle 1: Providing opportunities for pupils to express generality

Generality lies at the heart of mathematics. The teaching principle is to get pupils to *generalise* for themselves rather than just having generalisations presented to them. The advantages of this approach include the following.

- Pupils appreciate the purpose of algebra.
- Pupils are better able to understand the meaning of expressions if they have generated some for themselves.
- Knowing how expressions are built up helps to clarify the process of 'undoing', needed when solving equations.

### Principle 2: Asking pupils to 'find as many ways as you can'

This teaching principle requires that pupils are regularly asked to write algebraic expressions in different ways – to *construct* expressions or equations and to *transform* them. The benefits of this are many.

- Pupils appreciate that the same general relationship can be expressed in more than one way.
- They manipulate expressions to demonstrate that one expression is equivalent to another.
- They experience forming and transforming expressions in different ways.
- They have opportunities to discuss which transformations are the most efficient to use in a particular context, e.g. when solving an equation.

In addition, by paying careful attention to the 'using and applying' objectives set out in section 4.2 below, pupils are provided with opportunities to:

- *represent* problems in *symbolic* form;
- develop *algebraic reasoning*.

Through consistent application of these principles, pupils learn to construct and manipulate algebraic expressions and equations on the basis of their understanding of mathematical relationships, rather than being given a predetermined set of rules. This helps them to choose the methods and the sequence of operations needed to solve an equation.

## Teaching objectives

## Progression in the solution of linear equations

This table sets out objectives from the yearly teaching programmes that are addressed in the sessions.

OBJECTIVES		
Year 7	Year 8	Year 9
<p><b>Solving equations</b></p> <ul style="list-style-type: none"> <li>Construct and solve simple linear equations with integer coefficients (unknown on one side only) using an appropriate method (e.g. inverse operations).</li> </ul>	<ul style="list-style-type: none"> <li>Construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. inverse operations, transforming both sides in the same way).</li> </ul>	<ul style="list-style-type: none"> <li><b>Construct and solve linear equations with integer coefficients</b> (with and without brackets, negative signs anywhere in the equation, positive or negative solution), <b>using an appropriate method.</b></li> </ul>
<p><b>Using and applying</b></p> <ul style="list-style-type: none"> <li>Represent problems mathematically, making correct use of symbols.</li> <li>Present and interpret solutions in the context of the original problem; <b>explain and justify methods and conclusions.</b></li> <li>Suggest extensions to problems by asking 'What if...?'; begin to generalise.</li> </ul>	<ul style="list-style-type: none"> <li><b>Represent problems and interpret solutions in algebraic form.</b></li> <li><b>Use logical argument to establish the truth of a statement.</b></li> <li>Suggest extensions to problems and generalise.</li> </ul>	<ul style="list-style-type: none"> <li>Represent problems and synthesise information in algebraic form.</li> <li><b>Present a concise, reasoned argument, using symbols.</b></li> <li>Suggest extensions to problems and generalise.</li> </ul>
<p><b>Precursors</b></p> <ul style="list-style-type: none"> <li><b>Use letters or symbols to represent unknown numbers;</b> know the meanings of the words <i>term</i>, <i>expression</i> and <i>equation</i>.</li> <li><b>Understand that algebraic operations follow the same conventions and order as arithmetic operations.</b></li> <li>Simplify linear algebraic expressions by collecting like terms; begin to multiply a single term over a bracket.</li> </ul>	<ul style="list-style-type: none"> <li>Know that algebraic operations follow the same conventions and order as arithmetic operations.</li> <li><b>Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket.</b></li> <li>Add, subtract, multiply and divide integers.</li> </ul>	<ul style="list-style-type: none"> <li>Simplify or transform algebraic expressions by taking out single-term common factors.</li> </ul>

# Additive and multiplicative relationships

## Addition and subtraction

If  $A + B = C$ , then  $B + A = C$ ,  $A = C - B$  and  $B = C - A$

Substituting particular values for  $A$ ,  $B$  and  $C$  helps pupils to see the relationships within the family of equations.

The same principles apply to multiplication and division. With the conventions of algebraic notation:

If  $AB = C$ , then  $BA = C$ ,  $A = \frac{C}{B}$  ( $B \neq 0$ ) and  $B = \frac{C}{A}$  ( $A \neq 0$ )

Together with additive relationships, these constitute the basic transformations of elementary algebra.

A further development is to consider additive and multiplicative inverses. This involves a shift in perspective: instead of subtraction, the inverse of addition is seen as adding a negative number; instead of division, the inverse of multiplication is seen as multiplying by the reciprocal number. There are links to be made here with work in number, including calculating with negative numbers and using inverse multipliers.

## Extension to four terms

Write the equation  $3 + 4 = 5 + 2$  in *as many ways as you can*.

Using commutativity:

$$3 + 4 = 5 + 2$$

$$4 + 3 = 5 + 2$$

$$3 + 4 = 2 + 5, \text{ etc.}$$

Using inverse operations (putting brackets around numbers you want to think of as a single term):

$$(3 + 4) - 5 = 2$$

$$(3 + 4) - 2 = 5$$

$$3 = (5 + 2) - 4, \text{ etc.}$$

Try other examples, *working towards generality*:

$$A + B = C + D$$

## Multiplication and division

Pupils need to understand that  $A \div B = \frac{A}{B}$

## Multiplicative inverse

$$A \times B = C$$

$$B \times A = C$$

$$A = C \times \frac{1}{B}$$

$$B = C \times \frac{1}{A}$$

Reciprocal cards could be used to develop these relationships. The equivalence of  $\div 4$  and  $\times \frac{1}{4}$  for example, is an important link. If extended to non-integers it is easier to think, for example, of  $\frac{4}{3}$  as the inverse of  $\frac{3}{4}$ , rather than  $\frac{1}{\frac{3}{4}}$ .

### Extension to four factors

Write the equation  $3 \times 4 = 2 \times 6$  in *as many ways as you can*.

Using commutativity:

$$3 \times 4 = 2 \times 6 \qquad 4 \times 3 = 2 \times 6 \qquad 3 \times 4 = 6 \times 2, \text{ etc.}$$

Using inverse operations (bracket expressions to think of them as a single number):

$$(3 \times 4)/2 = 6 \qquad (3 \times 4)/6 = 2, \qquad \text{etc.}$$

Starting from  $4/2 = 6/3$ :

$$(4/2) = 6/3 \qquad (4/2) \times 3 = 6, \qquad \text{etc.}$$

Try other examples, *working towards generality*:

$$AB = CD \qquad \text{or} \qquad A/C = B/D$$

(Note the links to proportionality – see the Framework supplement of examples, page 137.)

### More complex equations

The following equations are from pages 123 and 125 of the Framework supplement of examples. Without expanding any brackets, pupils will benefit by writing them in *as many ways as they can*.

$$2(p + 5) = 24$$

$$4(n + 3) = 6(n - 1)$$

$$\frac{12}{(x + 1)} = \frac{21}{(x + 4)}$$

## The simple within the complex

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**Example 1**

- Rearrange the equation  $A = B - C$  in as many ways as you can.
- Now consider the following equation as being of the same form. Without removing brackets or solving, rearrange the equation in **as many ways as you can**:

$$7s = 45 - 3(12 - s)$$

**Example 2**

- Consider the following equation as being of the form  $A = \frac{B}{C}$ .

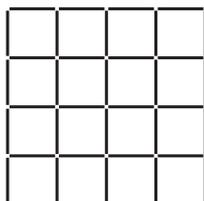
Without solving it, rearrange the equation in **as many ways as you can**.

$$\frac{2}{3} = \frac{5}{r}$$

## Constructing equations

The process of constructing expressions is an essential first stage in using algebra to model situations. The Framework gives examples of different puzzle contexts (arithmagons, pyramids, etc.) where pupils can form equations and gain a sense of the power of algebra in solving problems. Any of these contexts can be adapted to a level of difficulty suited to a particular class or year group and developed over a sequence of lessons.

### Matchstick grids



Construct a  $4 \times 4$  grid of squares from matchsticks.  
How many matchsticks are needed?  
Count systematically, *as many ways as you can*.

$$4 \times 4 + 3 \times 4 + 3 \times 4$$

'border and inside (rows and columns)'

$$4 \times 4 \times 4 - (3 \times 4 + 3 \times 4)$$

'individual squares, minus double-counted matches'

$$5 \times 4 + 5 \times 4$$

'verticals and horizontals'

$$4 + 3 \times 3 + 3(3 + 3 \times 2)$$

'square by square, row by row'

*Opportunity to generalise to a  $4 \times n$  rectangle:*

$$(2 \times 4 + 2n) + 4(n - 1) + 3n$$

'border and inside (rows and columns)'

$$4 \times 4 \times n - (4(n - 1) + 3n)$$

'individual squares, minus double-counted matches'

$$4(n + 1) + 5n$$

'verticals and horizontals'

$$4 + 3(n - 1) + 3(3 + 2(n - 1))$$

'square by square, row by row'

Use algebra to verify the equivalence of different expressions for the total number of matchsticks.

Using as a *source of equations*:

A rectangular grid of matches is 4 wide and uses 130 matches. How long is the grid?

## Solving equations

The use of inverse operations, probably the first approach to develop, involves 'undoing' a sequence of operations. It requires understanding of the order of precedence of operations and use of conventional signs and brackets.

A second approach involves transforming one or both sides and matching terms. Pupils will be used to interpreting the equals sign as meaning 'makes', so it is not an easy step to see an equation as a balance of sides. Activities such as 'Clouding the picture' help them to build this sense of balance and to read the equals sign as meaning 'is the same as'. Matching the sides of an equation reinforces this interpretation. Pupils see that equal terms appearing on both sides can be 'discarded' without upsetting the balance of the equation. The Year 7 and Year 8 course booklets support this progression.

In the spirit of '*as many ways as you can*', these approaches develop together and pupils should rearrange equations many different ways. This way they:

- discover and practise valid transformations, and
- develop judgement about which transformations will help solve a problem.

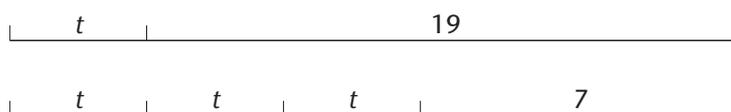
The ultimate aim is not to teach a set of rules, but to equip pupils with a repertoire of valid transformations to call on. As experience grows, they are able to examine an equation, decide the best approach to take and reduce the number of recorded steps by completing more of them mentally.

### Unknown on both sides of the equation

#### Preparatory work: mental approaches, illustrated on the number line

This can be developed as an introduction, before returning to written methods.

Use the number line in simple cases. For example, take  $3t + 7 = t + 19$ ; through matching segments obtain  $2t + 7 = 19$ , then  $2t = 19 - 7$ .



#### Developing symbolic argument: inverse operations and matching terms

Develop methods by first rearranging the equation in many different ways. Then identify transformations which help with solving the equation.

For example, rearrange  $5x - 3 = 7 + 2x$  *as many ways as you can*. Which arrangements help to solve the equation?

- Keeping the four *terms* intact and bracketing pairs you want to think of as a single term:

$$(5x - 3) - 7 = 2x$$

$$5x = (7 + 2x) + 3$$

$$(5x - 3) - 2x = 7, \text{ etc.}$$

- Keeping the *sides* intact:

$$5x - 3 = 7 + 2x$$

$$5x - 2 - 1 = 3 + 4 + 2x$$

$$3x + 2x - 3 = 7 + 2x$$

$$4x + x - 3 = 7 + 2x, \text{ etc.}$$

Identify rearrangements which are a useful step towards the solution. For example:

- Inverse operations:

$$5x - 3 = 7 + 2x$$

$$(5x - 3) - 2x = 7$$

$$3x - 3 = 7, \text{ etc.}$$

- Matching terms on each side:

$$5x - 3 = 7 + 2x$$

$$3x + 2x - 3 = 7 + 2x$$

$$3x + \cancel{2x} - 3 = 7 + \cancel{2x} \quad (\text{matching sides and noting that } 2x \text{ is added to both})$$

$$3x - 3 = 7, \text{ etc.}$$

Find *as many ways as you can*. Discuss what makes some ways more efficient than others.

### **Towards fluency: automating processes and reducing recorded steps**

Fluency is achieved over an extended period of time, not in a few lessons. It involves:

- seeing an order of steps that will lead efficiently to the correct answer;
- completing more steps mentally, with less recording.

$$5x - 3 = 7 + 2x$$

$$5x - 3 - 2x = 7$$

$$3x - 3 = 7$$

$$3x = 10$$

$$x = 3\frac{1}{3}$$

When asked, pupils should be able to explain each step.

## Dealing with errors by recovering understanding

Errors occur when pupils misapply rules they have learned. For example, when solving the equation:

$$22 - 5t = 7 - 2t$$

they might write:

$$7t = 29$$

'Add like terms'

$$22 - 7t = 7$$

'Do the same thing to both sides'

$$22 - 5t - 7 = 2t$$

'Change the side change the sign'

In preference to correcting the misapplied rule, offer fresh insight into the principles which underpin transforming equations.

### Inverse operations

'Think of  $7 - 2t$  as a single term, then  $22 - 5t = (7 - 2t)$  has the same form as  $C - B = A$ , where  $C = 22$ ,  $B = 5t$  and  $A = (7 - 2t)$ . If  $C - B = A$ , we know that  $C = A + B$ . So what is another way of writing the equation?'

$$22 = (7 - 2t) + 5t$$

'Does this help?'

'Try another approach, bracketing the LHS as a single term:  $(22 - 5t) = 7 - 2t \dots$ '

### Partitioning terms and matching the sides

'Think of subtracting  $5t$  as subtracting  $3t$  and subtracting  $2t$ .'

$$22 - 3t - 2t = 7 - 2t$$

'Does this help? Why? / Why not?'

'Try another approach, thinking of subtracting  $2t$  as subtracting  $5t$  and adding  $3t \dots$ '

'Or is  $15 + 7 - 3t = 7$  more useful? Why? / Why not?'

Objectives



- To consider Year 9 teaching objectives in constructing and solving linear equations
- To outline an effective progression and teaching approaches to help pupils construct and solve linear equations
- To develop lessons that incorporate activities from the session

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The simple within the complex: example 1



- Rearrange the equation  $A = B - C$  in as many ways as you can.
- Now consider the following equation as being of the same form. Without removing brackets or solving, rearrange the equation in **as many ways as you can**:

$$7s = 45 - 3(12 - s)$$

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The simple within the complex: example 2



Consider the following equation as being of the form

$$A = \frac{B}{C}$$

Without solving it, rearrange the equation in **as many ways as you can**:

$$\frac{2}{3} = \frac{5}{r}$$

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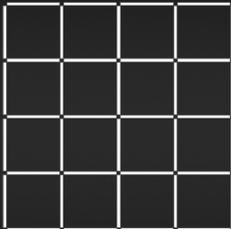


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Matchstick grids



Here is a  $4 \times 4$  square of matchsticks.

**As many ways as you can**

Find as many ways as you can of systematically counting the number of matches. Record numerical expressions of each way.

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Matchstick grids



**Opportunity to generalise**

Imagine constructing a  $4 \times n$  rectangle of matches.

Write expressions for the total number of matches, as many ways as you can.

**Solve an equation**

If a  $4 \times n$  rectangle needs 130 matches, what is the value of  $n$ ?

*Can you solve this more than one way?*

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Progression of pupils in Year 9



Consider two cases of what you may find when teaching algebra in Year 9:

**Flexible thinking:** Pupils have been taught many ways of transforming algebraic expressions, and will try different approaches to solving equations.

**Rigid thinking:** Pupils have been restricted to an approach where rules are demonstrated and illustrated with examples. Typically, they apply the rules to routine examples, but are less confident exploring and developing understanding of algebraic transformations.

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Some errors



Solve the equation:  $22 - 5t = 7 - 2t$

Examples of pupils' errors

$7t = 29$  'Add like terms'

$22 - 7t = 7$  'Do the same thing to both sides'

$22 - 5t - 7 = 2t$  'Change the side, change the sign'

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Responding to errors



In pairs:

- Examine the ways of transforming the equation suggested for discussion with pupils (**section 4.7** of the Year 9 booklet).
- Briefly, consider how this approach might apply to another equation, such as  

$$\frac{x}{2} + 3 = 5 - 4x$$
- Discuss the usefulness of developing a more exploratory approach with a Year 9 class.

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Examining a Year 9 algebra lesson plan



Working in pairs, consider a class for whom the lesson might be suitable (not necessarily Year 9):

- How could you adapt the lesson to be suitable for your class?
- How might you follow it up in a subsequent lesson or lessons?

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# 4.10–4.11

## Points for reflection



Two teaching principles discussed have been:

- providing opportunities for pupils to express generality;
- asking pupils to 'find as many ways as you can'.

Reflect on these questions.

- How are these principles exemplified in the lesson plan that you considered?
- How will you include these principles in your teaching of algebra?

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## Using the course booklets



Used with the Framework supplement of examples, the course booklets:

- assist teaching of constructing and solving equations, developing progression through KS3
- illustrate the application of teaching principles which are equally appropriate to other objectives

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## LESSON

# 9A.1

## Matching expressions

### OBJECTIVES

- Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket (Year 8); take out single-term common factors (Year 9).
- Present a concise, reasoned argument, using symbols.
- Suggest extensions to problems and generalise.

### STARTER

#### Vocabulary

equivalent expression  
linear expression  
common factor

#### Resources

Mini-whiteboards

Show some linear expressions such as  $8 - 4x$ .

Ask pupils to write an equivalent expression using a single bracket on their whiteboards and show the result. Discuss responses of the kind  $2(4 - 2x)$ , explaining that these are equivalent to  $8 - 4x$  but that the expression inside the bracket still has common factors and could be made simpler.

**Q Can you write an expression which could not be rewritten using a bracket?**

Take suggestions, with reasons.

### MAIN ACTIVITY

#### Vocabulary

equivalent expression  
linear expression  
common factor

#### Resources

Resource 9A.1a  
as a handout  
Large sheets of paper  
or acetate

#### Knowing and not knowing

Model the expansion and simplification of expressions of the type  $5 - 3(1 + 4x)$ .

Adjust the expression by changing the positive and negative signs.

**Q How many different expressions are possible just by changing the signs in this way? Which ones are equal?**

Follow a similar pattern of discussion for expressions of the type  $3(2 + 4x) - 7x$ .

Leave expansions and simplifications on display. Use pupils' annotations to make multiplication and collection of terms clear.

**Q What must you watch out for?**

Show similar expressions with hidden signs. Ask pupils to suggest, with reasons, what the sign must be if the expression has to have the same value as  $2 - 12x$ .

$$6 \text{ [ ] } 4 (1 \text{ [ ] } 3x)$$

After a full discussion organise pupils into pairs.

#### Task 1

Use **resource 9A.1a** as a handout. Point out the 'think' and 'talk' notes, reminding pupils that these paired discussions will prepare them to make a contribution to the mini-plenary.

#### Mini-plenary

Show a few examples of pupils' solutions choosing particularly those pairs developing strategies which would be interesting for the rest of the class to hear.

- Q What were your strategies for spotting the missing signs?
- Q Were you surprised that so many expressions were equivalent?
- Q Do you think there are more?

Show  $6 - 10x$ .

- Q How could we find an expression similar to those from resource 9A.1a which is equivalent to this expression?

Take suggestions and encourage pupils to come out to the front to record suggestions and describe their thinking.

Show  $6 - 10x = 6 - 3x - 7x = 3(2 - x) - 7x$ .

- Q Can you see that the  $-10x$  term is partitioned into  $-3x$  and  $-7x$ ? How else could we partition  $-10x$ ?
- Q Which of the following partitions help us to make a bracket?

$$6 - x - 9x \quad 6 - 6x - 4x \quad 6 - 5x - 5x \quad 6 - 2x - 8x$$

Discuss one or two of these and then give task 2.

### Task 2

Ask pupils to *find as many ways as they can* to write expressions with one bracket which are equivalent to  $6 - 10x$ .

Move pupils into pairs, working as checking partners – expanding and checking one another’s suggestions. Shortly before the end of this part of the lesson take round a big sheet of paper (or an acetate) and gather expressions from different pairs of pupils. Include a couple of wrong suggestions, perhaps invented by you.

### PLENARY

**Vocabulary**  
coefficient

**Resources**  
Prepared input from pupils (poster-sized working or acetate sheets)

Invite pupils to check some of the expressions on the large sheet, talking aloud as they do so. Prompt discussion about particular terms, using precise vocabulary.

Write on the board:

$$6 - 10x \quad 4(2 - 2x) - 2x$$

- Q How can I quickly see that these expressions cannot match?

Show the expression

$$10x \quad \left[ \right] \quad 2 \quad (3 \quad \left[ \right] \quad 4x)$$

- Q When will this generate a coefficient of  $x$  which is greater than 10?
- Q Could this expression generate a number term of 9?

### KEY IDEAS FOR PUPILS

- Pay attention to the effect of the plus and minus signs in expressions involving brackets.
- Check one or two of the terms after you have expanded and simplified. Ask yourself: ‘Is it what I expect?’

# Matching expressions to $6 - 8x$

Each of the expressions below has the same value as  $6 - 8x$ .

A plus or minus sign is hidden by each sticky note.

What is the hidden sign?

$$2 \text{ [Sticky Note]} 4 (1 \text{ [Sticky Note]} 2x)$$

$$10 \text{ [Sticky Note]} 4 (1 \text{ [Sticky Note]} 2x)$$

$$4 \text{ [Sticky Note]} 2 (1 \text{ [Sticky Note]} 4x)$$

$$8 \text{ [Sticky Note]} 2 (1 \text{ [Sticky Note]} 4x)$$

$$6 (1 \text{ [Sticky Note]} x) \text{ [Sticky Note]} 2x$$

$$6 (1 \text{ [Sticky Note]} x) \text{ [Sticky Note]} 14x$$

$$2 (3 \text{ [Sticky Note]} 2x) \text{ [Sticky Note]} 4x$$

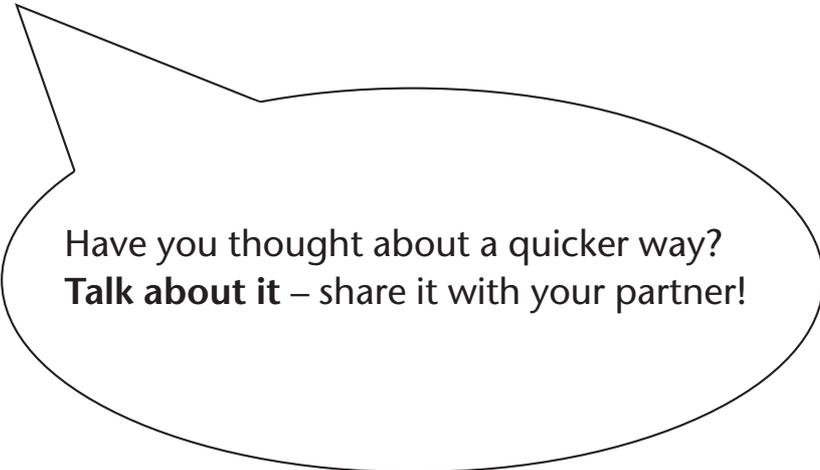
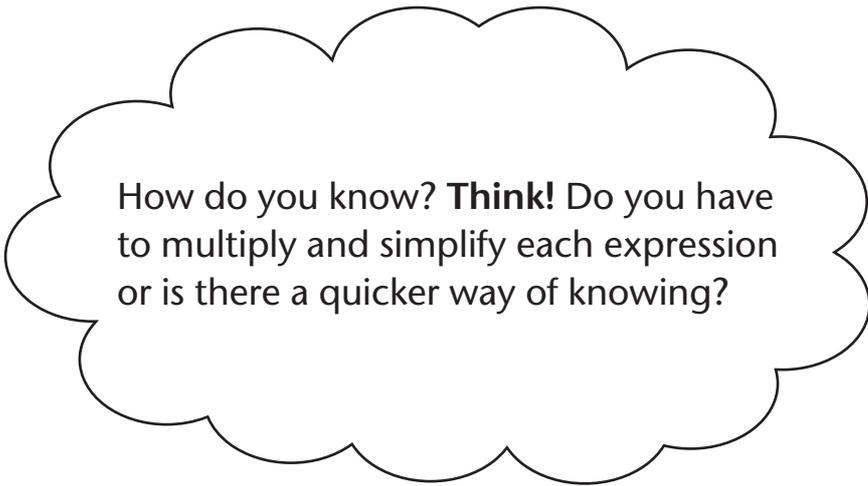
$$2 (3 \text{ [Sticky Note]} 2x) \text{ [Sticky Note]} 12x$$

$$3 (2 \text{ [Sticky Note]} x) \text{ [Sticky Note]} 5x$$

$$3 (2 \text{ [Sticky Note]} x) \text{ [Sticky Note]} 11x$$

$$2 (3 \text{ [Sticky Note]} x) \text{ [Sticky Note]} 6x$$

$$2 (3 \text{ [Sticky Note]} x) \text{ [Sticky Note]} 10x$$



## Solving pairs of linear equations

## OBJECTIVES

- Recognise that equations of the form  $y = mx + c$  correspond to straight-line graphs (Year 8).
- **Construct and solve linear equations with integer coefficients** (with and without brackets, negative signs anywhere in the equation, positive or negative solution), **using an appropriate method.**
- *Solve a pair of simultaneous equations.*

## STARTER

## Vocabulary

linear equation  
linear relationship  
linear function

## Resources

Prepared function cards (e.g. resource 9A.2a)  
Graph plotter or graphical calculators

Write some equations on large cards and pin them on the board (see **resource 9A.2a** for suggestions). Pupils respond to the following question with thumbs up for 'yes', down for 'no'. Position the cards under headings: 'linear', 'non-linear', 'not sure'.

**Q Is this relationship linear?**

Ask for explanations. Seek comments which express the idea that a linear relationship means that the graph will be a straight line, and that it is a linear function. Also that if  $x$  and  $y$  have a linear relationship, then  $y$  increases in equal steps for a constant-step change in  $x$  values.

**Q What do you notice about all the relationships we have said are linear?**

Use a whole-class graph plotter to display the graphs of one or two equations, in particular any in the 'not sure' category.

**Q We were 'not sure' about this equation. How should it be classified now?**

## MAIN ACTIVITY

## Vocabulary

linear equation  
simultaneous equations

## Resources

Resource 9A.2b (equation tree), with different starting values and at various stages of completion

**What's my equation?**

Construct a diagram similar to that on **resource 9A.2b**. Start with the middle box and ask:

**Q What equation could link  $x$  and  $y$  if they have the values shown in the box?**

Start one of the branches using a pupil's suggestion.

**Q How could we create an equivalent equation? What are we doing to one or both sides of the equation?**

Take some suggestions and write them along the same branch. If necessary, give an example to get the class started.

**Q Are there any other values of  $x$  and  $y$  which would satisfy all the equations along this branch?****Q Can you give another, different equation which the central values of  $x$  and  $y$  could satisfy?**

Start another branch holding a similar discussion. Note that there are many other values which could satisfy the equations on the second branch but only the central values satisfy the equations along *both* branches.

Tell pupils, working in pairs, to ask each other the same questions, starting new branches and completing different versions of resource 9A.2b. It might help to display the questions for reference. Start points and levels of completion of the branches can offer support, if needed, for some pupils.

## PLENARY

### Resources

Prepared equation tree  
(slide or poster)

Show an equation tree with all but one branch hidden and ask what the central values might be. Make a list.

**Q What could the central values of  $x$  and  $y$  be if this is one of the branches?**

Reveal another branch and ask the same question. Pupils may generate a whole new list or simply suggest that a value is chosen from the first list.

Reveal more of the tree and ask for confirmation that the central values satisfy all branches.

**Q Are there any other values of  $x$  and  $y$  which would satisfy all the equations along this branch?**

**Q How many branches do you need to see before you can be sure of the central values?**

A follow-up lesson may approach simultaneous equations from a graphical standpoint, i.e. the intersection of two straight-line graphs.

### KEY IDEAS FOR PUPILS

- There are many different pairs of  $x$  and  $y$  values which satisfy any one linear equation.
- Once we have two different linear equations there is at most one pair of values which will satisfy both equations.

## Which equations are linear?

$$y = 5 - \frac{x}{4}$$

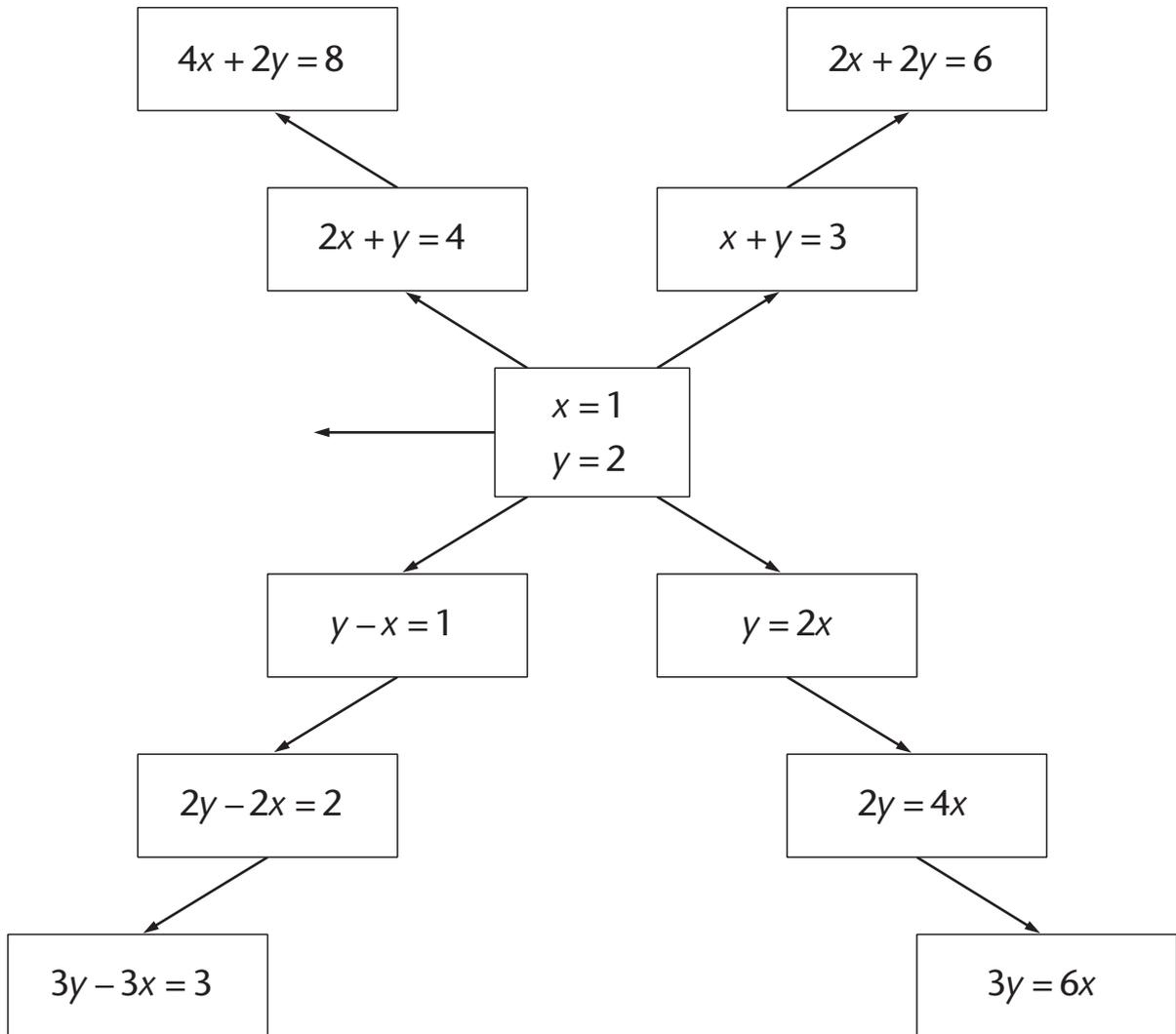
$$y + 2x - 7 = 0.3$$

$$y = x^2$$

$$y = \frac{3}{x}$$

$$y + x = x + 2$$

$$y = -0.6x - 6.4$$



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