The National Strategies
Secondary

Assessing pupils' progress in mathematics at Key Stage 3

Year 9 assessment package
Algebra

Teacher pack

department for
children, schools and families

Qualifications and
Curriculum Authority

Year 9 Algebra task: *Twelve sum* and *Three lines*

Levels 4/5/6

Note that for classes consisting only of pupils at level 4, teachers may wish to explore the material in lesson 1 more thoroughly, using the probing questions, rather than progressing to lesson 2.

The lesson plans in this pack are set out in two columns. The left-hand column has indicative times for activities, highlights the resource sheets required and also has some examples of questions which teachers may wish to use with pupils during the activities. The right-hand column describes each activity in detail.

APP ASSESSMENT CRITERIA
These lessons may generate evidence to help inform judgements against a number of assessment criteria, including the following:

Shape, space and measure

- level 5: reason about position and movement and transform shapes
- level 6: know that translations, rotations and reflections preserve length and angle and map objects onto congruent images

Algebra

- level 5: use and interpret coordinates in all four quadrants
- level 5: construct, express in symbolic form, and use simple formulae involving one or two operations
- level 6: plot the graphs of linear functions, where y is given explicitly in terms of x; recognise that equations of the form y = mx + c correspond to straight-line graphs

Using and applying mathematics

- level 6: present a concise, reasoned argument, using symbols, diagrams, graphs and related explanatory texts.

LESSON 1: *TWELVE SUM*
Resources

- The Flash resource:
  Algebra task
  should be loaded before the lesson to be shown on a large screen monitor or electronic whiteboard. Teachers who do not have access to the technology required can use the teacher resource OHT/whiteboard slides:
  Straight lines, teacher sheet 1 **(T12L1teacher1)**
  Straight lines, teacher sheet 2 **(T12L1teacher2)**
  Straight lines, teacher sheet 3 **(T12L1teacher3)**
  Twelve sum summary **(T12L1teacher4)**
- Each pupil, or each group of pupils, needs the following worksheet, depending on ability:
  Twelve sum **(T12L1pupil1)**
- Assessment sheets for pupils:
  Each pupil needs one of the following worksheets:
  Level 4 pupils: *Ten sum, sheet 1* **(T12L1assess1)**
  Level 5 pupils: *Ten sum, sheet 2* **(T12L1assess2)**
  Level 6 pupils: *Ten sum, sheet 3* **(T12L1assess3)**
- Graph or squared paper for any rough working
- Calculators may be used if needed, but not graphical calculators for the assessment activity

Lesson 1: with the Flash resource

<table>
<tr>
<td>
Starter
about 10 minutes

Flash demonstration
Algebra task

As x changes, what happens to y? How can you write this relationship in words? How can you write it as an equation?

What do you notice about the y value of each point? If x is a whole number, explain why y must also be a whole number. What would happen if x were not a whole number?

What symmetry is shown, and how does this explain why the equations of the lines are y = 2x and x = 2y?

Given the equation is y = 2x, why is the sum of integer pairs of ordinates always a multiple of 3? What if the equation had been y = 3x? Or y = 4x, etc?
[All coordinates that lie on the line $y = 2x$ must be of the form $(a, 2a)$, giving a sum of $3a$. If a is an integer, $3a$ must be a multiple of 3. Similarly if the ordinates are integers, the sum of the ordinates of points that lie on $y = 3x$ must be multiples of 4, and the sum of the ordinates of points that lie on $y = nx$ must be multiples of $n + 1$.]
</td>
<td>
Click on demonstration 1 (x, y axes appear followed by the line $y = x$ and two points on the line).

Ask the pupils for the equation of the straight line. Ask how they can be sure and show some other coordinates by clicking on any point on the line and holding the mouse button down.

If appropriate, discuss different ways of writing the equation. Clicking on the Equation button will show the equation of the line, written as $y = x$, on screen.

Now ask which point on the line $y = x$ has coordinates that sum to 12. When agreement is reached that (6, 6) is the correct point, click on the ‘Sum to 12’ button and the point and its coordinates will be shown in red.

Now click on the ‘forwards’ button, and the lines $y = 2x$ and $y = \frac{1}{2}x$ will appear, with one labelled point on each.

Confirm that both lines go through the origin, then ask the pupils for the equations of the lines, clicking on the lines and holding to see the coordinates of other points. If appropriate, discuss different ways of writing the equations, but agree on the form $y = 2x$ and $y = \frac{1}{2}x$ since these are the forms shown when the Equations button is clicked.

Ask which point on each line has coordinates that sum to 12, and display these by clicking on the ‘Sum to 12’ button. Lastly, press the ‘forwards’ button to repeat with $y = 3x$ and $y = \frac{1}{3}x$.
</td>
</tr>
<tr>
<td>
Group activity
about 10 minutes

T12L1pupil1

Think about the point that has coordinates that sum to 12 on each of the lines y = x, y = 2x, y = 3x, y = 4x and y = 5x. Only the point on the line y = 4x has non-integer values. Is there a reason for this?
[As before, the sum of integer ordinates on the line $y = nx$ must be multiples of $n + 1$. So $y = 4x$ has points with sums that are multiples of 5, which is the only one that is not a factor of 12.]

Do any other straight lines with equations of the form y = mx have integer values for the point that has coordinates that sum to 12?
[$y = 11x$, $y = 0x$ (i.e. $y = 0$)]
</td>
<td>
Pupils work in groups on the worksheet:
Twelve sum (T12L1pupil1)

(Note that some pupils may write the incorrect equations $y = 4x$ and $y = \frac{1}{4}x$, assuming the ‘pattern’ started continues.)

More able pupils should be asked if they can work out which point on the line $y = 4x$ has coordinates that sum to 12.
[Answer: (2.4, 9.6)]
</td>
</tr>
</table>

Mini-plenary about 5 minutes **Flash demonstration** ***Algebra task*** *Why is the equation of the* x*-axis not* x = 0*? Why is the equation of the* y*-axis not* y = 0*?*	Briefly discuss their findings, then show **demonstration 2**. Seven straight lines appear, with their equations. Click the 'Sum to 12' button and the set of points with coordinates that sum to 12 appear. What do they notice? Click the 'Show coordinates' button to see the coordinates of these points. Click 'Hide coordinates' to remove them. Click the 'forwards' button to show the straight line through these points. What can you say about all the points on this line? Lead to the equation $x + y = 12$, or whichever form you prefer, discussing alternative notation as appropriate. Discuss other points on the line, e.g. what are the coordinates of the points where this straight line intersects the x-axis and the y-axis? Note that you can click and hold any point on the line $x + y = 12$ to show its coordinates.
Assessment activity about 15 minutes **T12L1assess1** **T12L1assess2** **T12L1assess3**	Give out the assessment sheets as follows: Level 4 pupils: *Ten sum, sheet 1* **(T12L1assess1)** Level 5 pupils: *Ten sum, sheet 2* **(T12L1assess2)** Level 6 pupils: *Ten sum, sheet 3* **(T12L1assess3)**
Plenary about 10 minutes *How can you tell from straight-line equations whether two lines will be parallel? Why is it helpful to write both equations in the form* y = mx + c *to decide if they are parallel?* *What do you know about the gradients of lines that are perpendicular?* *Will the line* y = 9 + x *cross/intersect the line* x + y = 9*? How can we be certain?* [All non-parallel lines must intersect somewhere.] *Where will they cross?* [At (0, 9)] *How can you tell?*	Ask pupils to think about all the points whose coordinates sum to 9. What can they tell you about these points? Discuss the straight line formed and how it relates to $x + y = 12$ and $x + y = 10$, e.g. that these lines are parallel. Is the straight line with equation $x + y = 9$ perpendicular to the straight line with equation $y = x$? How do they know? Now ask if the point (4.7, 5.3) is on the line $x + y = 9$. How do they know? Show pupils the four equations $x + y = 9$, $y + x = 9$, $y = 9 + x$ and $y = 9 - x$. Which is the odd one out? Why? [$y = 9 + x$; the other three are different ways of labelling the same equation.] Now write the coordinates (3, 6) on the board. Ask how many straight lines go through the point with coordinates (3, 6). [An infinite number] Ask pupils to give as many **different** equations as they can for lines that go through (3, 6).

Lesson 1: without the Flash resource

Starter about 10 minutes **T12L1teacher1** **T12L1teacher2** **T12L1teacher3** *As* x *changes, what happens to* y*? How can you write this relationship in words? How can you write it as an equation?* *What do you notice about the* y *value of each point? If* x *is a whole number, explain why* y *must also be a whole number. What would happen if* x *were* ***not*** *a whole number?* *What symmetry is shown, and how does this explain why the equations of the lines are* y = 2x *and* x = 2y*?* *Given the equation is* y = 2x*, why is the sum of integer pairs of ordinates always a multiple of 3? What if the equation had been* y = 3x*? Or* y = 4x*, etc?* [All coordinates that lie on the line y = 2x must be of the form (a, 2a), giving a sum of 3a. If a is an integer, 3a must be a multiple of 3. Similarly if the ordinates are integers, the sum of the ordinates of points that lie on $y = 3x$ must be multiples of 4, and the sum of the ordinates of points that lie on $y = nx$ must be multiples of $n + 1$.]	Start the lesson by showing the OHT/whiteboard slide: *Straight lines, teacher sheet 1* **(T12L1teacher1)** Say that the straight line goes through the point (9, 9) and ask for coordinates of other points on the line (including (0, 0)). If appropriate, mark the coordinates of a few of these points. What is an equation of the straight line? Label the line $y = x$, discussing different ways of writing this equation. Now ask which point on the line $y = x$ has coordinates that **sum to 12**. Mark the point (6, 6) with its coordinates, if possible in red or some other colour, on the graph. Now show the OHT/whiteboard slide: *Straight lines, teacher sheet 2* **(T12L1teacher2)** Ask the pupils for the equations of the two black lines, discussing how they know. Discuss different ways of writing the equations, but label using the form $y = 2x$ and $y = \frac{1}{2}x$. Ask which point on the line $y = 2x$ has coordinates that sum to 12. Mark the point (4, 8). Repeat with the line $y = \frac{1}{2}x$. Mark the point (8, 4). Finally, show the OHT/whiteboard slide: *Straight lines, teacher sheet 3* **(T12L1teacher3)** Repeat all the stages above, labelling the lines $y = 3x$ and $y = \frac{1}{3}x$ and marking the points (3, 9) and (9, 3).
Group activity about 10 minutes **T12L1pupil1** *Think about the point that has coordinates that sum to 12 on each of the lines* y = x*,* y = 2x*,* y = 3x*,* y = 4x *and* y = 5x*. Only the point on the line* y = 4x *has non-integer values. Is there a reason for this?* [As before, the sum of integer ordinates on the line $y = nx$ must be multiples of $n + 1$. So $y = 4x$ has points with sums that are multiples of 5, which is the only one that is not a factor of 12.] *Do any other straight lines with equations of the form* y = mx *have integer values for the point that has coordinates that sum to 12?* [y = 11x, y = 0x (i.e. y = 0)]	Pupils work in groups on the worksheet: *Twelve sum* **(T12L1pupil1)** (Note that some pupils may write the incorrect equations $y = 4x$ and $y = \frac{1}{4}x$, assuming the 'pattern' started continues.) More able pupils should be asked if they can work out which point on the line $y = 4x$ has coordinates that sum to 12. [Answer: (2.4, 9.6)]

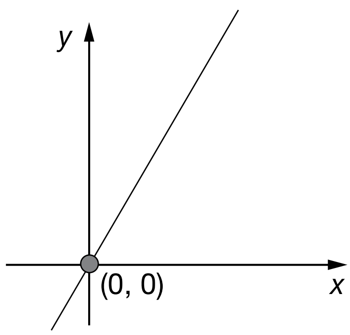
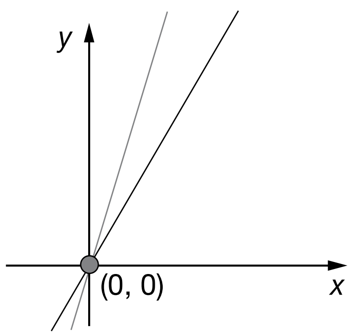
Mini-plenary about 5 minutes **T12L1teacher4** *Why is the equation of the* x*-axis not* x = 0*? Why is the equation of the* y*-axis not* y = 0*?*	Briefly discuss their findings, then show the OHT/whiteboard slide: *Twelve sum summary* **(T12L1teacher4)** Ask pupils what the points with coordinates shown represent, and then ask them what they notice about all these points. Draw a straight line, if possible in red or some other colour, through the points, extending it to intersect both axes. What can you say about all the points on this line? Lead to the equation $x + y = 12$, or whichever form you prefer, discussing alternative notation as appropriate. Discuss other points on the line, e.g. what are the coordinates of the points where this straight line intersects the x-axis and the y-axis? Choose a few other points on the red line, and ask pupils to check that the x value and the y value do add to 12.
Assessment activity about 15 minutes **T12L1assess1** **T12L1assess2** **T12L1assess3**	Give out the assessment sheets as follows: Level 4 pupils: *Ten sum, sheet 1* **(T12L1assess1)** Level 5 pupils: *Ten sum, sheet 2* **(T12L1assess2)** Level 6 pupils: *Ten sum, sheet 3* **(T12L1assess3)**
Plenary about 10 minutes *How can you tell from straight-line equations whether two lines will be parallel? Why is it helpful to write both equations in the form* y = mx + c *to decide if they are parallel?* *What do you know about the gradients of lines that are perpendicular?* *Will the line* y = 9 + x *cross/intersect the line* x + y = 9*? How can we be certain?* [All non-parallel lines must intersect somewhere.] *Where will they cross?* [At (0, 9)] *How can you tell?*	Ask pupils to think about all the points whose coordinates sum to 9. What can they tell you about these points? Discuss the straight line formed and how it relates to $x + y = 12$ and $x + y = 10$, e.g. that these lines are parallel. Is the straight line with equation $x + y = 9$ perpendicular to the straight line with equation $y = x$? How do they know? Now ask if the point (4.7, 5.3) is on the line $x + y = 9$. How do they know? Show pupils the four equations $x + y = 9$, $y + x = 9$, $y = 9 + x$ and $y = 9 – x$. Which is the odd one out? Why? [$y = 9 + x$; the other three are different ways of labelling the same equation.] Now write the coordinates (3, 6) on the board. Ask how many straight lines go through the point with coordinates (3, 6). [An infinite number] Ask pupils to give as many **different** equations as they can for lines that go through (3, 6).

LESSON 2: *THREE LINES*

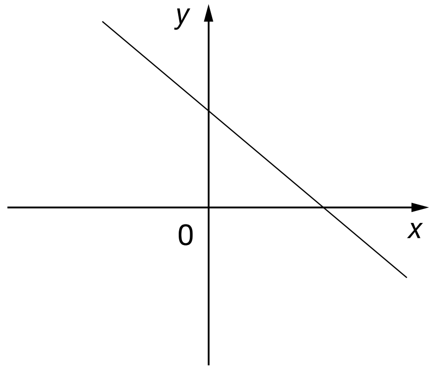
Resources

- The Flash resource:
  Algebra task
  should be loaded before the lesson to be shown on a large screen monitor or electronic whiteboard. Teachers who do not have access to the technology required will need to use any resources available to show lines rotating and translating, e.g. overlapping OHTs or board rules turned sideways
- Assessment sheets for pupils:
  Each pupil needs one copy of each of the following sheets:
  Level 4/5/6 pupils: *Reflecting triangle* **(T12L2assess1)**
  Level 5/6 pupils: *Rotating triangle* **(T12L2assess2)**
- Mirrors, tracing paper and protractors for the assessment (optional)
- Squared paper for the assessment as needed
- Graph or squared paper for any rough working
- Calculators may be used if needed, but not graphical calculators for the assessment activity

Lesson 2: with the Flash resource

Starter about 15 minutes **Flash demonstration** ***Algebra task*** *Could* m *be a negative number?* [No, since the line would then have a negative gradient and the line shown has a positive gradient.] *Where will the lines* y = 11x *and* y = 7x *meet? Which will be the steeper?* *Will the straight lines with equations* $y = \frac{1}{2}x$, $y = 0.3x$, $y = -5x$ *go through the origin? What about the line* $3x = y$ *or* $x = \frac{1}{4}y$*?* *What does the word 'gradient' mean? How do you find the gradient of a straight line?*	Start by showing **demonstration 3**. Say that the only thing that we know about this straight line is that it goes through the point (0, 0). What must be true about the equation of this straight line? [It is of the form $y = mx$, e.g. $y = 2x$, $y = 3x$, etc., but we do not know what value m has.]  Click the 'forwards' button to see a line with a greater gradient that also goes through the origin, i.e. Ask what is the same and what is different about these straight lines. [The same: they both go through the origin; different: gradients of the lines.]  Which has the steeper gradient? What does that tell us about the equations of these lines? [m is greater for the second line than for the first.] Now click again, and tell the pupils that this line is the line $y = x$. By clicking on one of the rotation buttons, show the line rotating anti-clockwise or clockwise about (0, 0). Stop the rotation at any point, but keep the gradient positive for now, and ask the pupils to suggest what the equation of the line might be. Ask volunteers to make the line show, for example, $y = 5x$, or $y = 50x$, or $y = 500x$.

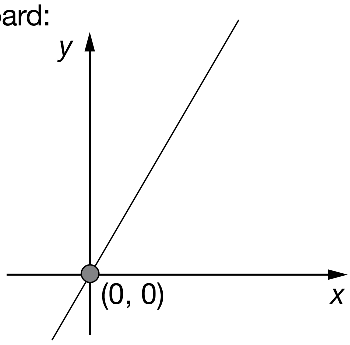
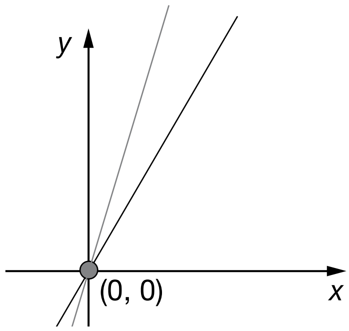
Why is the equation of the x*-axis* y *= 0 and not* x *= 0?* *Why is the equation of the* y*-axis* x *= 0 and not* y *= 0?*	Ask where the line with equation $y = 0x$ is. Discuss the fact that this is more usually written as $y = 0$ and that a gradient of zero means the line is horizontal. Discuss why, when the value of m is very large, the line is very close to the y-axis, but does not actually 'reach' the axis. [If $y = 1000000000000000000x$ for example, then $x = y \div 1000000000000000000$. As the gradient m increases, $y \div m$ tends to zero (though never gets there). Therefore $x = 0$ is the limiting case (or has the biggest gradient you could possibly try and imagine).] Ask where the line $y = -5x$ would be, or $y = -50x$, etc. What about $y = -\frac{1}{2}x$? Or $y = -\frac{1}{50}x$? When ready, progress to the next screen, where you can choose a value for m, e.g. $y = 2x$ (for now, make m a positive integer up to 5). The correct line will appear, with its equation. Click the 'forwards' button to see the question: Where will, e.g. $y = -2x$, be? (The demonstration changes only the sign, not the value of m previously entered.) Discuss, then click the 'forwards' button to show this line. Repeat as many times as you wish, discussing the symmetry.
Group activity about 5 minutes *What is the reflective symmetry linking* $y = 2x$ *and* $y = \frac{1}{2}x$*;* $y = 3x$ *and* $y = \frac{1}{3}x$*?* [Reflection in the line $y = x$] *If I showed you the lines* $y = x$ *and* $y = \frac{1}{7}x$*, where would you expect to find* $y = 7x$*?* [Reflection of $y = \frac{1}{7}x$ in $y = x$] *What does 'perpendicular' mean? Which pairs of lines are perpendicular?* [$y = 8x$ and $y = -\frac{1}{8}x$; $y = \frac{1}{8}x$ and $y = -8x$]	Pupils work in pairs, taking it in turn for one to be a 'computer screen' and the other to be the operator working at the 'computer'. The teacher asks all operators to position their 'computer' so that the arms show the equation $y = x$. Now show $y = 2x$, $y = 3x$ and so on. Swap roles. Make your 'computer' show $y = x$. Now show $y = \frac{1}{2}x$, $y = \frac{1}{3}x$, and so on. Swap roles. Make your 'computer' show $x = 0$. Swap roles. Make your 'computer' show $y = 0$. Swap roles. Make your 'computer' show $y = x$, then $y = 3x$, then $y = -3x$. Swap roles. Make your 'computer' show $y = -8x$. Now show $y = 8x$, $y = \frac{1}{8}x$, $y = -\frac{1}{8}x$. Swap roles. Your 'computer' needs to show two different lines, so each arm should be a line. Make your 'computer' show $y = 3x$ and $y = -3x$ and so on. Note that throughout this activity, more able groups can be asked to show equations in different forms, e.g. $x = 6y$.

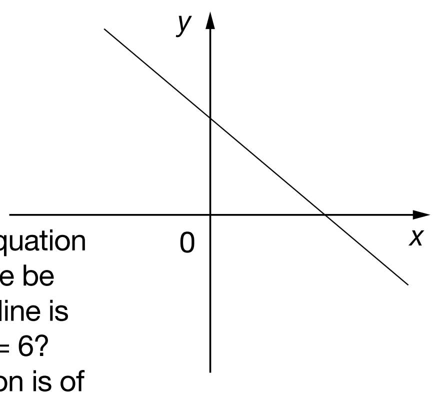
Group discussion about 5 minutes **Flash demonstration** ***Algebra task***	Now show **demonstration 4**.  Ask what we know about the equation of the straight line. Why can't we be certain that the equation of the line is of the form $x + y = c$, e.g. $x + y = 6$? What must be true if the equation is of this form? [The distance from the origin to the point where the line cuts the x-axis is the same as the distance from the origin to the point where the line cuts the y-axis and the scales on both axes are the same.] Tell them that the line has equation $x + y = 10$. Use the translation button to show the line moving; stop where you wish and ask the pupils what they think the equation of the line will be. Show some lines of the form $x + y = -c$ and discuss their equations with the class. Ask where the line with equation $x + y = 0$ will be. What is another way of writing this equation? [$y = -x$] Discuss the equivalence of these equations. When ready, click the 'forwards' button, and you will be asked to type in a numerical value, i.e. $x + y = \ldots$ (note that this is restricted to an integer value between –20 and 20 inclusive). Ask the pupils what the line with equation $x + y = -\ldots$ will look like, then click. Viewing both lines on screen at once helps pupils to make mathematical connections.
Assessment activity about 25 minutes **T12L2assess1** **T12L2assess2** *Note that in a further lesson, teachers may wish to revisit this triangle, asking how we can be certain it is right-angled and what the area of the triangle is. More able pupils can use Pythagoras, then check, by using 'counting' involving subtraction methods, that the area is exactly 8 square units.*	Give out the assessment sheets (all pupils do the same sheets though lower ability pupils may not progress beyond the first): Level 4/5/6 pupils: *Reflecting triangle* **(T12L2assess1)** Level 5/6 pupils: *Rotating triangle* **(T12L2assess2)** Note that the teacher should judge whether pupils should be given both at the same time, or whether it would be better to give the second, which is more challenging, after the first. Teachers should also judge whether pupils working at the lower levels would benefit from using mirrors and/or tracing paper, though this should be taken into account when assessing the work. Protractors may be also be needed, as well as spare squared paper. Suggest that pupils start the second sheet with easier rotations, e.g. multiples of 90°, but encourage more able pupils to do complex rotations, on squared paper if needed.

Plenary about 5 minutes **Flash demonstration** ***Algebra task*** *Describe the transformation from* $y = 2x$ *to* $y = 2x + 5$. *Where will the line* $y = 2x + 5$ *cross the* y-*axis?* *What transformation would change* $y = 2x$ *to* $y = 2x - 5$? *Where would* $y = 2x - 5$ *cross the* y-*axis?* *What line is parallel to* $y = 2x$ *and crosses the* y-*axis at (0, 4)? What would the equation be if it crossed at (0, –4) instead? Or (0, 3)? Or (0, –3)?* *When a straight-line equation is written in the form* $y = mx + c$, *what does the* c *value tell us?* *Do you agree that multiplying the gradient by –1 results in a reflection in the* y-*axis (or the* x-*axis)? (E.g.* $y = 2x$ *changing to* $y = -2x$*)* *Do you agree that changing to the reciprocal of the original gradient results in a reflection in the line* $y = x$? *(E.g.* $y = 2x$ *changing to* $y = \frac{1}{2}x$*)* *Do these facts help us explain why* $y = 2x$ *is perpendicular to* $y = -\frac{1}{2}x$?	**Show demonstration 5**. The line $y = 2x$ is shown. Ask the pupils to work out where the line $y = 2x + 5$ will be. Discuss the effect of adding a constant, linking if appropriate to work already done on equations of the form $y = mx + c$. Successive challenges are given, recapping work done earlier in the lesson. Note that some of the challenges are difficult and may not be appropriate for any but the more able pupils. Finish the lesson by asking pupils to use their arms (to be 'computer screens') and show the line with equation $y = 2x$. Then ask them to show $y = 2x - 3$. (Other transformations can be attempted depending on the ability level of the group, even starting with $y = x^2$.)

00588-2009PDF-EN-12

Lesson 2: without the Flash resource

Starter about 15 minutes	Start by showing this sketch on the board:  Say that the only thing that we know about this straight line is that it goes through the point (0, 0).
Could m *be a negative number?* [No, since the line would then have a negative gradient and the line shown has a positive gradient.] *Where will the lines* $y = 11x$ *and* $y = 7x$ *meet? Which will be the steeper?* *Will the straight lines with equations* $y = \frac{1}{2}x$, $y = 0.3x$, $y = -5x$ *go through the origin? What about the line* $3x = y$ *or* $x = \frac{1}{4}y$*?* *What does the word 'gradient' mean? How do you find the gradient of a straight line?*	What must be true about the equation of the straight line? [It is of the form $y = mx$, e.g. $y = 2x$, $y = 3x$, etc., but we do not know what value m has.] Now, on the same sketch but in a different colour, draw another line that goes through the origin but that has a greater gradient, e.g.  Ask what is the same and what is different about these straight lines. [The same: they both go through the origin; different: gradients of the lines.] Which has the steeper gradient? What does that tell us about the equations of these lines? [m is greater for the second line than for the first.] Now, using overlapping OHTs, or a metre rule, or whatever resources are available, demonstrate the line $y = x$ rotating about (0, 0). Ask pupils to tell you when they think the 'line' has equation, for example, $y = 5x$, or $y = 50x$, or $y = 500x$.
Why is the equation of the x*-axis* $y = 0$ *and not* $x = 0$*?*	Ask where the line with equation $y = 0x$ is. Discuss the fact that this is more usually written as $y = 0$ and that a gradient of zero means the line is horizontal.
Why is the equation of the y*-axis* $x = 0$ *and not* $y = 0$*?*	Discuss why, when the value of m is very large, the line is very close to the y-axis, but does not actually 'reach' the axis. [If $y = 1000000000000000000x$ for example, then $x = y \div 1000000000000000000$. As the gradient m increases, $y \div m$ tends to zero (though never gets there). Therefore $x = 0$ is the limiting case (or has the biggest gradient you could possibly try and imagine).] Ask where the line $y = -5x$ would be, or $y = -50x$, etc. What about $y = -\frac{1}{2}x$? Or $y = -\frac{1}{50}x$? When ready, sketch the line $y = 2x$. Ask the pupils where the line $y = -2x$ will be and show on the graph. Change the equations as many times as you wish, discussing symmetry.

Group activity about 5 minutes *What is the reflective symmetry linking* $y = 2x$ *and* $y = \frac{1}{2}x$*;* $y = 3x$ *and* $y = \frac{1}{3}x$*?* [Reflection in the line $y = x$] *If I showed you the lines* $y = x$ *and* $y = \frac{1}{7}x$*, where would you expect to find* $y = 7x$*?* [Reflection of $y = \frac{1}{7}x$ in $y = x$] *What does 'perpendicular' mean? Which pairs of lines are perpendicular?* [$y = 8x$ and $y = -\frac{1}{8}x$; $y = \frac{1}{8}x$ and $y = -8x$]	Pupils work in pairs, taking it in turn for one to be a 'computer screen' and the other to be the operator working at the 'computer'. The teacher asks all operators to position their 'computer' so that the arms show the equation $y = x$. Now show $y = 2x$, $y = 3x$ and so on. Swap roles. Make your computer show $y = x$. Now show $y = \frac{1}{2}x$, $y = \frac{1}{3}x$, and so on. Swap roles. Make your 'computer' show $x = 0$. Swap roles. Make your 'computer' show $y = 0$. Swap roles. Make your 'computer' show $y = x$, then $y = 3x$, then $y = -3x$. Swap roles. Make your 'computer' show $y = -8x$. Now show $y = 8x$, $y = \frac{1}{8}x$, $y = -\frac{1}{8}x$. Swap roles. Your 'computer' needs to show two different lines, so each arm should be a line. Make your 'computer' show $y = 3x$ and $y = -3x$ and so on. Note that throughout this activity, more able groups can be asked to show equations in different forms, e.g. $x = 6y$.
Group discussion about 5 minutes	Show this sketch on the board:  Ask what we know about the equation of the straight line. Why can't we be certain that the equation of the line is of the form $x + y = c$, e.g. $x + y = 6$? What must be true if the equation is of this form? [The distance from the origin to the point where the line cuts the x-axis is the same as the distance from the origin to the point where the line cuts the y-axis and the scales on both axes are the same.] Tell them that the line has equation $x + y = 10$. Now, using whatever resources are available, show the line being translated (i.e. staying parallel to the original line). Stop where you wish and ask the pupils what they think the equation of the line might be. Show some lines of the form $x + y = -c$, discussing their equations. Ask where the line with equation $x + y = 0$ will be. What is another way of writing this equation? [$y = -x$] Discuss the equivalence of these equations. Look again at the line with equation $x + y = 10$. Ask the pupils where the line with equation $x + y = -10$ will be and show this on the same graph. Discuss the relationship between them.

00588-2009PDF-EN-12

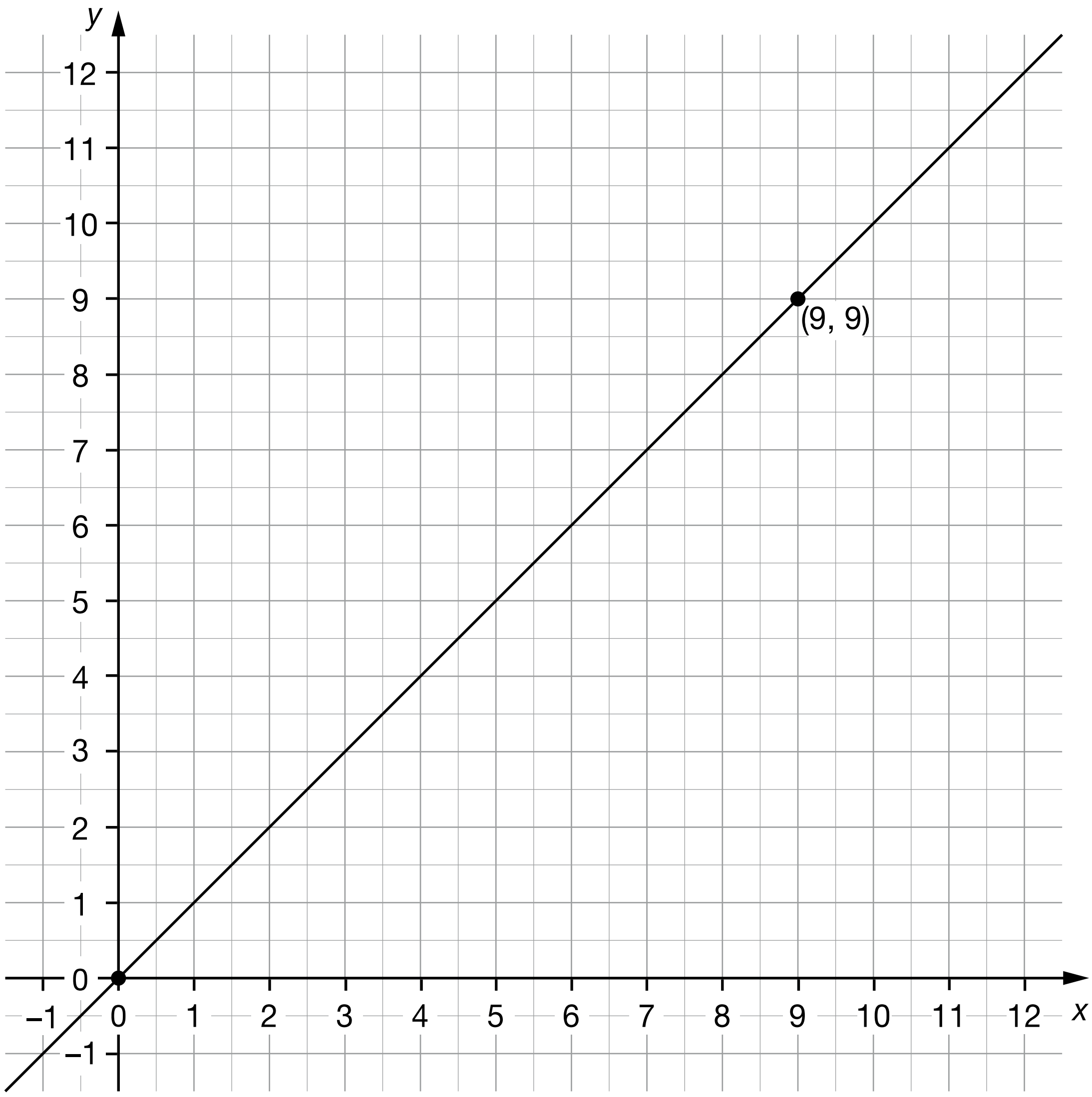
Assessment activity about 25 minutes **T12L2assess1** **T12L2assess2** *Note that in a further lesson, teachers may wish to revisit this triangle, asking how we can be certain it is right-angled and what the area of the triangle is. More able pupils can use Pythagoras, then check, by using 'counting' involving subtraction methods, that the area is exactly 8 square units.*	Give out the assessment sheets (all pupils do the same sheets though lower ability pupils may not progress beyond the first): Level 4/5/6 pupils: *Reflecting triangle* **(T12L2assess1)** Level 5/6 pupils: *Rotating triangle* **(T12L2assess2)** Note that the teacher should judge whether pupils should be given both at the same time, or whether it would be better to give the second, which is more challenging, after the first. Teachers should also judge whether pupils working at the lower levels would benefit from using mirrors and/or tracing paper, though this should be taken into account when assessing the work. Protractors may also be needed, as well as spare squared paper. Suggest that pupils start the second sheet with easier rotations, e.g. multiples of 90°, but encourage more able pupils to do complex rotations, on squared paper if needed.
Plenary about 5 minutes *Describe the transformation from* $y = 2x$ *to* $y = 2x + 5$. *Where will* $y = 2x + 5$ *cross the* y*-axis?* *What transformation would change* $y = 2x$ *to* $y = 2x - 5$*?* *Where will* $y = 2x - 5$ *cross the* y*-axis?* *What line is parallel to* $y = 2x$ *and crosses the* y*-axis at (0, 4)? What would the equation be if it crossed at (0, –4) instead? Or (0, 3)? Or (0, –3)?* *When a straight-line equation is written in the form* $y = mx + c$*, what does the* c *value tell us?* *Do you agree that multiplying the gradient by –1 results in a reflection in the* y*-axis (or the* x*-axis)? (E.g.* $y = 2x$ *changing to* $y = -2x$*)* *Do you agree that changing to the reciprocal of the original gradient results in a reflection in the line* $y = x$*? (E.g.* $y = 2x$ *changing to* $y = \frac{1}{2}x$*)* *Do these facts help us explain why* $y = 2x$ *is perpendicular to* $y = -\frac{1}{2}x$*?*	Show the pupils the line $y = 2x$. Ask the pupils to work out where the line $y = 2x + 5$ will be. Discuss the effect of adding a constant, linking if appropriate to work already done on equations of the form $y = mx + c$. Remove, or rub out, the line $y = 2x$, leaving only the line $y = 2x + 5$. Ask where the line $y = 3x + 5$ will be. Remove, or rub out, the line $y = 2x + 5$, leaving only the line $y = 3x + 5$. Ask where the line $y = 3x - 5$ will be. Continue, using different equations according to the ability level of the class. More able pupils can be challenged with equations such as $10y + x = -5$. Finish the lesson by asking pupils to use their arms (to be 'computer screens') and show the line with equation $y = 2x$. Then ask them to show $y = 2x - 3$. (Other transformations can be attempted depending on the ability level of the group, even starting with $y = x^2$.)

Teacher resource sheets

00588-2009PDF-EN-12

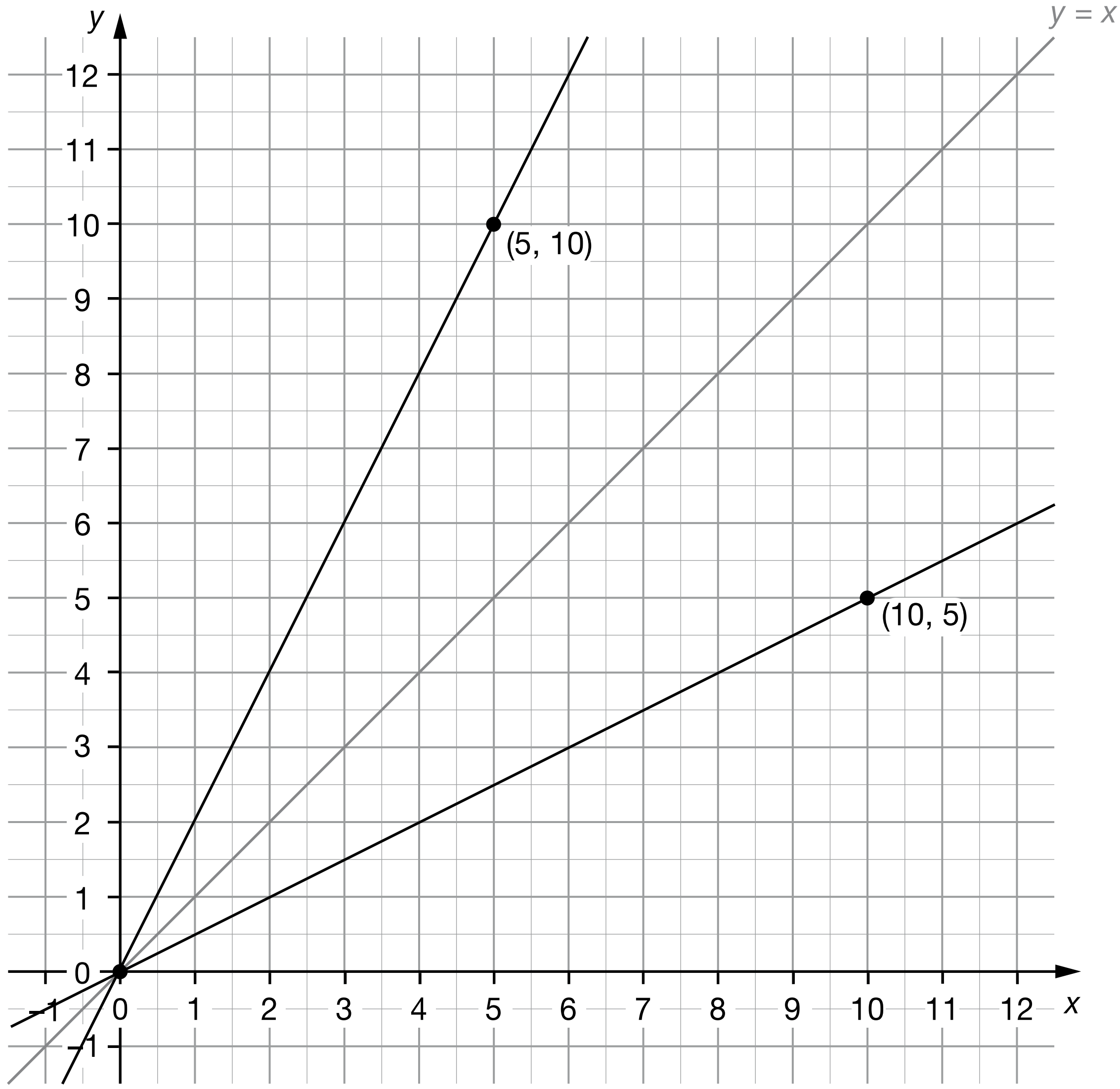
T12L1teacher1

Straight lines, teacher sheet 1
(for teachers who do not have access to the Flash resource)



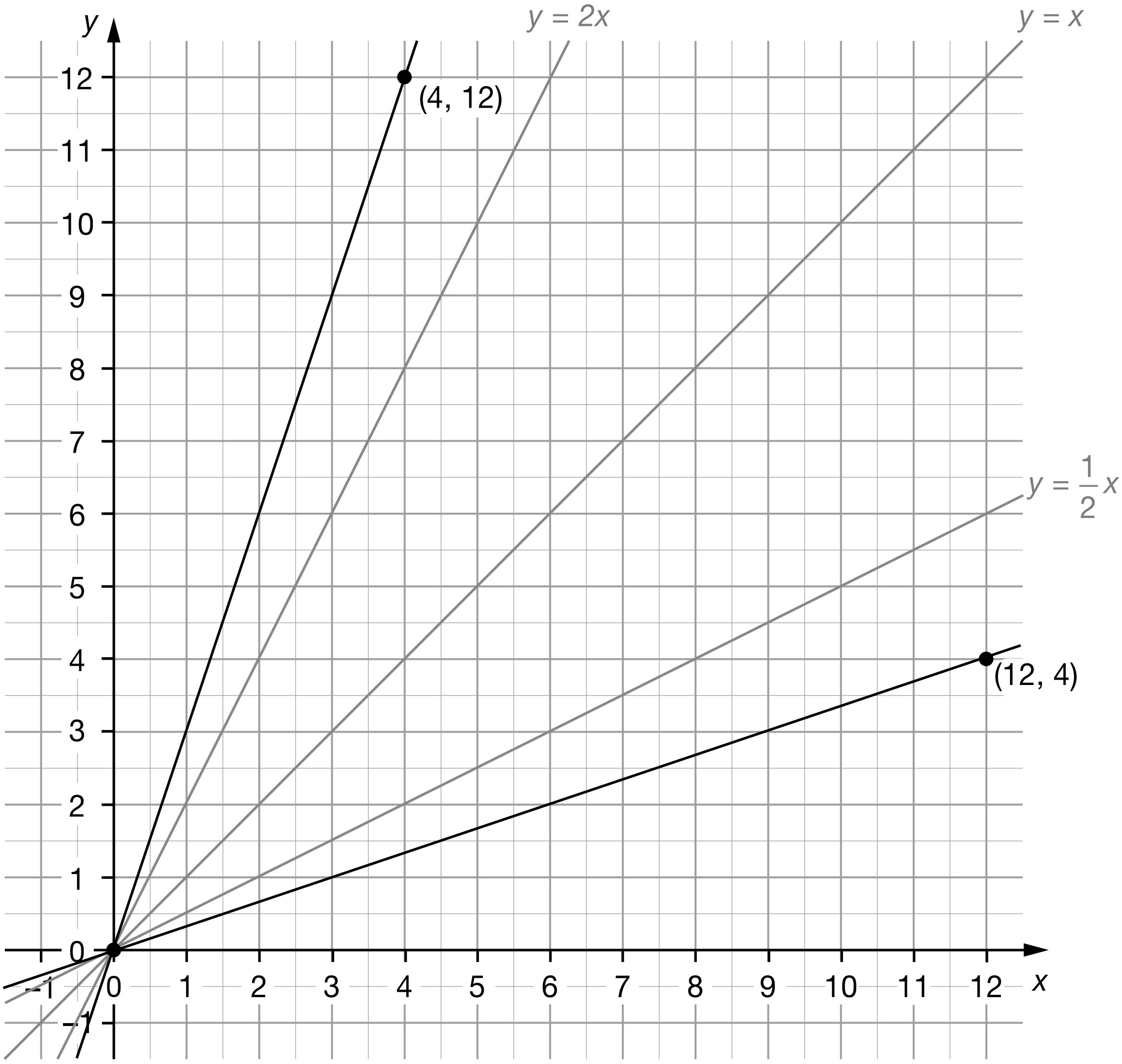
T12L1teacher2

Straight lines, teacher sheet 2
(for teachers who do not have access to the Flash resource)



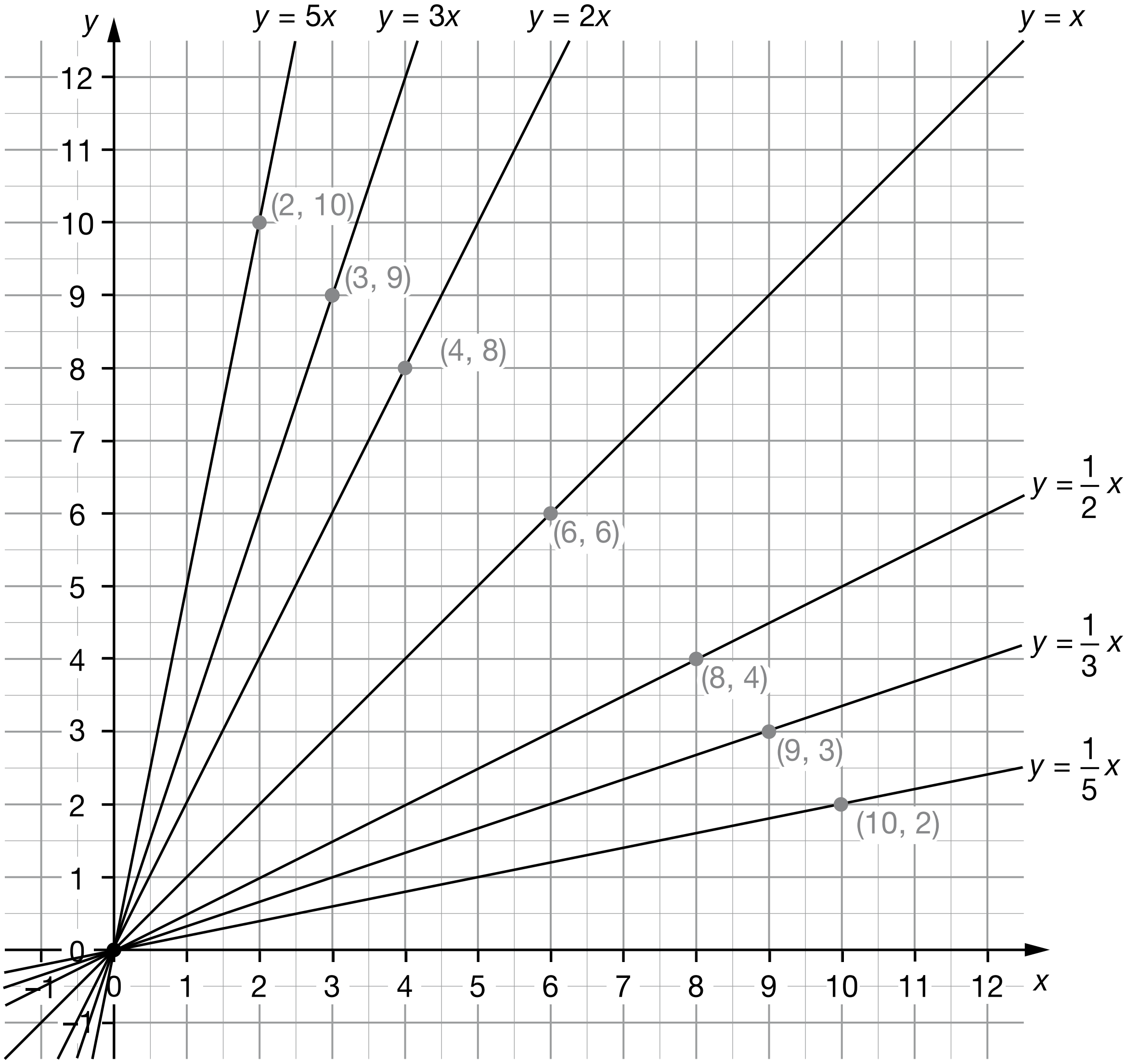
T12L1teacher3

Straight lines, teacher sheet 3
(for teachers who do not have access to the Flash resource)



T12L1teacher4

Twelve sum summary
(for teachers who do not have access to the Flash resource)



00588-2009PDF-EN-12

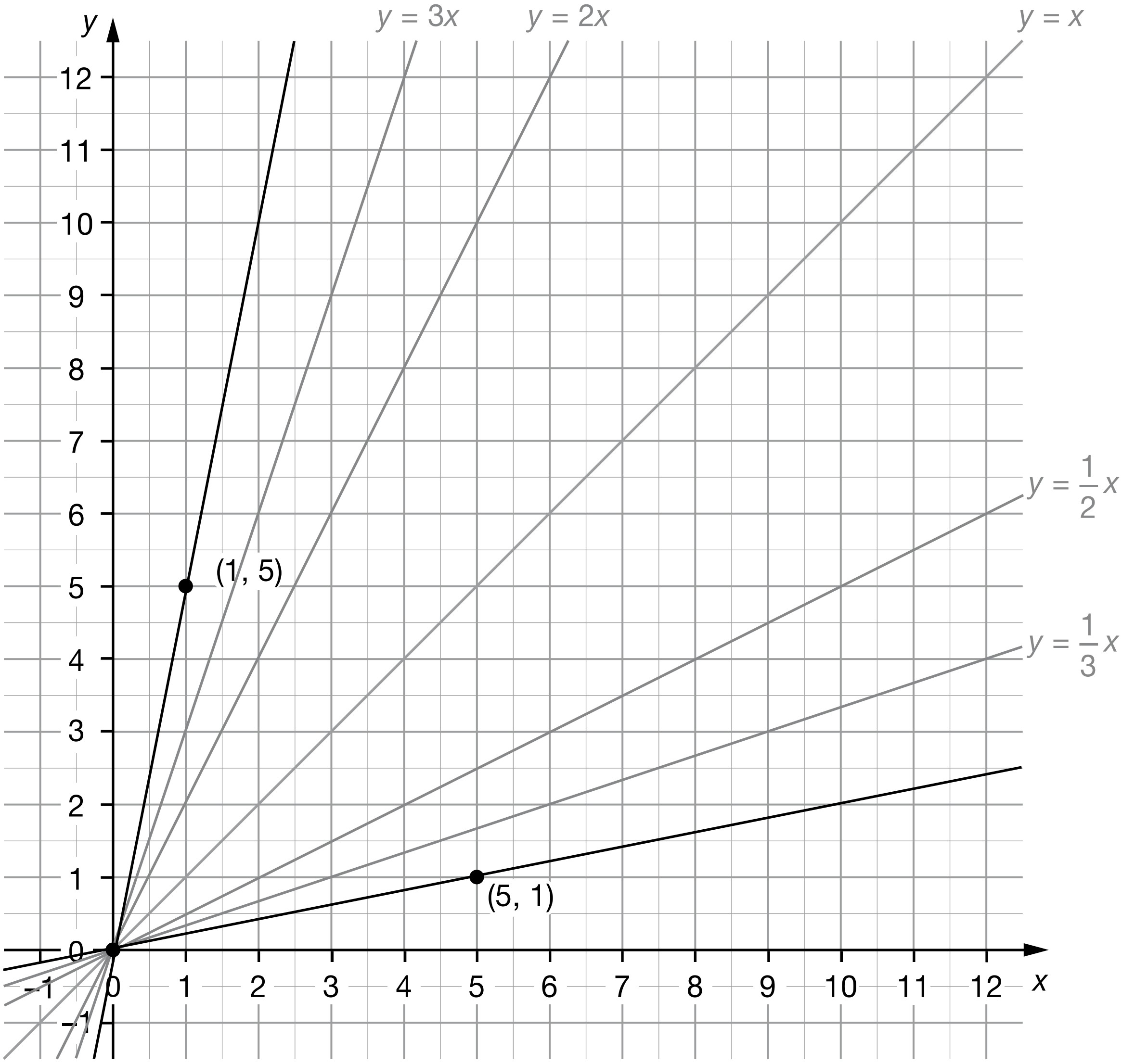
Pupil sheets

T12L1pupil1

Twelve sum Name(s): ________________________________

Write the equations of the black lines.

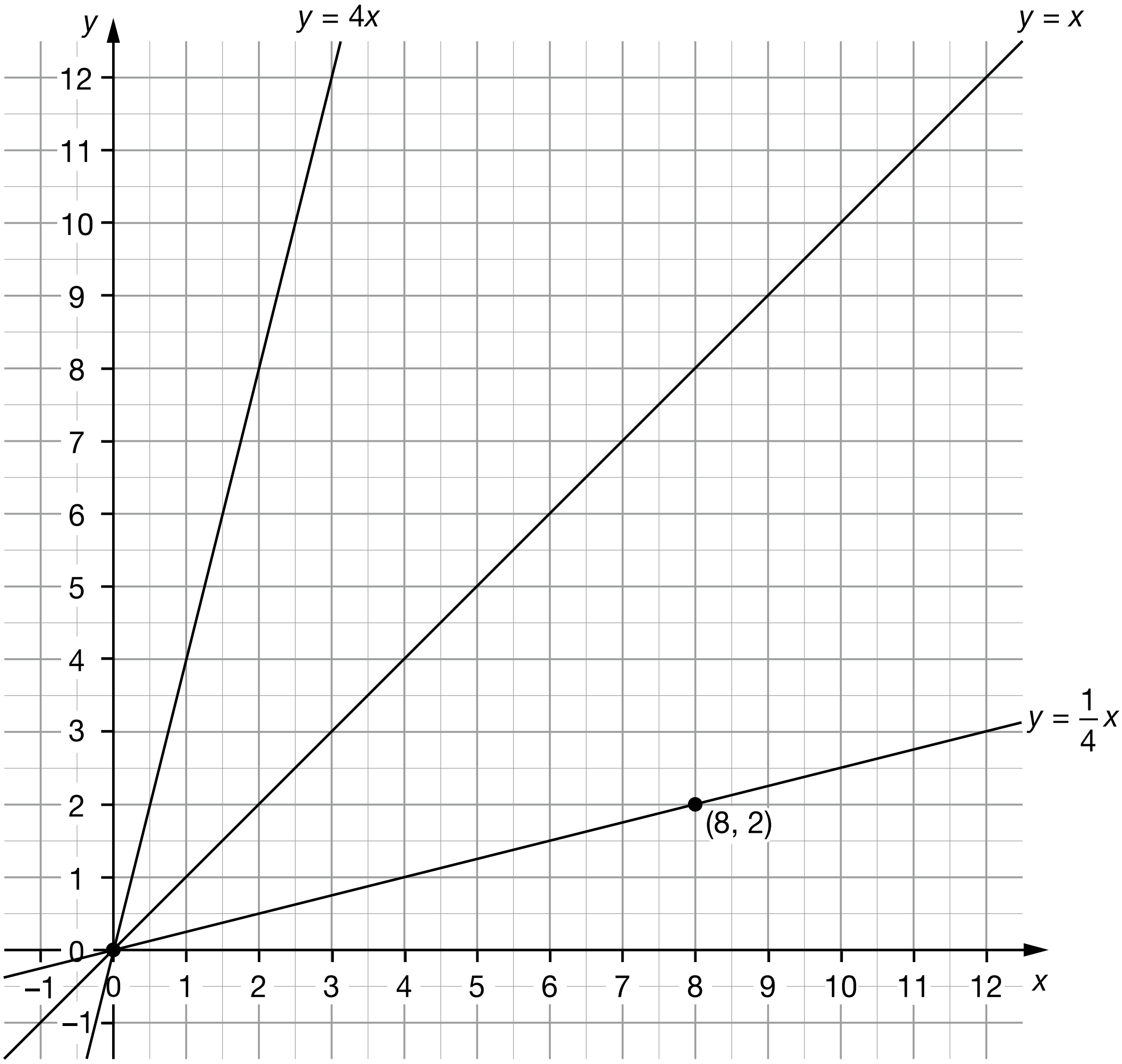
Then mark the point on each line that has **coordinates that sum to 12**
Write the coordinates next to each point.



00588-2009PDF-EN-12

T12L1assess1

Ten sum, sheet 1 Name: ________________________



On each line, label the point that has **coordinates that sum to 10**

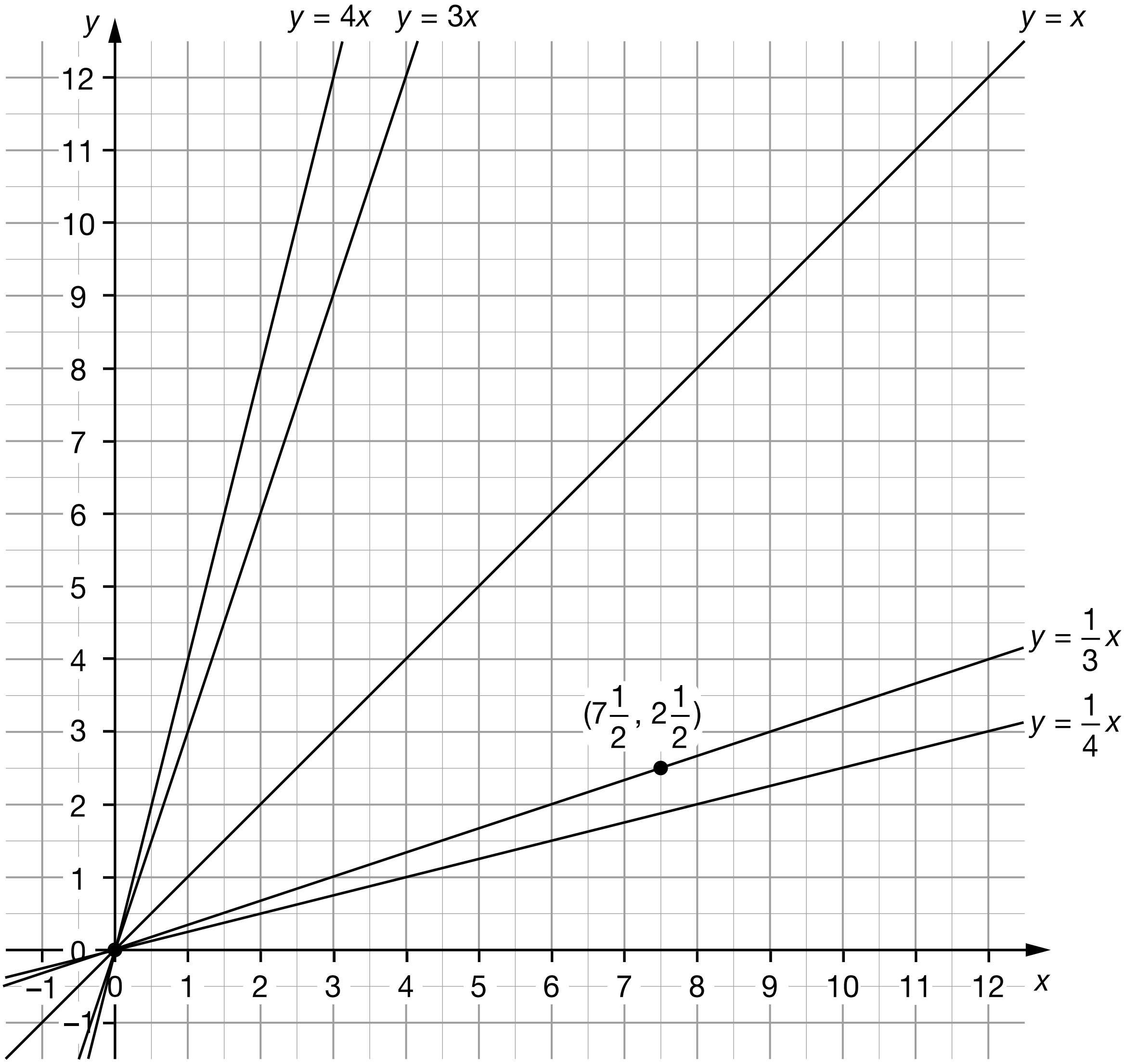
The point on the line $y = \frac{1}{4}x$ has been done for you.

Now draw a straight line through these three points.

What is an equation of this straight line?

On the graph, draw the straight line with equation $\boldsymbol{x + y = 8}$

Ten sum, sheet 2 Name: ______________________________



On each line, label the point that has **coordinates that sum to 10**

The point on the line $y = \frac{1}{3}x$ has been done for you.

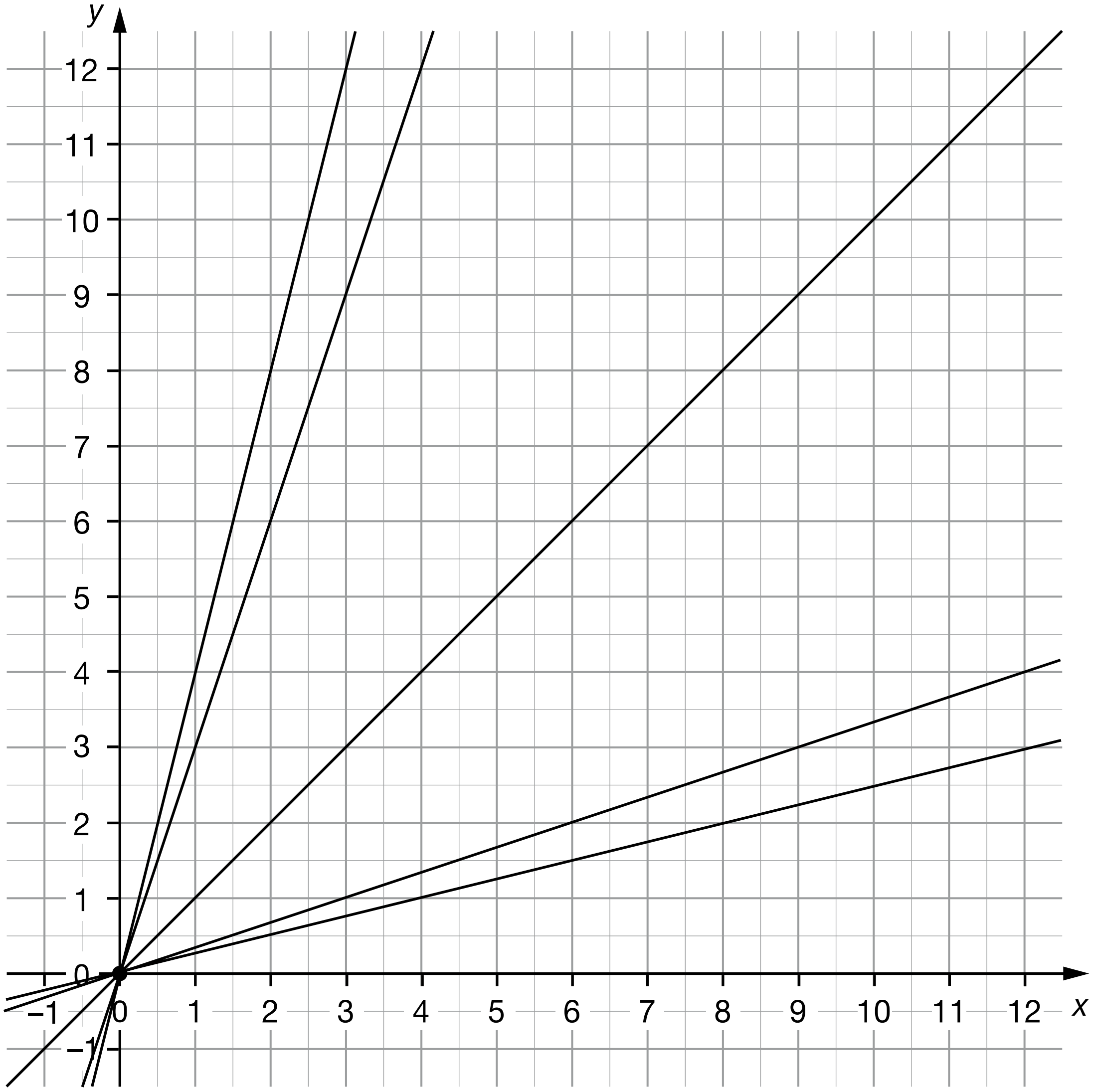
Now draw a straight line through these five points.

What is an equation of this straight line?

Where will the straight lines $y = 3x$ and $x + y = 8$ meet? How do you know?

T12L1assess3

Ten sum, sheet 3 Name: ______________________________



The graph shows five straight lines. Label the lines to show their equations.

For each line, label the point that has **coordinates that sum to 10**

What is an equation of the straight line through these points?

Where will the straight lines $y = 3x$ and $x + y = 8$ meet? How do you know?

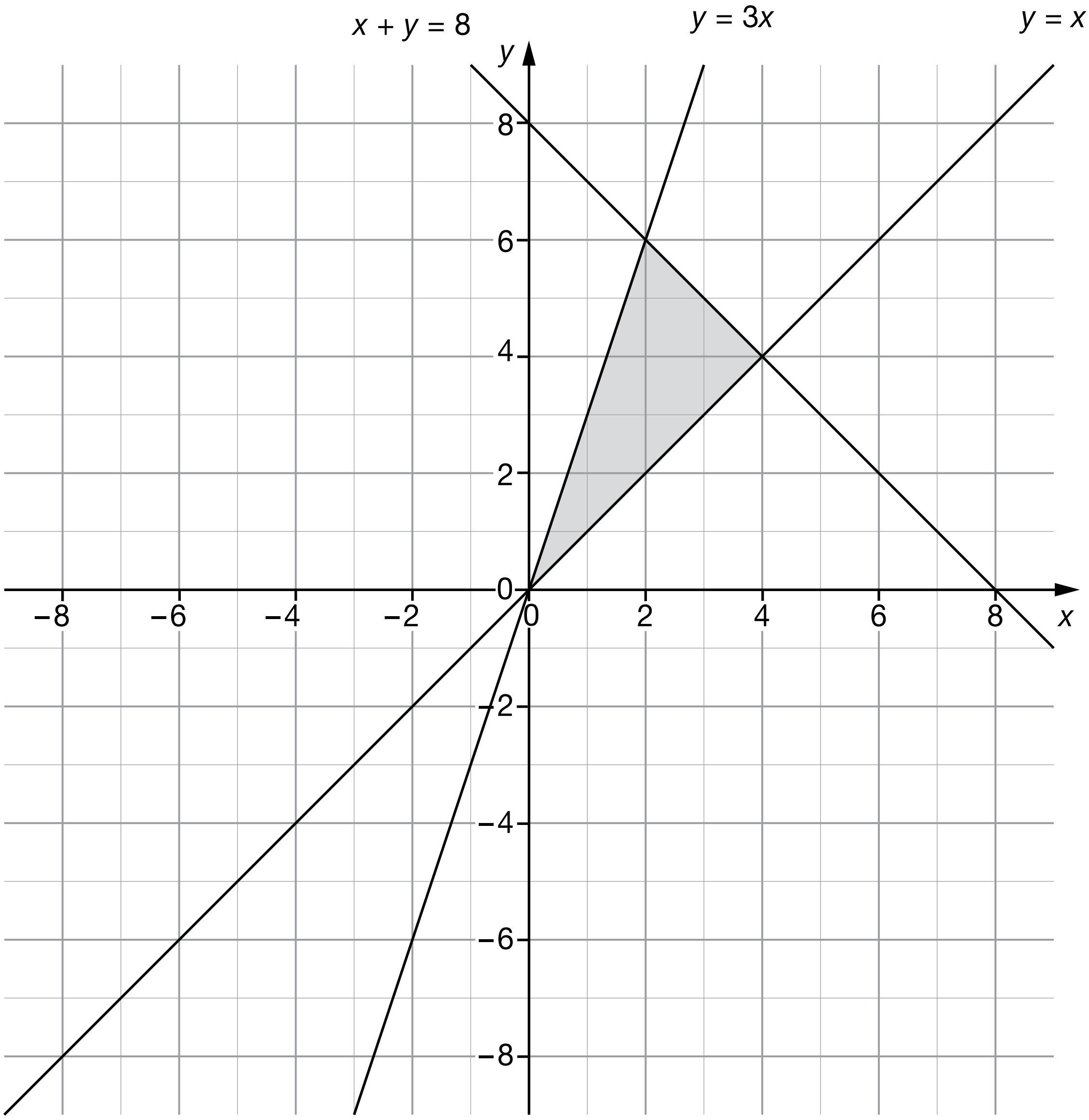
Where will the straight lines $y = 2x$ and $x + y = 21$ meet? How do you know?

T12L2assess1

Reflecting triangle Name: ______________________________

Reflect this picture in the y-axis.

Then reflect the complete picture in the x-axis.



What are the equations of the lines in your picture?

Label as many of them as you can.

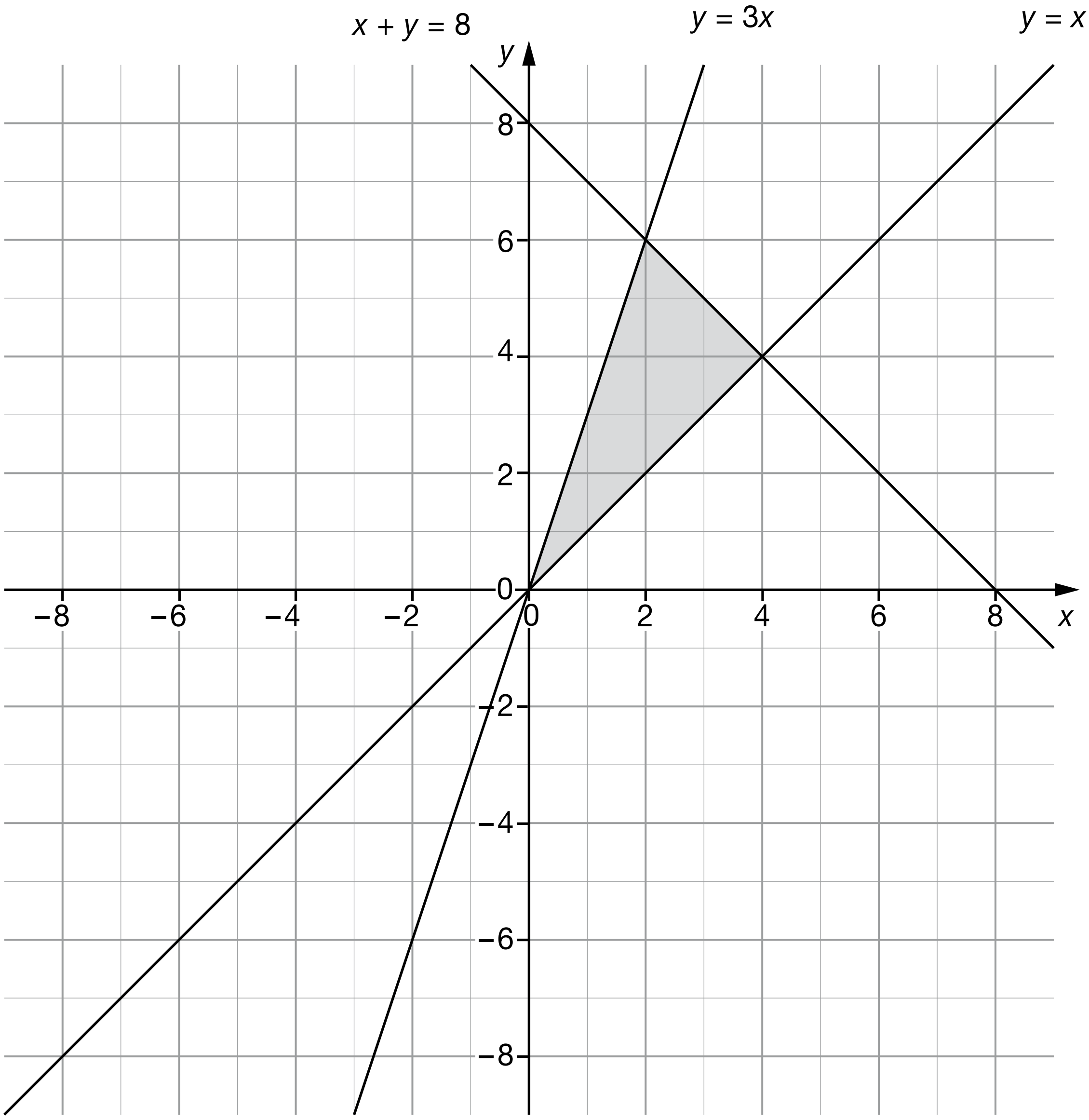
00588-2009PDF-EN-12

T12L2assess2

Rotating triangle Name: ______________________________

Using (0, 0) as the centre of rotation, rotate the shaded triangle.

Draw the new position of the triangle accurately.



What are the equations of the lines that form the new triangle?

Label as many of them as you can.

Repeat as many times as you like.

Solutions and performance indicators

Ten sum, sheet 1 (target level 4)	**T12L1assess1**
Solutions	Notes
Points (5, 5) and (2, 8) labelled and a straight line drawn through all three points on the graph	**Good** responses mark the correct points on the graph. **Better** responses also label the points with the correct coordinates.
A correct equation of the straight line, e.g. • $x + y = 10$ • $y = 10 - x$ • $x = 10 - y$	**Good** responses show understanding that the equation is linked to the fact that the coordinates sum to 10. **Better** responses give the correct equation using conventional algebraic notation.
The line $x + y = 8$ drawn on the graph, i.e. the line that passes through points (8, 0) and (0, 8)	**Good** responses plot some points with coordinates that sum to 8. **Better** responses show and label the correct straight line spanning the whole grid.
Ten sum, sheet 2 (target level 5)	**T12L1assess2**
Solutions	Notes
Points (5, 5), ($2\frac{1}{2}$, $7\frac{1}{2}$), (2, 8) and (8, 2) labelled and a straight line drawn through all five points on the graph	**Good** responses mark some of the correct points on the graph. **Better** responses label all correct points with the correct coordinates.
A correct equation of the straight line, e.g. • $x + y = 10$ • $y = 10 - x$ • $x = 10 - y$	**Good** responses show understanding that the equation is linked to the fact that the coordinates sum to 10. **Better** responses give the correct equation using conventional algebraic notation.
The point of intersection (2, 6) indicated Evidence of a correct method, e.g. • $x + y = 8$ drawn correctly on the graph • $3 \times 2 = 6$ and $2 + 6 = 8$ • 'It's the only point where the first number times 3 gives the second number and where the coordinates sum to 8' • $x + 3x = 8$ $4x = 8$ $x = 2$, so $y = 6$	**Good** responses show understanding that the point lies on the line $y = 3x$ and has coordinates that sum to 8. **Better** responses give the correct coordinates and a correct graphical or algebraic method.

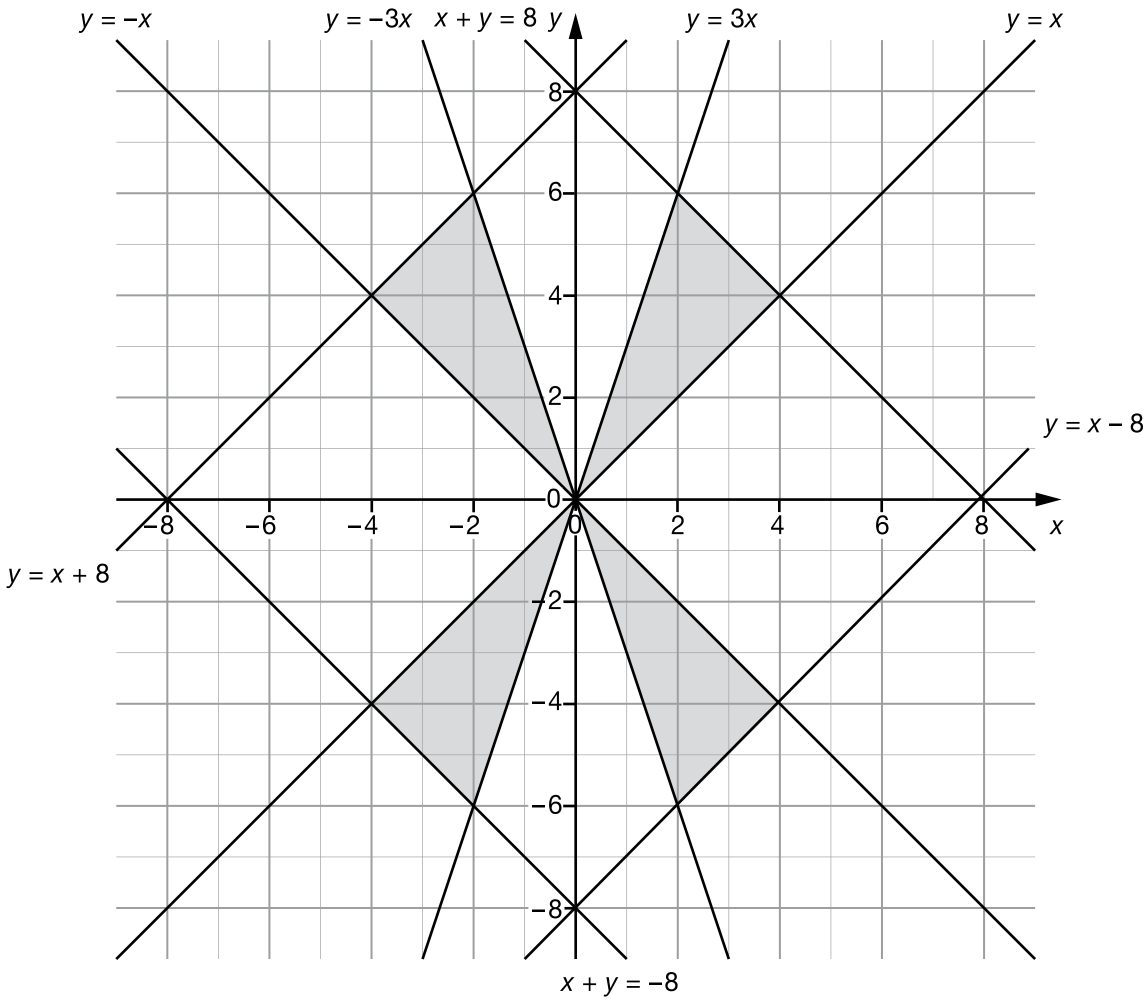
Ten sum, sheet 3 (target level 6)	**T12L1assess3**
Solutions	Notes
Points (5, 5), $(2\frac{1}{2}, 7\frac{1}{2})$, $(7\frac{1}{2}, 2\frac{1}{2})$, (2, 8) and (8, 2) labelled and a straight line drawn through all five points on the graph	**Good** responses mark some of the correct points on the graph. **Better** responses label all correct points with the correct coordinates.
A correct equation of the straight line, e.g. • $x + y = 10$ • $y = 10 - x$ • $x = 10 - y$	**Good** responses show understanding that the equation is linked to the fact that the coordinates sum to 10. **Better** responses give the correct equation using conventional algebraic notation.
The point of intersection (2, 6) indicated Evidence of a correct method, e.g. • $x + y = 8$ drawn correctly on the graph • $3 \times 2 = 6$ and $2 + 6 = 8$ • 'It's the only point where the first number times 3 gives the second number and where the coordinates sum to 8' • $x + 3x = 8$ $4x = 8$ $x = 2$, so $y = 6$	**Good** responses show understanding that the point lies on the line $y = 3x$ and has coordinates that sum to 8. **Better** responses give the correct coordinates and a correct graphical or algebraic method.
The point of intersection (7, 14) indicated Evidence of a correct method, e.g. • $2 \times 7 = 14$ and $7 + 14 = 21$ • 'It's the only point where the first number times 2 gives the second number and where the coordinates sum to 21' • $x + 2x = 21$ $3x = 21$ $x = 7$, so $y = 14$	**Good** responses show understanding that the point has a y-coordinate that is double the x-coordinate and has coordinates that sum to 21. **Better** responses give the correct coordinates and a correct algebraic method.

LESSON 1: *TWELVE SUM* **Performance indicators**

Note that performance indicators involving an element of 'Using and applying mathematics' are given in **bold**.

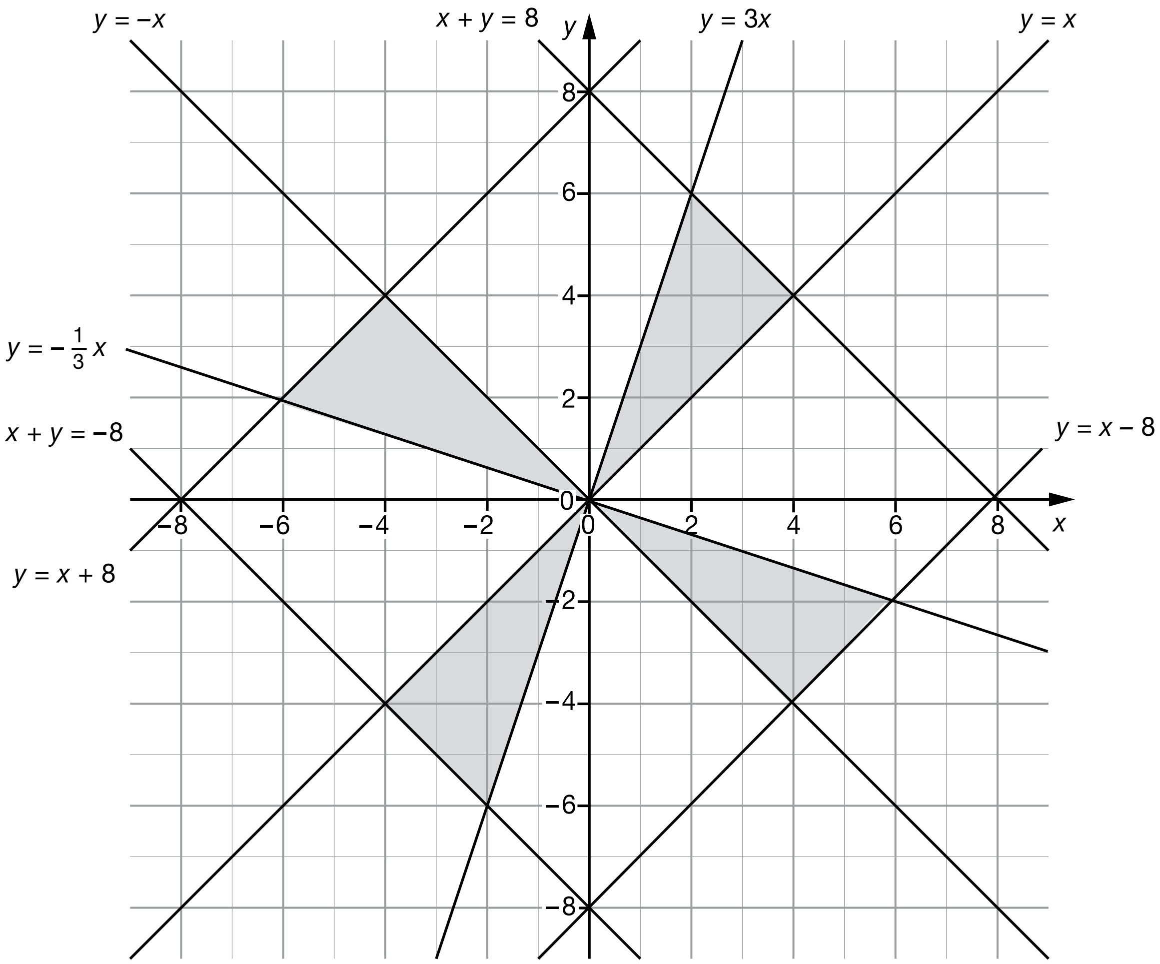
Worksheet	Performance indicators
Ten sum, sheet 1 (target level 4) **T12L1assess1**	**Level 4:** At this level, pupils are generally able to: • give correct coordinates of points on straight lines whose coordinates sum to a given value; • give a correct equation of the resultant line, e.g. of the form $x + y = 10$; • plot correct points on the line given a simple equation such as $x + y = 8$. However, they are less likely to be able to: • give correct coordinates of the point of intersection of two straight lines by drawing them on a graph, given their equations; • **show understanding that the coordinates of the point of intersection must 'work' in both equations**; • label simple straight lines correctly with their equations.
Ten sum, sheet 2 (target level 5) **T12L1assess2**	**Level 5:** At this level, pupils are generally able to: • give correct coordinates of the point of intersection of two straight lines by drawing them on a graph, given their equations; • **show understanding that the coordinates of the point of intersection must 'work' in both equations**; • label simple straight lines correctly with their equations. However, they are less likely to be able to: • give correct coordinates of the point of intersection of two straight lines without drawing them on a graph, given their equations.
Ten sum, sheet 3 (target level 6) **T12L1assess3**	**Level 6:** At this level, pupils are generally able to: • give correct coordinates of the point of intersection of two straight lines without drawing them on a graph, given their equations. However, they are less likely to be able to: • give correct coordinates of the point of intersection of two straight lines showing an algebraic method, given their equations. **Above level 6:** At these levels, pupils are generally able to: • give evidence for the performance indicators listed previously for pupils working at level 6; plus • give correct coordinates of the point of intersection of two straight lines showing an algebraic method, given their equations.

Reflecting triangle (target level 4/5/6)	**T12L2assess1**
Solutions	Notes
Diagram reflected correctly in the y-axis, then the complete picture reflected correctly in the x-axis, with lines labelled correctly with their equations, i.e. •	**Good** responses complete some correct reflections and label some lines correctly. **Better** responses complete the reflections accurately and use conventional algebraic notation to label lines correctly.



00588-2009PDF-EN-12

Rotating triangle (target level 5/6)	**T12L2assess2**
Solutions	Notes
Diagram rotated correctly about (0, 0), with lines labelled correctly with their equations, and the process repeated using angles of the pupil's choice, e.g. •	**Good** responses complete some correct rotations and label some lines correctly. **Better** responses complete several rotations accurately and use conventional algebraic notation to label lines correctly.



LESSON 2: ***THREE LINES*** **Performance indicators**

Note that performance indicators involving an element of 'Using and applying mathematics' are given in **bold**.

Worksheet	Performance indicators
Reflecting triangle (target level 4/5/6) **T12L2assess1** *Rotating triangle* (target level 5/6) **T12L2assess2**	**Level 4:** At this level, pupils are generally able to: • reflect a triangle accurately on a grid using x- and y-axes as mirror lines [a Shape, Space and Measures skill only and should not be used as evidence for the outcomes of this task within Ma2]; • recognise that extending a given line does not change its equation; • give a simple equation of a straight line based on one given, e.g. $y = -x$ given $y = x$, or $y = -3x$ given $y = 3x$, or $x + y = -8$ given $x + y = 8$. However, they are less likely to be able to: • give more than one of the simple equations of straight lines that can be easily derived from those given. **Level 5:** At this level, pupils are generally able to: • give more than one of the simple equations of straight lines that can be easily derived from those given; • rotate a triangle accurately on a grid using a simple angle such as 90° or 180° [a Shape, Space and Measures skill only and should not be used as evidence for the outcomes of this task within Ma2]. However, they are less likely to be able to: • give the equation of a straight line that is harder to derive from those given, e.g. $y = x + 8$ or $y = x - 8$; • give the equation of a straight line with a gradient not seen before, e.g. $y = -\frac{1}{3}x$. **Level 6:** At this level, pupils are generally able to: • give the equation of a straight line that is harder to derive from those given, e.g. $y = x + 8$ or $y = x - 8$; • give the equation of a straight line with a gradient not seen before, e.g. $y = -\frac{1}{3}x$. However, they are less likely to be able to: • use angles other than 90° or 180° for their rotations and **calculate associated equations of lines**. **Above level 6:** At these levels, pupils are generally able to: • give evidence for the performance indicators listed previously for pupils working at level 6; plus • use angles other than 90° or 180° for their rotations and **calculate associated equations of lines**.

00588-2009PDF-EN-12

Audience: Teachers
Date of issue: 06-2009
Ref: **00588-2009PDF-EN-12**

Copies of this publication may be available from:
www.teachernet.gov.uk/publications

You can download this publication and obtain further information at: **www.standards.dcsf.gov.uk**

department for
children, schools and families