Guidance

Curriculum and Standards

Key Stage 3
National Strategy

Teachers of mathematics

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Interacting with mathematics in Key Stage 3

Constructing and solving linear equations

Year 8 booklet

department for
education and skills
creating opportunity, releasing potential, achieving excellence

Introduction

This booklet is to be used with the *Framework for teaching mathematics: Years 7, 8 and 9.* It provides additional guidance on developing progression in the teaching of *constructing and solving linear equations*. Although specific in this focus, it illustrates an approach that is designed to serve the broader purpose of developing the teaching of all aspects of algebra. The booklet:

- supports the training session 'Constructing and solving linear equations in Year 8';
- provides a resource for mathematics departments to use in collaborative planning for the teaching of algebra.

Contents

SECTION

3.1 Key teaching principles

Principle 1: Providing opportunities for pupils to express generality

Generality lies at the heart of mathematics. The teaching principle is to get pupils to *generalise* for themselves rather than just having generalisations presented to them. The advantages of this approach include the following.

- Pupils appreciate the purpose of algebra.
- Pupils are better able to understand the meaning of expressions if they have generated some for themselves.
- Knowing how expressions are built up helps to clarify the process of 'undoing', needed when solving equations.

Principle 2: Asking pupils to 'find as many ways as you can'

This teaching principle requires that pupils are regularly asked to write algebraic expressions in different ways – to *construct* expressions or equations and to *transform* them. The benefits of this are many.

- Pupils appreciate that the same general relationship can be expressed in more than one way.
- They manipulate expressions to demonstrate that one expression is equivalent to another.
- They experience forming and transforming expressions in different ways.
- They have opportunities to discuss which transformations are the most efficient to use in a particular context, e.g. when solving an equation.

In addition, by paying careful attention to the 'using and applying' objectives set out in section 3.2 below, pupils are provided with opportunities to:

- *represent* problems in *symbolic* form;
- develop *algebraic reasoning*.

Through consistent application of these principles, pupils learn to construct and manipulate algebraic expressions and equations on the basis of their understanding of mathematical relationships, rather than being given a predetermined set of rules. This helps them to choose the methods and the sequence of operations needed to solve an equation.

SECTION

3.2 Teaching objectives

Progression in the solution of linear equations

The following table sets out objectives from the yearly teaching programmes that are addressed in the training session.

OBJECTIVES

Year 7	Year 8	Year 9
Solving equations • Construct and solve simple linear equations with integer coefficients (unknown on one side only) using an appropriate method (e.g. inverse operations).	• Construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. inverse operations, transforming both sides in the same way).	• **Construct and solve linear equations with integer coefficients** (with and without brackets, negative signs anywhere in the equation, positive or negative solution), **using an appropriate method.**
Using and applying • Represent problems mathematically, making correct use of symbols. • Present and interpret solutions in the context of the original problem; **explain and justify methods and conclusions.** • Suggest extensions to problems by asking 'What if…?'; begin to generalise.	• **Represent problems and interpret solutions in algebraic form.** • **Use logical argument to establish the truth of a statement.** • Suggest extensions to problems and generalise.	• Represent problems and synthesise information in algebraic form. • **Present a concise, reasoned argument, using symbols.** • Suggest extensions to problems and generalise.
Precursors • **Use letters or symbols to represent unknown numbers**; know the meanings of the words *term*, *expression* and *equation.* • **Understand that algebraic operations follow the same conventions and order as arithmetic operations.** • Simplify linear algebraic expressions by collecting like terms; begin to multiply a single term over a bracket.	• Know that algebraic operations follow the same conventions and order as arithmetic operations. • **Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket.** • Add, subtract, multiply and divide integers.	• Simplify or transform algebraic expressions by taking out single-term common factors.

SECTION

3.3 Expressions and terms

$-3ahd$ 5.8 $-3a + hd$ $2(2a - c) + 18$

$2(2a - c)$ $2(2a - c + 9)$ $5 + 0.8$ $4a - 2c$

a^2 $a^2 + b^2$ $6a + 2a$

SECTION

3.4 The grid method and brackets

Multiplying a single term over a bracket

Remind pupils of how 7×24 is calculated, introducing bracket notation (Framework supplement of examples, pages 116–117):

$$\begin{aligned}7 \times 24 &= 7(20 + 4)\\ &= 7 \times 20 + 7 \times 4\end{aligned}$$

	20	4
7	7×20	7×4

Opportunity to generalise

Draw attention to the fact that unknown numbers (represented by letters) prevent the calculation from being evaluated numerically. The result is the expression in an equivalent form, reading the equals sign as meaning ‘is the same as’.

$8a = (3 + 5)a = 3a + 5a$

	3	5
a	$3a$	$5a$

$7(x + 8) = 7x + 56$

	x	8
7	$7x$	7×8

$a(b + c) = ab + ac$

	b	c
a	ab	ac

Factorising

$6a - 8 = \square(3a + \square)$ consider

	$3a$	?
?	$6a$	-8

$6n + 24 = \square(\square + \square)$ consider

	?	?
?	$6n$	24

Extension: double brackets (Year 9 Framework)

Example 1

	v	w
d	$v \times d$	$w \times d$
c	$v \times c$	$w \times c$

Example 2

	x	3
x	$x \times x$	$3 \times x$
3	$x \times 3$	3×3

Example 3

	$2v$	$3w$
d	$2v \times d$	$3w \times d$
c	$2v \times c$	$3w \times c$

'As many ways as you can'

	v	v	w	w	w
d	vd	vd	wd	wd	wd
c	vc	vc	wc	wc	wc

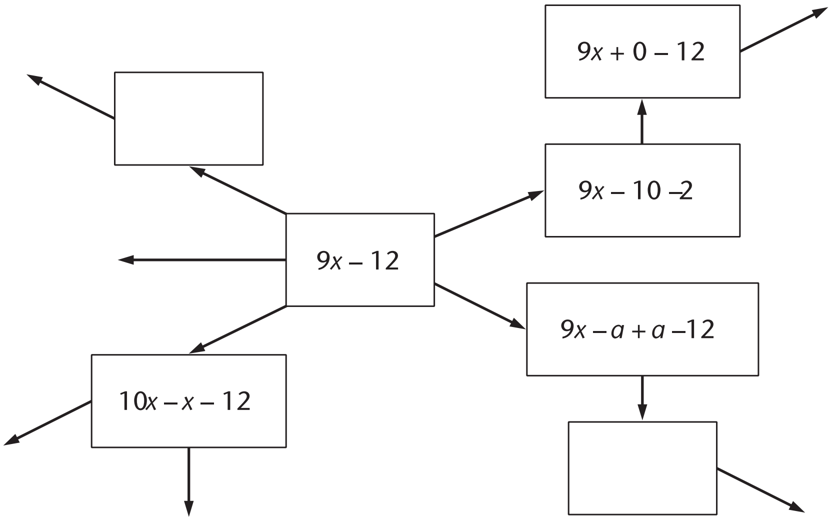
SECTION

3.5 Clouding the picture

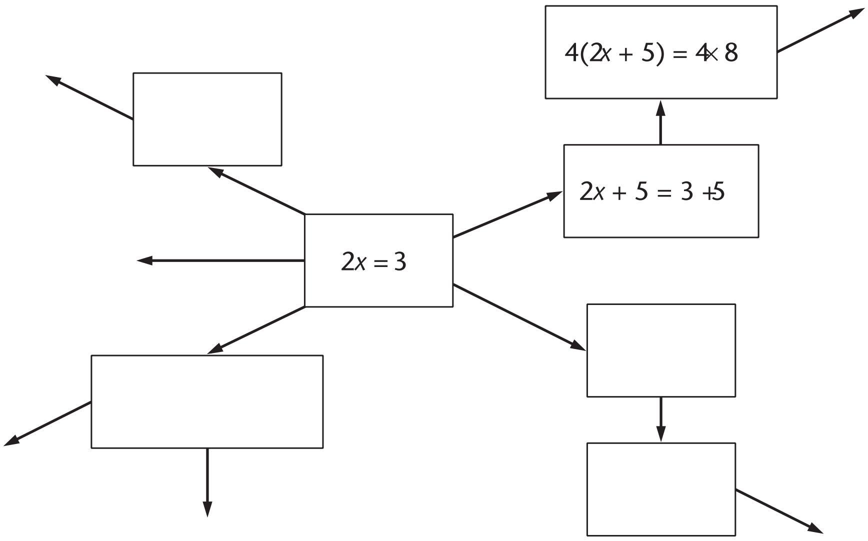
Complicate the central expression or equation in *as many ways as you can*.

- Start by giving another couple of examples along each branch.
- Stop after a few examples and try to explain what is happening along the branch (*generalise the process*).
- Can you start a new branch that does something different to complicate the expression or equation?

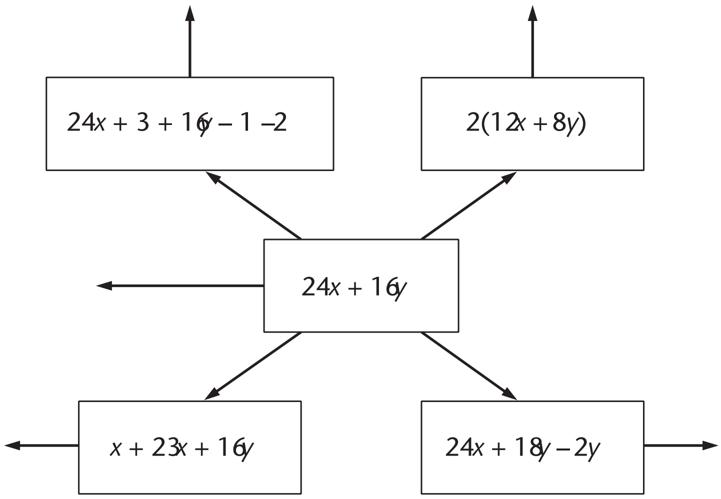
Expression



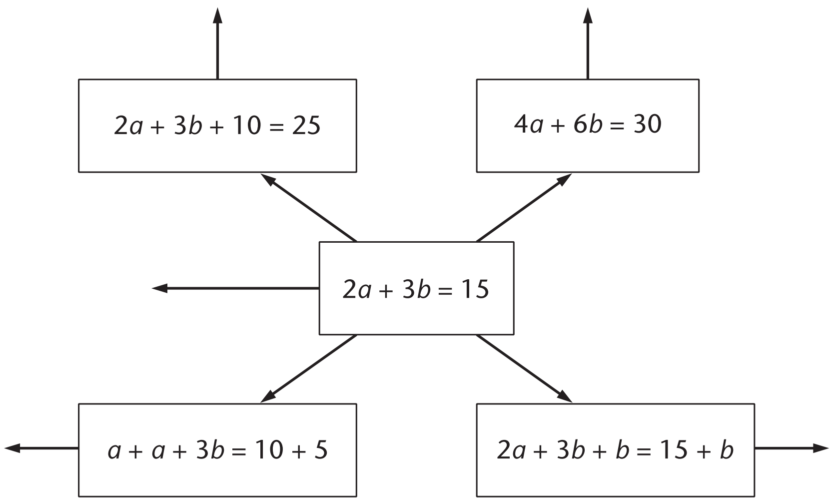
Equation



Expression

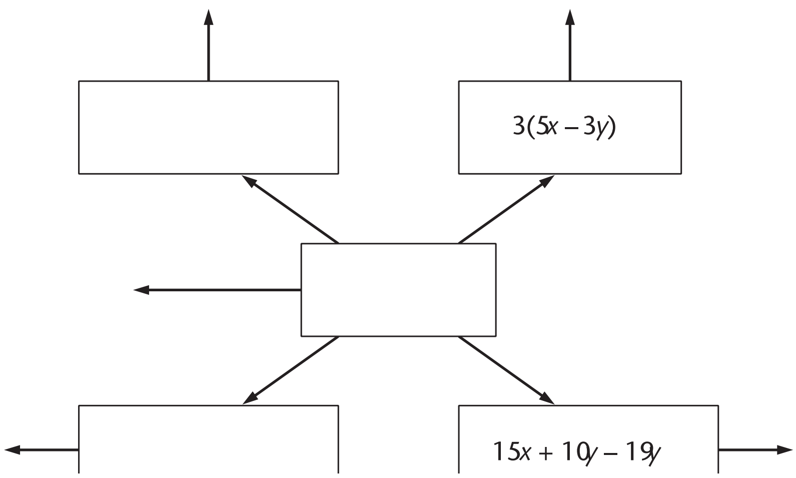


Equation



Clarifying the picture

In the central box write the expression which is the same as the two expressions shown but is written in the simplest possible way. Add some more equivalent expressions to each of the branches.



SECTION

3.6

Constructing linear equations: ‘Pyramids’

Find the number n that will give the same value for the upper cells in both pyramids.

n	–2	15

n	3	$2n$

Rule: Add adjacent expressions to give the result in the cell above.

When using ‘Pyramids’ to construct and solve equations:

- adapt the level of difficulty to suit your class;
- develop the activity over a sequence of lessons.

Bigger numbers

Introduce bigger numbers to encourage pupils to move beyond trial and improvement.

28	n	41
$28 + n$	$n + 41$	
	133	

$(28 + n) + (n + 41) = 133$

Opportunity to generalise

a	n	b
?	?	
	c	

$n = \tfrac{1}{2}(c - a - b)$

More layers

12	*n*	21	8	13
?	?	?	?	
?	?	?		
?	?			
251				

Double pyramid

23	*n*	13
?	?	
?		
?	?	
2	4*n*	4

$(23 + n) + (n + 13) = (2 + 4n) + (4n + 4)$

Change the rules

For example:

- Add twice the number above left to the number above right.
- Subtract the number above right from the number above left.

Example of subtraction rule

17	*n*	13
$17 - n$	$n - 13$	
18		

$(17 - n) - (n - 13) = 18$

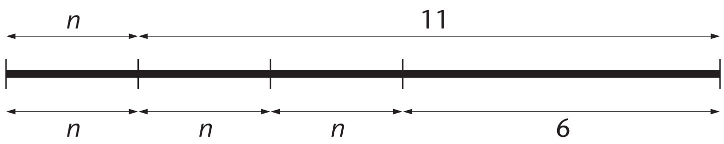
SECTION

3.7 Solving equations: number line

Using a number line to solve equations

$n + 11 = 3n + 6$

$n + 11$



$3n + 6$

Use a number line to solve:

$x + 13 = 2x + 7$

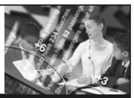
$4x - 3 = 2x + 18$

$3x + 2 = x - 4$

Solving equations: matching method

SLIDES 3.1–3.3

Objectives



- To consider Year 8 teaching objectives in constructing and solving linear equations
- To outline an effective progression and teaching approaches to help pupils construct and solve linear equations
- To develop lessons that incorporate activities from the session

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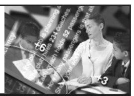
Expressions and terms



$-3ahd$ 5.8 $-3a + hd$

$2(2a - c) + 18$ $2(2a - c)$

$2(2a - c + 9)$ $5 + 0.8$ $4a - 2c$

a^2 $a^2 + b^2$ $6a + 2a$

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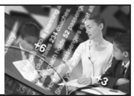
Factors



$6a - 8 = \square(3a + \square)$ consider

	$3a$	?
?	$6a$	–8

$6n + 24 = \square(\square + \square)$ consider

	?	?
?	$6n$	24

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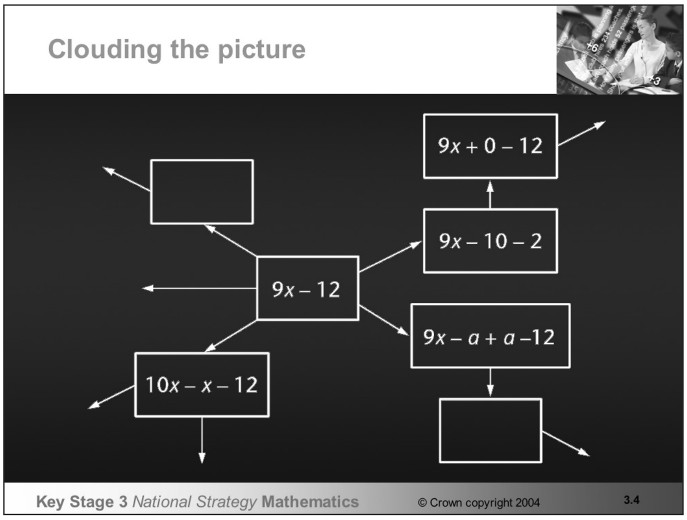
SLIDES 3.4–3.6


Clouding the picture
9x + 0 – 12
9x – 10 – 2
9x – 12
9x – a + a –12
10x – x – 12
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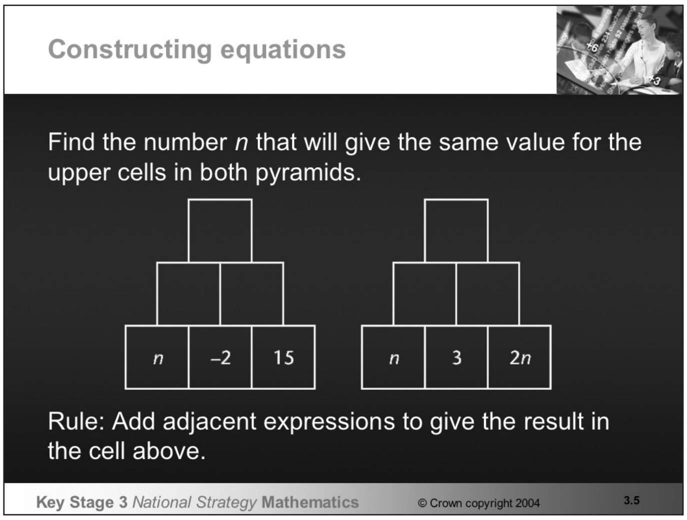
3.4


Constructing equations
Find the number n that will give the same value for the upper cells in both pyramids.
n
–2
15
n
3
2n
Rule: Add adjacent expressions to give the result in the cell above.
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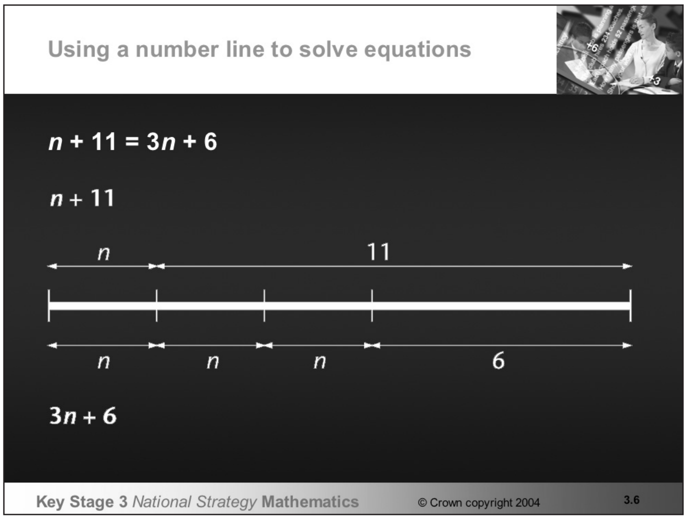
3.5


Using a number line to solve equations
n + 11 = 3n + 6
n + 11
n
11
n
n
n
6
3n + 6
Key Stage 3 National Strategy Mathematics
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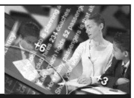
3.6

Using a number line to solve equations

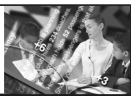


$x + 13 = 2x + 7$

$4x - 3 = 2x + 18$

$3x + 2 = x - 4$

Using the matching method



$x + 13 = 2x + 7$

$3x + 2 = x - 4$

$16 - 3x = x - 20$

LESSON

8A.1

Solving linear equations using a number line

OBJECTIVES

- Construct and solve linear equations with integer coefficients (unknown on either or both sides, without and with brackets) using appropriate methods (e.g. transforming both sides in the same way).
- **Represent problems and interpret solutions in algebraic form.**
- Suggest extensions to problems and generalise.

STARTER

Vocabulary
equation
unknown
number line
interpretation
representation
matching

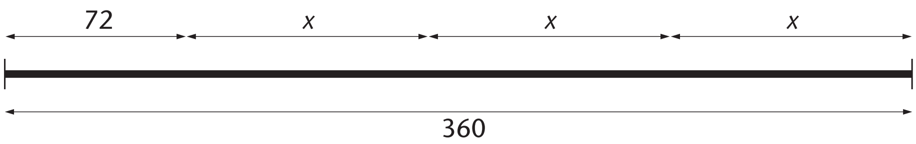
Write on the board the equation $360 = 72 + 3x$.

Q Can anyone describe a situation that could lead to this equation?

Encourage a variety of explanations including the reference to the angles of a quadrilateral or sectors on a pie chart.

Q Who can solve this equation?

Ask for volunteers and encourage a variety of responses. Some pupils will be able to use 'matching' or 'balancing' methods. Others may still need the support of a number line, for example:



By matching equal lengths show that:

$360 = 72 + x + x + x$

Q So what must $x + x + x$ be equal to?

$288 = x + x + x$

Q And how do I now calculate the value of x?

$96 = x$

Check that pupils understand what x represents.

Q Does the solution make sense in the context of the question?

Q How many different solutions are there?

Establish there is only one solution.

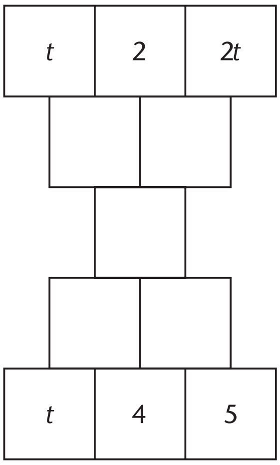
MAIN ACTIVITY

Vocabulary
solve
equation
unknown
equivalent equation
matching
like terms

Resources
OHT 8A.1a
Resource 8A.1b as a handout

Display and explain the question on **OHT 8A.1a**.

Find the number, t, that will give the same value for the central cell using either the upper or lower pyramid.



Rule: Add adjacent expressions to give the result in the cell above (or below for the upper pyramid).

If pupils have had experience of pyramids before, ask them to work in pairs to arrive at the equation and to share their result with another pair. If they have not, model the process, with inputs from pupils, to arrive at the equation:

$3t + 4 = t + 13$

Q What do you notice about the equation?

Take feedback, and point out that the equation has the unknown on both sides. (Note: this question is taken from the 2003 National Curriculum test.)

Q Does anyone know the value of *t*?

Take any answers with explanations, pointing out that these equations are generally too complicated to do in our heads.

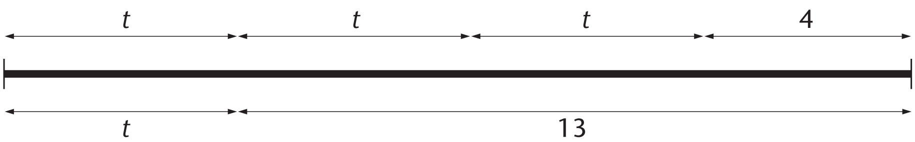
Q What could we do to help find the value of *t*?

Consider any responses from pupils and if necessary follow up with:

Q Could we use the number line as before?

Pupils could be given the opportunity to work in pairs on OHTs and then share their responses with the class. Alternatively, volunteers could work through their method at the front of the class or you could model a solution.

Point out that by matching like terms we can, in steps, arrive at simpler equivalent equations and finally at one we can solve. Say that you expect the steps to be written down (this is necessary to explain your thinking and for more difficult equations in the future). In this case:



$3t + 4 = t + 13$
$t + t + t + 4 = t + 13$
$t + t + 4 = 13$
$t + t + 4 = 4 + 9$
$t + t = 9,$ or $2t = 9$
$t = 4.5$

Note: Not all pupils will need all these steps, but the steps lay the foundation for the matching method used without a number line.

The solution should be checked in the pyramid.

Ask pupils to answer similar questions using a number line if necessary – for example, those shown below (reproduced on **resource 8A.1b**). Emphasise that intermediate steps must be shown.

1. $2x + 11 = 17$
2. $m + 9 = 2m + 4$
3. $5g + 6 = 11 + 3g$
4. $4x + 5 = x + 2.4 + 2x$
5. $2r + 15 = 3(r + 1)$

Adjust the difficulty of the questions for the different ability groups but all coefficients and number terms should be positive with no minus operations.

PLENARY

Resources
OHT 8A.1a
Resource 8A.1b as handout

Q Was the number line useful in solving these equations?

Take feedback with explanations.

Ask pupils to choose one of their solutions, cover up the number line and see if they understand what they have done by just looking at the working. Repeat this for the pyramid question you modelled earlier (OHT 8A.1a). Make the point that clear recording is very important if we want to be able to follow and share our thinking.

Ask for two volunteers to share their solutions with the rest of the class, giving pupils the opportunity to offer different steps. Get pupils to mark their neighbour's work.

For homework, ask pupils to look at the second half of **resource 8A.1b**. Ask them to use a number line to solve question 1 and then to spend 10 minutes on questions 2 and 3.

1. $4x + 7 = 2x + 13$
2. $2x + 3 = 2x + 7$
3. $2x - 1 = x + 9$

Note: In the next lesson you might discuss the solution of the equations set for homework and the difficulties that arise with questions 2 and 3 when using a number line. Pupils need to move on to 'matching' (lesson 8A.2) or 'balancing' methods. The aim is that pupils become proficient in manipulating algebraic expressions and equations.

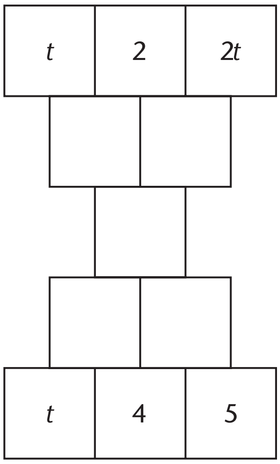
KEY IDEAS FOR PUPILS

- The number line can help us solve equations.
- Clear recording is necessary if we want to check and share our thinking.

OHT

8A.1a

Find the number, *t*, that will give the same value for the central cell using either the upper or lower pyramid.



Rule: Add adjacent expressions to give the result in the cell above (or below for the upper pyramid).

RESOURCE

8A.1b

Use a number line to solve these equations.

1 $2x + 11 = 17$

2 $m + 9 = 2m + 4$

3 $5g + 6 = 11 + 3g$

4 $4x + 5 = x + 2.4 + 2x$

5 $2r + 15 = 3(r + 1)$

Homework

Use a number line to solve question 1 and then spend 10 minutes on questions 2 and 3.

1 $4x + 7 = 2x + 13$

2 $2x + 3 = 2x + 7$

3 $2x - 1 = x + 9$

LESSON 8A.2 Solving equations using the matching method

OBJECTIVES

- Construct and solve linear equations with integer coefficients (unknown on both sides, without and with brackets) using appropriate methods (e.g. transforming both sides in the same way).
- Know that algebraic operations follow the same conventions and order as arithmetic operations.

STARTER

Vocabulary
quadrilateral
rectangle
expression
term
equivalent
infinite

Resources
OHT 8A.2a

Draw a rectangle with the dimensions shown:

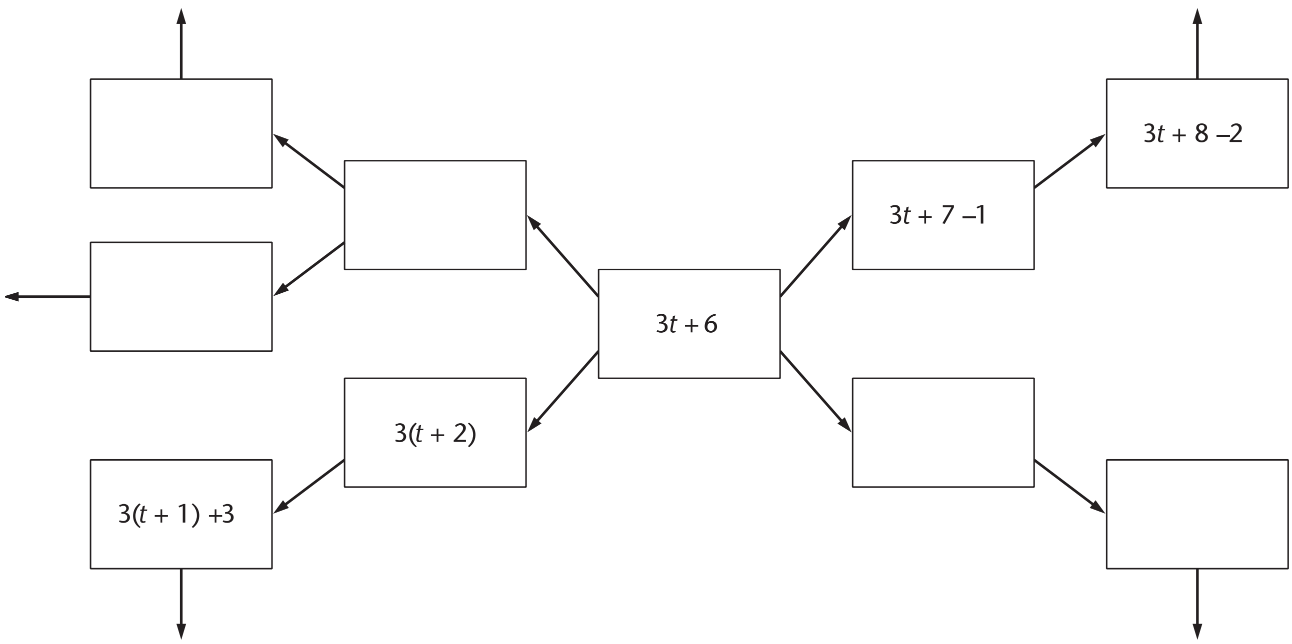
$3t + 6$ centimetres

Establish that it is a rectangle and that the number represented by the unknown t has the same value in the expressions for the length and width.

Q How big is the rectangle?

Take some possible dimensions for different values of t, emphasising the infinite number of possibilities.

Now focusing on the side $3t + 6$, show the diagram on **OHT 8A.2a**.



Q In how many different ways can we write this expression?

Explain the process used along the given branches. Ask for volunteers to start a new branch or to continue a branch. Explanations must be given and the same process or rule must be followed along a branch. Confirm that there is an infinite number of equivalent terms but the central one is the most compact.

MAIN ACTIVITY

Vocabulary
expression
term
like terms
equivalent
equation
matching

Resources
OHT 8A.2b
Resource 8A.2c as a handout

Go back to the original rectangle.

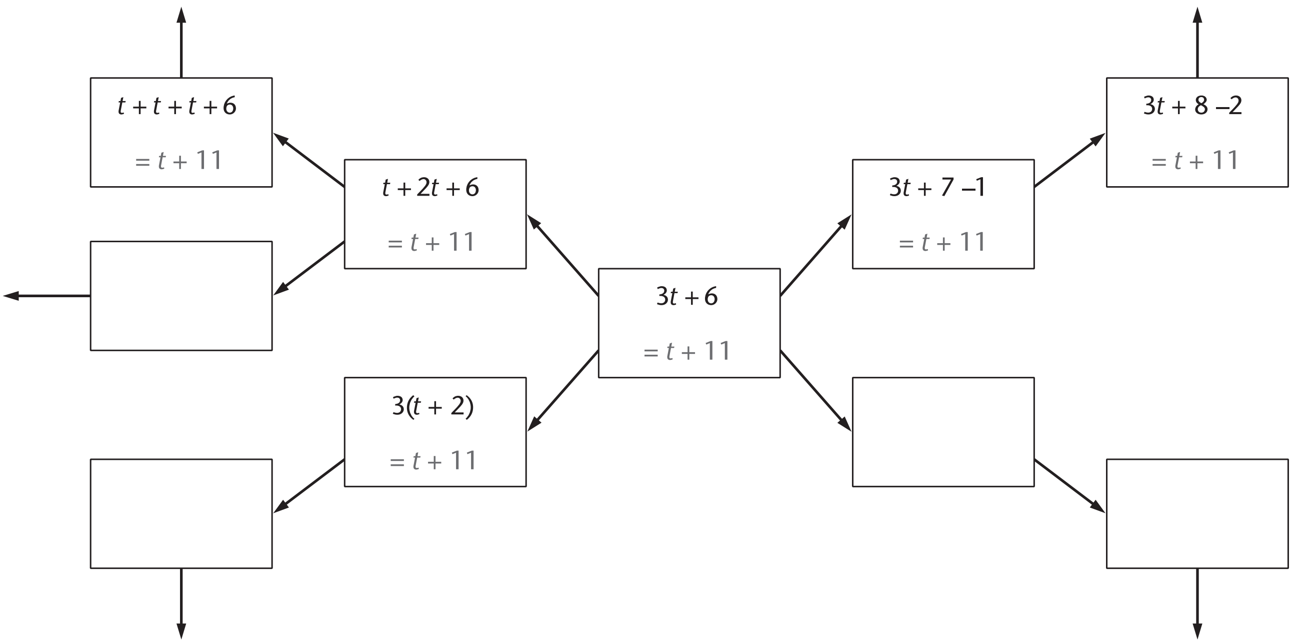
Q If this rectangle is a square what do we know?

Come to an agreement that:

$3t + 6 = t + 11$

Q What are values of *t* that make this equation true?

Ask that all solutions are accompanied by an explanation whether or not the correct solution is given. Say you are going to model a new method of solution. Refer back to the equivalent-expression diagram (OHT 8A.2a) and say that the left-hand side (LHS) has been written in different ways. This means the equation can now be written in a number of different ways by simply writing '$t + 11$', the RHS, equal to each of these equivalent expressions (**OHT 8A.2b**).



Q Which version of the equation is the easiest to work with to find the value of *t*?

If necessary, rewrite the equations outside the diagram underneath each other.

LHS		RHS
$3t + 6$	=	$t + 11$
$t + t + t + 6$	=	$t + 11$
$3t + 7 - 1$	=	$t + 11$
$3t + 8 - 2$	=	$t + 11$
$3(t + 2)$	=	$t + 11$

Let pupils work through their suggestions in front of the class and if necessary model the following by matching equal terms:

$\cancel{t} + t + t + 6 = \cancel{t} + 11$
$t + t + 6 = 11$
$t + t + \cancel{6} = 5 + \cancel{6}$
$t + t = 5$
$t = 2.5$

Note that pupils will not necessarily need the same steps or number of steps.

Check the solution in the equation and in the original rectangle.

Q Is it a square?

Confirm that this is the only solution.

Now repeat for the equation:

$x + 13 = 2x + 7$

Model a solution without the use of the diagram (if appropriate) by partitioning and matching equal terms.

- Write the left-hand expression in as many ways as possible.
- Write the right-hand expression in as many ways as possible.
- Pick out expressions that have matching terms.
- Form equivalent equations that can be simplified, for example:

$\cancel{x} + 13 = \cancel{x} + x + 7$	$x + 4 + \cancel{7} = 2x + \cancel{7}$	$\cancel{x} + 4 + \cancel{7} = \cancel{x} + x + \cancel{7}$
$13 = x + 7$	$x + 4 = 2x$	$4 = x$

If necessary the matching method (or another appropriate method) can be used to find 'easier' equivalent equations to solve.

Note that with practice pupils will partition the LHS and RHS in such a way as to target particular matches that leave a simpler equivalent equation to solve.

Ask pupils to use the matching method to solve the following equations (reproduced on **resource 8A.2c**).

1. $2x + 7 = 19$
2. $2m + 9 = 3m + 4$
3. $2y - 5 = y + 7$
4. $4t + 5 = t + 2.4 + t$
5. $17 = 2(4d - 2)$

PLENARY

Ask pupils to explain their solutions to their neighbour. Confirm the correct solutions and ask two pupils to share their methods with the class for questions 1 and 3, if appropriate.

Q Is it different when solving an equation which includes a minus sign?

Responses may confirm that the same method can be used but partitioning may be more difficult.

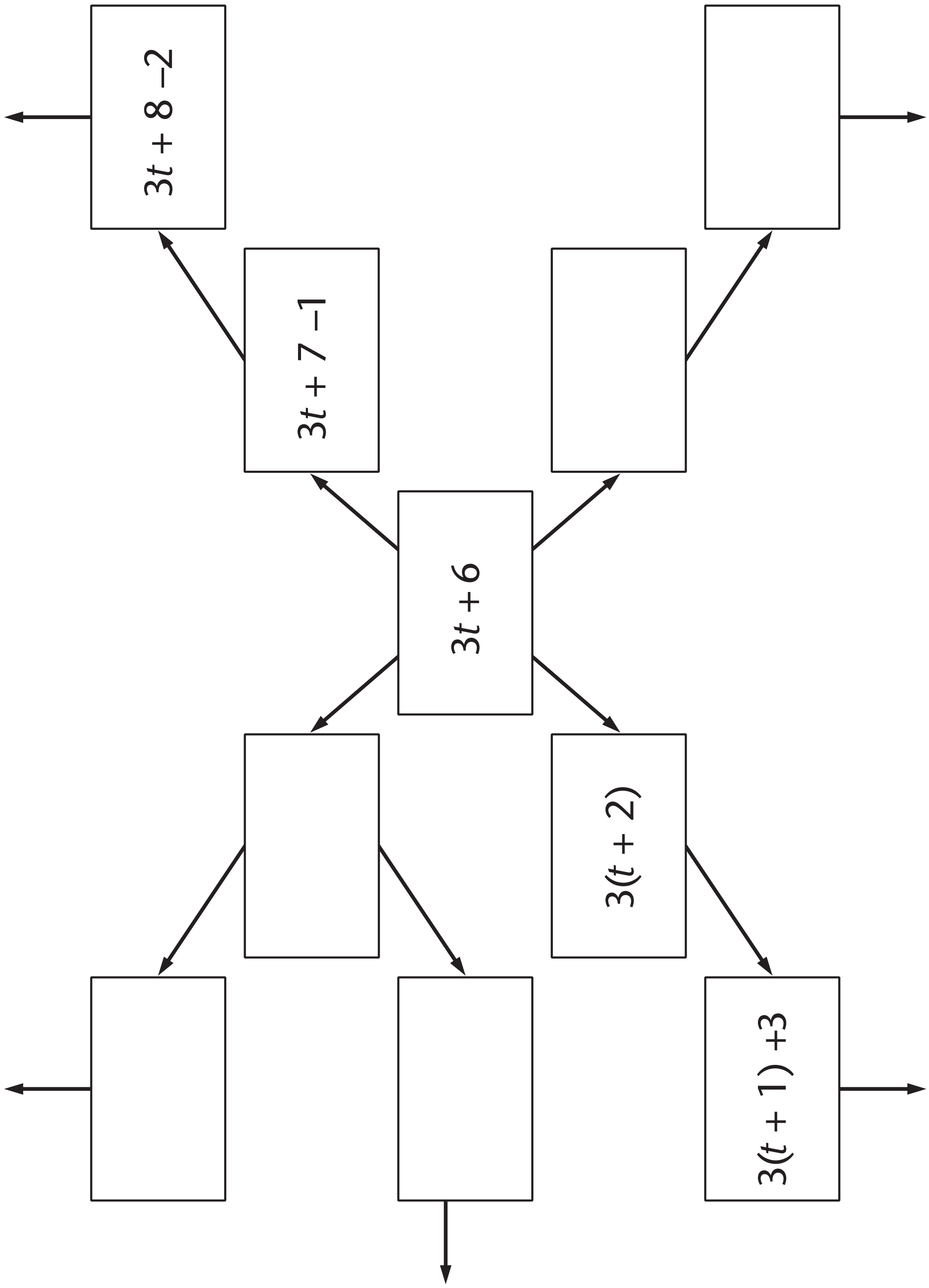
Note: It is important not to rush the partitioning and matching. Pupils' confidence and competence should increase with experience and ultimately provide understanding.

KEY IDEAS FOR PUPILS

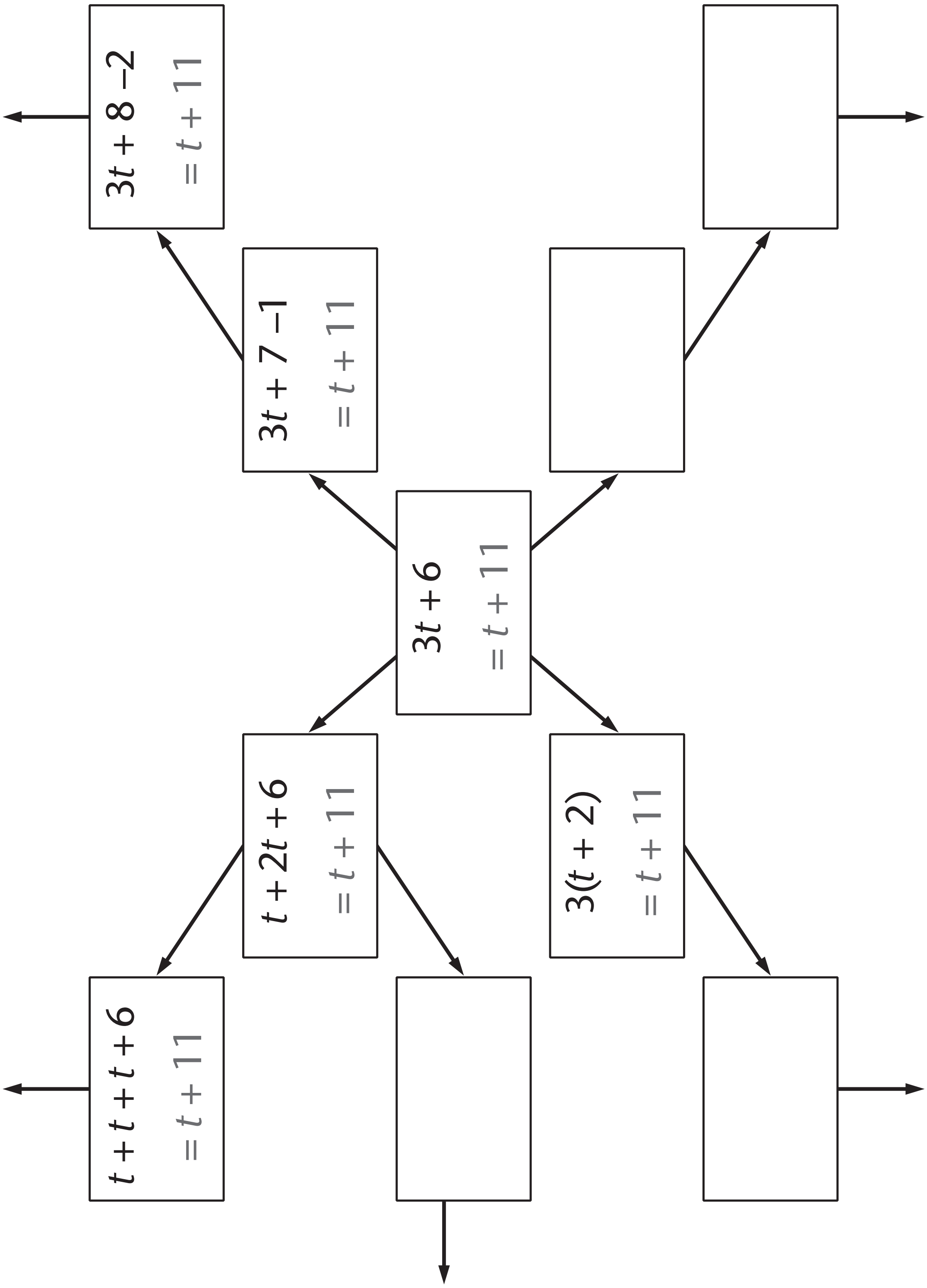
- The matching method can lead to many different equivalent equations. Try to find the easiest one to solve.
- Try to partition the two sides of the equation to make a match.
- It is important to record your thinking clearly.

OHT

8A.2a



OHT



RESOURCE

8A.2c

Use the matching method to solve these equations.

Record your working clearly.

1 $2x + 7 = 19$

2 $2m + 9 = 3m + 4$

3 $2y - 5 = y + 7$

4 $4t + 5 = t + 2.4 + t$

5 $17 = 2(4d - 2)$

Key Stage 3
National Strategy

Constructing and solving linear equations
Year 8 booklet

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