## The National Strategies



## Assessing Pupils' Progress

Focused assessment materials: Level 4

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## Acknowledgement

The National Strategies are grateful for the many contributions from teachers, consultants and students that helped to make these materials possible. Particular thanks are due to colleagues from Gloucestershire Local Authority for their contributions.

These materials are based on the APP assessment criteria and organised in the National Curriculum levels. There is a set for each of levels 4 to 8.

The focused assessment materials include for each assessment criterion:

- Examples of what pupils should know and be able to do so teachers have a feel for how difficult the mathematics is intended to be. These are not activities or examples that will enable an accurate assessment of work at this level. To do this, you need a broad range of evidence drawn from day-today teaching over a period of time; this is exemplified in the Standards files, which are provided as part of the overall APP resources.
- Some probing questions for teacher to use with pupils in lessons to initiate dialogue to help secure their assessment judgement.


## Numbers and the number system

## Examples of what pupils should know and be able to do

## Recognise and describe number patterns

Describe sequences in words, e.g.:
$8,16,24,32,40, \ldots$
$5,13,21,29,37, \ldots$
$89,80,71,62,53, \ldots$
Continue sequences including those involving decimals.

Find missing numbers in sequences.

Probing questions

## Recognise and describe number relationships including multiple, factor and square

Use the multiples of 4 to work out the multiples of 8. Identify factors of two-digit numbers.
Know simple tests for divisibility for 2, 3, 4, 5, 6, 8, 9
Find the factors of a number by checking for divisibility by primes, e.g. to find the factors of 123 check mentally or otherwise, if the number divides by 2 , then $3,5,7,11 \ldots$

What do you notice about this sequence of numbers?
Can you describe a number sequence (without showing it to me) so that I could write down the first ten numbers?
How do you go about finding missing numbers in a sequence?

| by 2, then $3,5,7,11 .$. | 500 that is divisible by 3 ? How do you know? <br> How do you know if a number is divisible by 6 ? <br> Can you give me an example of a number greater that 100 that is divisible by 5 and also by 3 ? How do you know? <br> Is there a quick way to check if a number is divisible by 25 ? |
| :---: | :---: |

## Use place value to multiply and divide whole numbers by 10 or 100

Respond to oral and written questions such as:

- How many times larger is 2600 than 26 ?
- How many $£ 10$ notes are in $£ 120, £ 1200$ ? How many $£ 1$ coins, 10 p coins, 1 p coins?
- Tins of dog food at 42p each are put in packs of 10. Ten packs are put in a box. How much does one box of dog food cost? 10 boxes? 100 boxes?

Work out mentally the answers to questions such as:

| $329 \times 100=\square$ | $8000 \div 100=\square$ |
| :--- | :--- |
| $56 \times \square=5600$ | $7200 \div \square=72$ |
| $420 \times \square=4200$ | $3900 \div \square=390$ |

Complete statements such as:

- $4 \times 10=\square$
- $4 \times \square=400$
- $\square \div 10=40$
- $\square \times 1000=40000$
- $\square \times 10=400$

Why do $6 \times 100$ and $60 \times 10$ give the same answer?
What about $30 \div 10$ and $300 \div 100$ ?
I have 37 on my calculator display. What single multiplication should I key in to change it to 3700? Explain why this works.
Can you tell me a quick way of multiplying by 10 , by 100?

Can you tell me a quick way of dividing by 10 , by 100 ?

## Recognise approximate proportions of a whole and use simple fractions and percentages to describe these

Recognise simple equivalence between fractions, decimals and percentages, e.g. $1 / 2,1 / 4,1 / 10,3 / 4$.

Convert mixed numbers to improper fractions and vice versa.

Express a smaller number as a fraction of a larger one e.g. What fraction of:

- 1 m is 35 cm
- 1 kg is 24 g
- 1 hour is 33 minutes?

What fractions/percentages can you easily work out in your head? Talk me through a couple of examples.

Talk me through how you know that, e.g. 90 is $3 / 4$ of 120.
Approximately what fraction of, e.g. this shape is shaded? Would you say it is more or less than this fraction? How do you know?

When calculating percentages of quantities, what percentage do you usually start from? How do you use this percentage to work out others?
‘To calculate $10 \%$ of a quantity, you divide it by 10 . So to find $20 \%$, you must divide by $20^{\prime}$. What is wrong with this statement?

Using a 1-100 grid, $50 \%$ of the numbers are even. How would you check? Give me a question with the answer $20 \%$ (or other simple percentages and fractions).

## Order decimals to three decimal places

Place these numbers in order of size, starting with the greatest: $0.206,0.026,0.602,0.620,0.062$

Place these numbers on a line from 6.9 to 7.1: 6.93, 6.91, 6.99, 7.01, 7.06

Put these in order, largest/smallest first: 1.5, 1.375, 1.4, 1.3, 1.35, 1.425

Put these in order, largest/smallest first: 7.765, 7.675, 6.765, 7.756, 6.776

## Begin to understand simple ratio

Use the vocabulary of ratio to describe the relationships between two quantities within a context.

- Given a bag of 4 red and 20 blue cubes, write the ratio of red cubes to blue cubes, and the ratio of blue cubes to red cubes.

Solve simple problems using informal strategies.

- A girl spent her savings of $£ 40$ on books and clothes in the ratio 1:3. How much did she spend on clothes?

Scale numbers up or down, e.g. by converting recipes for, say, six people to recipes for two people:

- In a recipe for six people you need 120 g flour and 270 ml of milk. How much of each ingredient does a recipe for two people require?

What do you look for first when you are ordering numbers with decimals?

Which part of each number do you look at to help you? Which numbers are the hardest to put in order? Why? What do you do when numbers have the same digit in the same place?

Give me a number between 0.12 and 0.17 . Which of the two numbers is it closer to? How do you know?

Can you talk me through how you solved this problem?
What do you see as the important information in this problem? How do you use it to solve the problem?

## Calculating

\section*{| Examples of what pupils should know and be able to |
| :--- | do}

## Use a range of mental methods of computation with all operations

Calculate mentally a difference such as
8006 - 2993 by 'counting up' or by considering
the equivalent calculation of $8006-3000+7$.
Work out mentally that:
4005-1997 = 2008 because it is
$4005-2000+3=2005+3=2008$.
Work out mentally by counting up from the smaller to the larger number:
$8000-2785$ is $5+10+200+5000=5215$.
Calculate complements to 1000.

Which of these calculations can you do without writing anything down? Why is it sensible to work this out mentally? What clues did you look for?

How did you find the difference? Talk me through your method.
Give me a different calculation that has the same answer, an answer that is ten times bigger, etc. How did you do it?

## Recall multiplication facts up to $10 \times 10$ and quickly derive corresponding division facts

Use their knowledge of tables and place value in calculations with multiples of 10 such as $180 \div 3$. Respond rapidly to oral and written questions like:

- Nine eights
- How many sevens in 35 ?
- 8 times 8
- 6 multiplied by 7
- Multiply 11 by 8
- 7 multiplied by 0

Respond quickly to questions like:

- Divide 3.6 by 9
- What is 88 shared between 8 ?
- 0.6 times 7 times 2

Know by heart or derive quickly:

- doubles of two-digit whole numbers or decimals
- doubles of multiples of 10 up to 1000
- doubles of multiples of 100 up to 10000
- and all the corresponding halves

If someone has forgotten the eight times table, what tips would you give them to work it out? What other links between tables are useful?

If you know that $4 \times 7=28$, what else do you know?
Start from a two-digit number with at least six factors, e.g. 56. How many different multiplication and division facts can you make using what you know about 56? How have you identified the divisions?

The product is 40 . Make up some questions. How are these different questions linked?

The quotient is 5 . Make up some questions. How did you go about devising these questions?

## Use efficient written methods of addition and subtraction and of short multiplication and division

Calculate $1202+45+367$ or $1025-336$.
Work with numbers to two decimal places, including sums and differences with different numbers of digits, and totals of more than two numbers, e.g.:

- 671.7-60.2
- $543.65+45.8$
- 1040.6-89.09
- 764.78-56.4
- $76.56+312.2+5.07$

Use, for example, the grid method before moving on to short multiplication.
Use efficient methods of repeated subtraction, by subtracting multiples of the divisor, before moving to short division.

Give pupils some examples of work with errors for them to check, e.g.:

$$
\begin{gathered}
12.3+ \\
9.8 \\
\hline 21.11
\end{gathered}
$$

4.07+
$-1.5$
3.57
36.2
x $\quad 8$
$\overline{288.16}$
Which are correct/incorrect? How do you know? Explain what has been done wrong and correct the answers.

## Multiply a simple decimal by a single digit

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Calculate:
- 2.4\times7
- 4.6 x 8
- }9.3\times
```

What would you estimate the answer to be? Is the
accurate answer bigger or smaller than your estimate?
Why?
How would you help someone to understand that, e.g.
$0.4 \times 7=2.8,2.4 \times 7=16.8$ ?
Which of these calculations are easy to work out in your
head, and why?
$0.5 \times 5$
$0.5 \times 8$
$1.5 \times 5$
Talk me through your method.

## Solve problems with or without a calculator

Interpret a calculator display of 4.5 as $£ 4.50$ in the context of money.

Use a calculator and inverse operations to find missing numbers, including decimals as, e.g.:
$6.5-9.8=\square$
$4.8 \div \square=0.96$
$1 / 8$ of $\square=40$
Use inverses to check results, e.g.:
$703 / 19=37$ appears to about right because
$36 \times 20=720$
Carry out simple calculations involving negative numbers in context.

What would 0.6 mean on a calculator display if the units were pounds, metres, hours, cars?

What is the important information in this problem?
Show me a problem where you would use a calculator to work out the answer. Show me a problem where you would not use a calculator. How do you decide?

Is it always quicker to use a calculator?
What key words tell you that you need to add, subtract, multiply or divide?

How would you use a calculator to solve this problem?

Choose a number to put into a calculator. Add 472 (or multiply by 26 , etc.). What single operation will get you back to your starting number?

Will this be the same for different starting numbers? How do you know?

## Check the reasonableness of results with reference to the context or size of numbers

Use rounding to approximate and judge whether the answer is the right order of magnitude, e.g.:

2605-1897 is about $3000-2000$
$12 \%$ of 192 is about $10 \%$ of 200
Discuss questions such as:

- A girl worked out the cost of eight bags of apples at 47 p a bag. Her answer was $£ 4.06$. Without working out the answer, say whether you think it is right or wrong.
- A boy worked out how many 19p stamps you can buy for $£ 5$. His answer was 25 . Do you think he was right or wrong? Why?
- A boy worked out $£ 2.38+76$ p on a calculator. The display showed 78.38. Why did the calculator give the wrong answer?

Roughly what answer do you expect to get? How did you come to that estimate?

Do you expect your answer to be less than or greater than your estimate? Why?

## Algebra

Examples of what pupils should know and be able to do

## Begin to use formulae expressed in words

Explain the meaning of and substitute integers into formulae expressed in words, or partly in words, such as the following:

- number of days $=7$ times the number of weeks
- cost $=$ price of one item $\times$ number of items
- age in years $=$ age in months $\div 12$
- pence $=$ number of pounds $\times 100$
- area of rectangle $=$ length $\times$ width
- cost of petrol for a journey
$=$ cost per litre $\times$ number of litres used.
Use formulae expressed in words, e.g. for a phone bill based on a standing charge and an amount per unit.

Recognise that a formula expressed in words needs to include an answer, e.g.:

- 'I think of a number and double it', is different from
- 'I think of a number and double it. The answer is 12'.

Working with two sets of cards, one with formulae in words and the other with a range of calculations that match the different formulae (more than one for each formula in words), sort the cards into pairs.

Show me an example of a formula expressed in words.
How can you change 'Cost of plumber's bill = $£ 40$ per hour' to include a $£ 20$ call-out fee?

I think of a number and add 12 - do you know what my number is? Why or why not?
'I think of a number and add 12. The answer is 17.' Do you know what my number is? Why?

How do you know this calculation is for this rule/ formula?

Why is it possible for more than one calculation to match with the same rule? Could there be others?

What's the same and what's different about the calculations for the same rule/formula?

## Use and interpret coordinates in the first quadrant

Given the coordinates of three vertices of a rectangle drawn in the first quadrant, find the fourth.

What are the important conventions when describing a point using coordinates?

I'm thinking of a point that I want you to plot. I can only answer 'yes' and 'no'. Ask me some questions so you can plot the point.
How do you use the scale on the axes to help you to read the coordinates of a point that has been plotted?

How do you use the scale on the axes to help you to plot a point accurately?
If these three points are vertices of a rectangle, how will you find the coordinates of the fourth vertex?

## Shape, space and measures

Examples of what pupils should know and be able to do

Probing questions

## Use the properties of 2-D and 3-D shapes

Recognise and name most quadrilaterals, e.g. trapezium, parallelogram, rhombus.
Recognise right-angled, equilateral, isosceles and scalene triangles and know their properties.
Use mathematical terms such as horizontal, vertical, parallel, perpendicular.
Understand properties of shapes, e.g. why a square is a special rectangle.

Visualise shapes and recognise them in different orientations.

Can you describe a rectangle precisely in words so someone else can draw it?
What mathematical words are important when describing a rectangle?

What properties do you need to be sure a triangle is isosceles; equilateral; scalene?
Show a mix of rectangles including squares.

- Can you tell me which of the shapes are square? Convince me.
- Can you tell me which shapes are rectangles? Convince me.


## Make 3-D models by linking given faces or edges and draw common 2-D shapes in different

 orientations on gridsIdentify and make up the different nets for an open cube.
Complete a rectangle which has two sides drawn at an oblique angle to the grid.

## When presented with a net:

- Which edge will meet this edge?
- Which vertices will meet this one?

What can you tell me about a 3-D shape from its 2-D net?
How would you plot a rectangle that has no horizontal sides on a square grid? How would you convince me that the shape is a rectangle?

## Reflect simple shapes in a mirror line, translate shapes horizontally or vertically and begin to rotate a simple shape or object about its centre or a vertex

Use a grid to plot the reflection in a mirror line inclined at $45^{\circ}$, both when the shape touches the mirror line and where it does not.

Begin to use the distance of vertices from the mirror line to reflect shapes more accurately.

Give me instructions to reflect this shape into this mirror line. What would you suggest I do first?
How do the squares on the grid help when reflecting? Show me.

Make up a reflection that is easy to do.
Make up a reflection that is hard to do. What makes it hard?

How can you tell if a shape has been reflected or translated?

## Choose and use appropriate units and instruments

Know metric conversions: mm/cm; cm/m; m/km; $\mathrm{mg} / \mathrm{g} ; \mathrm{g} / \mathrm{kg} ; \mathrm{ml} / \mathrm{l}$.

How do the names of units like millimetres, centimetres, metres, kilometres help you to convert from one unit to another?

How do you go about finding the perimeter of a rectangle when one side is measured in centimetres and the other in millimetres?

When is it essential to use a ruler rather than a straight edge?

## Interpret, with appropriate accuracy, numbers on a range of measuring instruments

Measure and draw lengths and angles accurately $\left( \pm 2 \mathrm{~mm} \pm 5^{\circ}\right.$ ).

Read and interpret scales on a range of measuring instruments, including:

- vertical scales, e.g. thermometer, tape measure, ruler
- scales around a circle or semicircle, e.g. for measuring time, mass, angle.

What is the first thing you look for when you are reading a scale on measuring equipment?

How do you decide what each division on the scale represents?

How would you measure 3.6 cm if the zero end of your ruler was broken?

## Find perimeters of simple shapes and find areas by counting squares

Find perimeters and areas of shapes other than rectangles. Focus on having a feel for the perimeter and area - not calculating them.

Work out the perimeter of some shapes by measuring, in millimetres.
Use the terms 'area' and 'perimeter' accurately and consistently.

Find areas by counting squares and part squares of shapes drawn on squared paper.
Begin to find the area of shapes that are made from joining rectangles.

Use 'number of squares in a row times number of rows' to find the area of a rectangle.

How do you go about finding the perimeter of a shape?

How are the perimeter of a shape and the area of a shape different? How do you remember which is which?

Would you expect the area of a paperback book cover to be: $200 \mathrm{~cm}^{2}, 600 \mathrm{~cm}^{2}$, or $6000 \mathrm{~cm}^{2}$ ? Explain why.

Would you expect the area of a digit card to be: $5 \mathrm{~cm}^{2}$, $50 \mathrm{~cm}^{2}$ or $100 \mathrm{~cm}^{2}$ ? Explain why.
Suggest 2-D shapes/objects where the area could be measured in $\mathrm{cm}^{2}$.

## Handling data

Examples of what pupils should know and be able to do

## Probing questions

## Collect and record discrete data

Record discrete data using a frequency table.
Structure data into a frequency table for enquiries such as:

- the number of goals scored during one season by a hockey team.

How did you decide on how to structure your table of results?
Why did you choose these items? Might there be others?

How did you go about collecting the data for this enquiry?
What made the information easy or difficult to record?

## Group data, where appropriate, in equal class intervals

Decide on a suitable class interval when collecting or representing data, e.g.:

- pupils' time per week spent watching television - using intervals of one hour
- how long pupils take to travel to school using intervals of five minutes.

Why did you choose this group size for organising the data? What would happen if you chose a different group size?
How do you know these are equal class intervals?
Why is it important to use equal class intervals?

## Continue to use Venn and Carroll diagrams to record their sorting and classifying of information

Using this Carroll diagram for numbers, write a number less than 100 in each space.


Use a Venn diagram to sort by two criteria, e.g. sorting numbers using the properties 'multiples of 8 ' and 'multiples of 6 '.

Give me an example of a Venn diagram that can be used to sort the numbers 1-50. Which criteria have you used and why?

Give me an example of a Carroll diagram - with four cells - that can be used to sort the numbers 1-50.

Which criteria have you used and why?
What are the important steps when completing a Carroll diagram? What strategies do you use to check your Carroll diagram is complete?
What are the important steps when completing a Venn diagram? What strategies do you use to check your Venn diagram is complete?
How is a Venn diagram different from a Carroll diagram?

## Construct and interpret frequency diagrams and simple line graphs

Suggest an appropriate frequency diagram to represent particular data, e.g. decide whether a bar chart, Venn diagram or pictogram would be most appropriate. For pictograms use one symbol to represent several units.

Decide on an appropriate scale for a graph, e.g. labelled divisions, representing $2,5,10,100$.

Interpret simple pie charts.
Interpret the scale on bar charts and line graphs, reading between the labelled divisions, e.g. reading 17 on a scale labelled in fives.

Interpret the total amount of data represented, compare data sets and respond to questions, e.g. how does our data about favourite television programmes compare with the data from Year 8 pupils?

For a given graph/table/chart, make up three questions that can be answered using the information represented. What makes the information easy or difficult to represent? How do you decide on the scale to use on the vertical axis? How would a different scale change the graph?

## Understand and use the mode and range to describe sets of data

Use mode and range to describe data relating to, e.g. shoe sizes in the pupils' class and begin to compare their data with data from another class.

Discuss questions such as:

- How can we find out if this is true?
- What information should we collect?
- How shall we organise it?
- What does the mode tell us?
- What does the range tell us?

List a small set of data that has a mode of 5 . How did you do it?
List a small set of data that has a mode of 5 and a range of 10. How did you work this out?

Can you find two different small sets of data that have the same mode and range? How did you do it?

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