PD3 • Looking at learning activities

Purpose

To encourage participants to:

- explore the five different types of mathematical activity contained in the resource.
  1. **Classifying mathematical objects**: reflecting on the properties of mathematical objects; sameness and difference; definitions.
  2. **Interpreting multiple representations**: interpreting concepts from a variety of perspectives; making links.
  3. **Evaluating mathematical statements**: testing generalisations, generating examples and counterexamples.
  4. **Creating problems**: creative thinking, 'doing and undoing' mathematical processes.
  5. **Analysing reasoning and solutions**: comparing different methods for doing problems, organising solutions and/ or diagnosing the causes of errors in solutions.

Materials required

For each participant you will need:

- Sheet PD3.1 – *Types of activity used in the resource*;
- Sheet PD3.2 – *Classifying mathematical objects*;
- Sheet PD3.3 – *Interpreting multiple representations*;
- Sheet PD3.4 – *Evaluating mathematical statements*;
- Sheet PD3.5 – *Creating problems: using an exam question creatively*;
- Sheet PD3.6 – *Creating problems: doing and undoing processes*.

For each pair of participants you will need:

- Card set PD3.7 – *Looking at reasoning*;
- scissors.

Participants should work in pairs with one session for each type of activity, e.g.

Type 1	**SS1**	**Classifying shapes**
Type 2	**A1**	**Interpreting algebraic expressions**
Type 3	**S2**	**Evaluating probability statements**
Type 4	**A2**	**Creating and solving equations**
Type 5	**N6**	**Developing proportional reasoning**
Type 5	**C5**	**Finding stationary points of cubic functions**

Supporting materials

To support the session you may wish to use:

- extract from the DVD-ROM in *Planning learning/Session 1/Excerpt 2*;
- extract from the DVD-ROM in *Planning learning/Session 3/Excerpt 3*;
- extract from the DVD-ROM in *Thinking about learning/Using misconceptions/Example*;
- PowerPoint presentation in *Materials/Professional development* on the DVD-ROM. This will be useful when running the session and includes slides of the aims, and of appropriate handouts and tasks.

Time needed

Allow at least 30 minutes to explore each type of activity.

You are likely to need two sessions to ensure that all participants have the opportunity to explore each type of activity.

Suggested activities

Give each participant a copy of Sheet PD3.1 – *Types of activity used in the resources*. Explain each type briefly and tell participants that they will have the opportunity to experience an example of each type for themselves.

Note
There is one section for each type of activity. The five sections can be used in any order.

1. Classifying mathematical objects

Mathematics is full of conceptual 'objects' such as numbers, shapes, and functions. In this type of activity, learners examine such objects carefully, and classify them according to their different attributes. Learners have to select an object, discriminate between that object and other similar objects ('what is the same and what is different?') and create and use categories to build definitions. This type of activity is therefore powerful in helping learners understand what is meant by different mathematical terms and symbols, and the process through which they are developed.

Give each participant a copy of Sheet PD3.2 – *Classifying mathematical objects*. Ask participants to work in pairs on the 'odd-one-out' activity, and then to share their ideas with the whole group. Encourage participants to find several reasons for each mathematical object being different from the others. Make the point that, on each attempt, participants are recognising properties of the objects.

Introduce a session from the resource that requires this type of thinking, for example **SS1 Classifying shapes**. Work through the session together and discuss the learning implications.

If there is time, show the group the film sequence *Planning learning/Session 1/Excerpt 2* on the DVD-ROM. This is an example of a classification activity in an A level session on Functions.

Challenge participants to devise a classification activity using other mathematical objects. For example, they may choose to classify numbers, equations or functions in different ways.

2. Interpreting multiple representations

Mathematical concepts have many representations: words, diagrams, algebraic symbols, tables, graphs and so on. These activities are intended to allow these representations to be shared, interpreted, compared and grouped in ways that allow learners to construct meanings and links between the underlying concepts.

If there is time, show the film sequence *Planning learning/Session 3/Excerpt 3* on the DVD-ROM. Here you will find an example of a teacher using a 'multiple representations' activity with a group of level 2 learners. You might want to use this sequence as a stimulus for discussion or simply to set the scene for the activity below. You can also hear Samina and the learners reflecting on the activity in the same section of the DVD-ROM.

Ask participants to work together in groups of two or three on session **A1 Interpreting algebraic expressions**, or on an alternative 'multiple representations' session. Encourage participants to act the role of learners. Discuss how the activity confronts and exposes common misinterpretations and misconceptions.

Give each participant a copy of Sheet PD3.3 – *Interpreting multiple representations* and invite them to create their own (small) set of cards that would encourage learners to interpret other representations in mathematics. Words, algebraic symbols, pictures, graphs, tables, and/or geometric shapes could be used. Participants should include cards that force learners to distinguish representations that are frequently confused (such as $(3n)^2$ and $3n^2$). Share these new cards among the group for comments and suggestions.

3. Evaluating mathematical statements

These activities offer learners a number of mathematical statements or generalisations. Learners are asked to decide on their validity and give explanations for their decisions. Explanations usually involve

generating examples and counterexamples to support or refute the statements. In addition, learners may be invited to add conditions or to otherwise revise the statements so that they become 'always true'.

Ask participants to work together in groups of two or three on session **S2 Evaluating probability statements**, or on an alternative session that involves evaluating mathematical statements. Discuss the misconceptions described in the teacher's notes to the session and share further examples from participants' experience. Discuss ways of confronting and overcoming these misconceptions. If there is time you might want to show them a film example of this session. You will find this in *Planning learning/Session 2* on the DVD-ROM.

Give each participant a copy of Sheet PD3.4 – *Evaluating mathematical statements*. This handout contains a range of statements taken from other sessions in the resource. Ask participants to devise further statements at a level suitable for their own learners. Share these ideas in the group.

If you have time you might like to show the short video sequence *Thinking about learning/Using misconceptions/Example* on the DVD-ROM. This is an example of a teacher working with learners, exploring and moving towards resolving misconceptions.

4. Creating problems

In this type of activity, learners are given the task of devising their own mathematical problems. They try to devise problems that are both challenging and that they know they can solve correctly. Learners first solve their own problems and then challenge other learners to solve them. During this process, they offer support and act as 'teachers' when the problem-solver gets stuck.

(i) Developing an examination question

Give each participant a copy of Sheet PD3.5 – *Creating problems: using an exam question creatively* or, if you prefer, use an alternative question from a recent examination paper of your own. The example in Sheet PD3.5 is taken from session **N10 Developing an examination question: number**.

Ask participants to work in pairs. They should answer the question, then, without changing the information given, they should write down further questions that could be asked about the situation. For the 'Van hire' question in Sheet PD3.5, this will generate a list such as the following:

- Over what distance is Bujit's more expensive than Hurt's? Over what distance are Hurt's vans more expensive than Bujit's?
- Where is the cross-over point?
- Can you make a table showing Bujit's prices so that prices are easier to compare with Hurt's?
- Can you make a graph showing how the two companies' prices vary with the miles covered?
- Can you write a formula to show each company's prices?

Discuss various ways of answering these new questions. What makes them more or less interesting?

Ask participants to change the question itself to make it more interesting and more challenging. They must do this by filling in the blanks in the 'Car hire' question in Sheet PD3.5. As they do this, ask them to reflect on the type of thinking involved.

Allow time for participants to solve each other's questions.

Explain that, by using this type of activity, learners begin to take ownership of questions, become more aware of their structure and learn to see each question as an example of a broader class of questions that could be asked.

Invite participants to consider how this approach can be used with other examination questions.

(ii) Exploring the doing and undoing processes in mathematics

Explain that creating and solving problems can also be used to illustrate doing and undoing processes in mathematics. For example, one learner might draw a circle and calculate its area. This learner then passes the result to a neighbour, who must try to reconstruct the circle from the given area. The two learners then collaborate to check their answers and see where mistakes have arisen.

Give each participants a copy of Sheet PDF3.6 – *Creating problems: doing and undoing processes* and discuss the examples given. Ask participants to generate two further examples of their own and write these in the blank spaces.

Issue each pair of participants with a copy of session **A2 Creating and solving equations**. Allow them time to work through it together.

5. Analysing reasoning and solutions

These activities are designed to shift the emphasis away from 'getting the answer' and towards a situation where learners are able to evaluate and compare different forms of reasoning.

(i) Comparing different solution strategies

Issue pairs of participants with copies of session **N6 Developing proportional reasoning**. Explain that, in this session, learners are expected to try to solve four proportion problems from different parts of the mathematics curriculum, then compare the methods they have used. They are also invited to mark work produced by other learners.

(ii) Evaluating reasoning

Ask participants to work in pairs. Issue Card set PD3.7 – *Looking at reasoning* to each pair. Ask them to cut out the cards and arrange them to create two logical proofs.

Explain that this type of activity can be used to develop chains of reasoning, particularly at higher levels. Participants may wish to look at **C5 Finding stationary points of cubic functions for an example**.

Sheet PD3.1 – *Types of activity used in the resource*

Classifying mathematical objects

Learners devise their own classifications for mathematical objects, and apply classifications devised by others. They learn to discriminate carefully and recognise the properties of objects. They also develop mathematical language and definitions.

Interpreting multiple representations

Learners match cards showing different representations of the same mathematical idea. They draw links between different representations and develop new mental images for concepts.

Evaluating mathematical statements

Learners decide whether given statements are always, sometimes or never true. They are encouraged to develop rigorous mathematical arguments and justifications, and to devise examples and counterexamples to defend their reasoning.

Creating problems

Learners devise their own problems or problem variants for other learners to solve. This offers them the opportunity to be creative and 'own' problems. While others attempt to solve them, they take on the role of participant and explainer. The 'doing' and 'undoing' processes of mathematics are vividly exemplified.

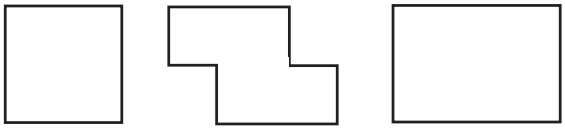
Analysing reasoning and solutions

Learners compare different methods for doing a problem, organise solutions and/ or diagnose the causes of errors in solutions. They begin to recognise that there are alternative pathways through a problem, and develop their own chains of reasoning.

Sheet PD3.2 – *Classifying mathematical objects*

Odd one out

In the triplets below, how can you justify each of (a), (b), (c) as the odd one out?

(a) a fraction (b) a decimal (c) a percentage	(a) sin 60° (b) cos 60° (c) tan 60°
(a) (b) (c) 	(a) $y = x^2 - 6x + 8$ (b) $y = x^2 - 6x + 9$ (c) $y = x^2 - 6x + 10$
(a) (b) (c) 	(a) 20, 14, 8, 2, … (b) 3, 7, 11, 15, … (c) 4, 8, 16, 32, …

Now make up some triplets of your own.

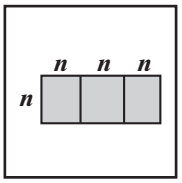
Sheet PD3.3 – *Interpreting multiple representations*

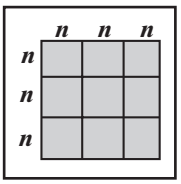
These cards focus learners' attention on a specific aspect of algebraic notation. Learners are expected to interpret each representation and match them together if they have an equivalent meaning.

Your task is to create a different set of cards that will encourage learners to interpret some other representations in mathematics.

These may include words, algebraic symbols, pictures, graphs, tables, geometric shapes, etc.

Try to create cards that require learners to distinguish between representations that they often confuse (such as $(3n)^2$ and $3n^2$ in the example).





Square n then multiply your answer by 3

Multiply n by 3 then square your answer

$9n^2$

$(3n)^2$

$3n^2$

Square n then multiply your answer by 9

Sheet PD3.4 – *Evaluating mathematical statements*

Classify each statement as always, sometimes or never true.

If you think it is always or never true, then try to explain how you can be sure.

If you think it is sometimes true, then try to define exactly when it is true and when it is not.

Number operations

The square root of a number is less than or equal to the number.	The square of a number is greater than or equal to the number.

Directed numbers

If you subtract a positive number from a negative number you get a negative answer.	If you subtract a negative number from a negative number you get a positive answer.

Perimeter and area

When you cut a piece off a shape, you reduce its area and perimeter.	If a square and a rectangle have the same perimeter, the square has the smaller area.

Equations, inequations, identities

$p + 12 = s + 12$	$3 + 2y = 5y$

Write four statements that your learners would benefit from discussing. Write statements that focus on particular misconceptions or difficulties.

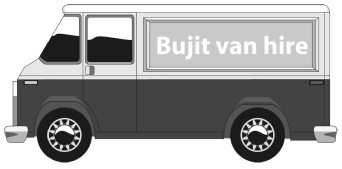
Sheet PD3.5 – *Creating problems: using an exam question creatively*

Van hire

Sanjay wants to hire a van to move some furniture.

He obtains the following information from two hire companies.

Bujit's Van Hire



£30 for the first 50 miles.

Every mile after that costs an extra 20p.

Hurt's Vans

You only pay for the miles you travel.

Miles travelled	50	100	150	200
Hire charge	£16	£32	£48	£64

1. How much do Hurt's vans cost per mile?

2. Sanjay expects to travel 175 miles.
   Which company has the lower charge for this distance?
   You must show all your working.

Car hire

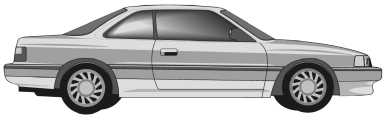
Cath wants to hire a car for a weekend.

She obtains the following information from two hire companies.

..... Car Hire

£ .....for the first .....miles.
Every mile after that costs an extra ..... p.

..... Car Hire



Miles travelled				
Hire charge				

. . . . . .

. . . . . .

. . . . . .

Sheet PD3.6 – *Creating problems: doing and undoing processes*

Doing: the problem poser…	**Undoing: the problem solver…**
• creates an equation step-by-step, starting with a value for x and 'doing the same to both sides'.	• solves the resulting equation.
• draws a rectangle and calculates its area and perimeter.	• tries to draw a rectangle with the given area and perimeter.
• writes down an equation of the form $y = mx + c$ and plots a graph.	• tries to find an equation that fits the resulting graph.
• expands an algebraic expression such as $(x + 3)(x - 2)$.	• factorises the resulting expression: $x^2 + x - 6$.
• writes down a polynomial and differentiates it.	• integrates the resulting function.
• writes down five numbers and finds their mean, median and range.	• tries to find five numbers with the given mean, median and range.
•	•
•	•

Sheet PD3.7 – *Looking at reasoning*

Cut up the following cards. Rearrange them to form two proofs.

The first should prove that: **If n is an odd number, then n^2 is an odd number**

The second should prove that: **If n^2 is an odd number, then n is an odd number**.
You may not need to use all the cards.

If n is odd	**So n is odd**
$n = 2m + 1$ **for some integer m**	$= 2k$ **where $k = 2m^2$**
$(2m + 1)^2 = 4m^2 + 4m + 1$	**But n^2 is odd**
$(2m)^2 = 4m^2$	**So n^2 is odd**
If n is even	$n = 2m$ **for some integer m**
So n^2 is even	$= 2k + 1$ **where $k = 2m(m + 1)$**
If n^2 is odd	$n^2 = 2m + 1$ **for some integer m**