The National Strategies
Secondary

Assessing pupils' progress in mathematics at Key Stage 3

Year 8 assessment package
Number/handling data

Teacher pack

department for
children, schools and families

QCA Qualifications and Curriculum Authority

Year 8 Number/Handling data task: *Powerful stuff* and *The first hundred*

Levels (3)/4/5/6

Structured for a Year 8 mixed ability class, but would be easy to modify for classes grouped by ability.

The lesson plans in this pack are set out in two columns. The left-hand column has indicative times for activities, highlights the resource sheets required and also has some examples of questions which teachers may wish to use with pupils during the activities. The right-hand column describes each activity in detail.

APP ASSESSMENT CRITERIA

These lessons may generate evidence to help inform judgements against a number of assessment criteria, including the following:

Numbers and the number system

- level 4: recognise and describe number relationships including multiple, factor and square
- level 5: recognise and use number patterns and relationships

Handling data

- level 3: collect and record discrete data
- level 4: interpret graphs and diagrams, including pie charts, and draw conclusions
- level 4: ask questions, plan how to answer them and collect the data required
- level 6: communicate interpretations and results of a statistical survey using selected tables, graphs and diagrams in support

Using and applying mathematics

- level 5: show understanding of situations by describing them mathematically using symbols, words and diagrams
- level 5: draw simple conclusions of their own and give an explanation of their reasoning
- level 6: use logical argument to establish the truth of a statement.

LESSON 1: *POWERFUL STUFF*

Resources

- Each pupil, or group of pupils, needs the following worksheets:
  How many factors? sheet 1 **(T8L1pupil1)**
  How many factors? sheet 2 **(T8L1pupil2)**
  How many factors? sheet 3 **(T8L1pupil3)**
  Preparation: To save paper, each page has two copies of the worksheet. These pages should be cut in half before the lesson
- Assessment sheets for pupils:
  Each pupil needs one of the following worksheets:
  Level (3/)4 pupils: *Powerful stuff sheet 1* **(T8L1assess1)**
  Level 5/6 pupils: *Powerful stuff sheet 2* **(T8L1assess2a)**
  Pupils who complete their assessment with confidence may wish to attempt the next in the series of worksheets. For pupils working at level 6 or above, the following worksheet is available:
  Extension activity: *Powerful stuff sheet 2 (continued)* **(T8L1assess2b)**
- Teacher resource OHT/whiteboard slide:
  The first twenty **(T8L1teacher1)**
- Paper for any rough work and for the extension activity
- Pupils will need access to calculators throughout the activity

<table>
<tr>
<td>Starter
about 5 minutes

Why does 4 have more factors than 5?

What other numbers have only two factors? [Include the numbers 2 and 3; the fact that 2 is even helps to highlight the misconception that all primes are odd.]

What are numbers with exactly two factors called?

How many odd primes are there?
Are all primes odd?
Are all odd numbers prime? Give an example of an odd number that is not prime.

Can you find out what the biggest prime number ever found is? Is it likely that a bigger one will be found at some point in the future?</td>
<td>Write the numbers 1, 2, 3, 4 and 5 on the board and ask which of these numbers has the most factors.

Check that all pupils understand the term ‘factor’.

An initial response may be ‘5’ since 5 is the largest number in the list. While it is true that larger numbers tend to have more factors (probing question: why?), there is no simple relationship between size and number of factors. Some large numbers have only a small number of factors (e.g. 101 has only two factors).</td>
</tr>
<tr>
<td>Group activity
about 15 minutes

T8L1pupil1
T8L1pupil2
T8L1pupil3</td>
<td>Give each pupil, or each group of pupils, either of the worksheets:
How many factors? sheet 1 (T8L1pupil1)
How many factors? sheet 2 (T8L1pupil2)

Ensure that some pupils are working on each so answers can be pooled. Pupils who work quickly can attempt both sheets.

When groups have completed the sheet(s), they can progress, if there is time, to the next in the series:
How many factors? sheet 3 (T8L1pupil3)</td>
</tr>
<tr>
<td>Mini-plenary
about 5 minutes</td>
<td>Write the results from sheet 1 on the board, i.e.

<table>
<tr><td>Number</td><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td></tr>
<tr><td>How many factors?</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
</table>

Ask the pupils how the table would continue [i.e. the number 32 has six factors, and so on], and why.

Show the factors of each number, i.e.

<table>
<tr><td>Number</td><td>Factors</td></tr>
<tr><td>1</td><td>1</td></tr>
<tr><td>2</td><td>1, 2</td></tr>
<tr><td>4</td><td>1, 2, 4</td></tr>
<tr><td>8</td><td>1, 2, 4, 8</td></tr>
<tr><td>16</td><td>1, 2, 4, 8, 16</td></tr>
<tr><td>32</td><td>1, 2, 4, 8, 16, 32</td></tr>
</table>

How does this list help to show why the number of factors goes up by one each time?</td>
</tr>
</table>

How would you describe the sequence of numbers on How many factors? sheet 2 *and* How many factors? sheet 3? *[Powers of 3 and powers of 4 respectively.]*

What about powers of 10?

Number	10^0	10^1	10^2	10^3	10^4	10^5
How many factors?	*1*	*4*	*9*	*16*	*25*	*36*

What if the numbers had been squares? Is there a predictable pattern?

Square number	*1*	*4*	*9*	*16*	*25*	*36*
How many factors?	*1*	*3*	*3*	*5*	*3*	*9*

Why do square numbers have an odd number of factors? Can you think of a non-square integer with an odd number of factors?

Is the following hypothesis true: 'As the numbers get bigger, the number of factors increases'? What sets of numbers show that this is not true? [E.g. squares and primes.]

Ask the pupils for a number that has:

… exactly seven factors [64] … exactly ten factors [512].

How many factors does the number 2048 have? [12]

Write the sequence 1, 2, 4, 8, 16, … on the board.
Ask pupils what the rule for generating new terms could be.

Initial responses are likely to focus on the term-to-term doubling, but use questioning to help the pupils identify the terms as powers of 2. More able pupils may be able to write the *n*th term as 2^{n-1}.

Ask similar questions about the sequences on sheets 2 and 3, i.e.

1, 3, 9, 27, 81, … (powers of 3). Note that the completed table is:

Number	1	3	9	27	81
How many factors?	1	2	3	4	5

1, 4, 16, 64, 256, … (powers of 4). Note that the completed table is:

Number	1	4	16	64	256
How many factors?	1	3	5	7	9

Assessment activity
about 20 minutes

T8L1assess1
T8L1assess2a
T8L1assess2b

Give pupils the following assessment sheets:

Level (3/)4 pupils:
Powerful stuff sheet 1 **(T8L1assess1)**

Level 5/6 pupils:
Powerful stuff sheet 2 **(T8L1assess2a)**

More able pupils working at level 6 or above can continue with the extension worksheet:
Powerful stuff sheet 2 (continued) **(T8L1assess2b)**

Other pupils who complete their assessment can be asked to find how many factors different integers have, e.g. integers up to 20 or 30. This will be useful for the plenary and for the next lesson.

Plenary about 5 minutes **T8L1teacher1** *How could we use the chart to find all the prime numbers between 1 and 20? What do they all have in common?* *What is the relationship between any EVEN number and its second-highest factor?* [For even numbers, the smallest possible multiple above 1 is 2; therefore the second-highest factor of any even number (including prime 2) will always be half the original number being factorised.] *Explain why this is always true.* *Find three examples in the table where the second-highest factor of a number is one-third of the original number. What is the same about these numbers?* [The numbers are odd multiples of 3, i.e. 3, 9, 15 from the table.]	Show the pupils the teacher resource OHT/whiteboard slide: *The first twenty* **(T8L1teacher1)** Explain that the factors of each number from 1 to 20 are shaded. Ask a pupil to choose a number from 1 to 20, and then ask the pupils to read off the factors of that number. Ask how the table shows how many factors each number has [either by counting the shaded boxes in the relevant row, or by the entry in the column that is on the furthest right]. What patterns can they see? Why are these patterns occurring? How would the table continue? End the lesson by explaining that in the next lesson pupils will be exploring these data further.

LESSON 2: *THE FIRST HUNDRED*

Resources

- Each pupil needs a copy of the pupil resource sheet:
  The first hundred **(T8L2pupil1)**
- Assessment sheets for pupils:
  Each pupil needs one copy of the assessment resource sheet which contains both the following tasks (level 3/4/5/6):
  Task one – The first hundred: a frequency table
  Task two – The first thirty: a hypothesis **(T8L2assess1)**
- Graph, squared, plain and lined paper should be available for the assessment
- Paper for rough work
- Pupils may need access to calculators for the assessment activity

Starter including group activity about 10 minutes

T8L2pupil1

How can we be sure that no number other than 1 will have only one factor? [All other numbers have at least 1 and the number itself as factors.]

How can we be sure that there must be a number that has more than 12 factors? [The number 60 has 12 factors. The number 60 × any number must have more than 12 factors because the larger number produced must be a factor of itself but wasn't a factor of 60.]

Is it possible to find the number with the greatest number of factors? [No – suppose a number A has the greatest so far. Then A × any number will have at least $A + 1$ factors.]

Remind the pupils that in the last lesson they looked at how many factors the numbers from 1 to 20 have.

This lesson they are going to be working with the numbers from 1 to 100.

Give each pupil a copy of the pupil resource sheet:
The first hundred **(T8L2pupil1)**

Ask the pupils for some questions that they could answer using the information on the sheet. For example:

For the numbers 1 to 100, what is the smallest/greatest number of factors? [smallest: 1, greatest: 12]

How many numbers have exactly three factors? [4]

Lead the discussion towards the question 'What is the modal number of factors?' Check all the pupils understand the term 'modal', then ask how they would find the mode. Discuss why this is difficult to do directly and suggest that a frequency table would be a good method for summarising the data first.

Show an incomplete frequency table on the board, e.g.

Number of factors	Frequency
1	1
2	?
3	4

Ask pupils to share the work between them to find the frequencies. For example, ask pairs of pupils to work on each row of data needed, then pool the results.

After a few minutes, ask for the results and write them in the frequency table, i.e.

Number of factors	Frequency
1	1
2	25
3	4
4	32
5	2
6	16

	<table><tr><td>7</td><td>1</td></tr><tr><td>8</td><td>10</td></tr><tr><td>9</td><td>2</td></tr><tr><td>10</td><td>2</td></tr><tr><td>11</td><td>0</td></tr><tr><td>12</td><td>5</td></tr></table> Ask what checks can be done to remove errors. What should the total of the frequencies be? [100] Why? Correct any errors. Now ask again for the modal number of factors [four] and reinforce how helpful the frequency table is for finding it.
Assessment activity about 10 minutes for task one, about 20 minutes for task two **T8L2assess1** **T8L2pupil1**	Tell the pupils that the assessment has two tasks, so they need to plan their time to do both. Note that more time is needed for task two than for task one. Give each pupil a copy of the assessment resource sheet showing the two tasks (level 3/4/5/6): *The first hundred: a frequency table* *The first thirty: a hypothesis* **(T8L2assess1)** Tell the pupils that the frequency table is a summary of the information shown on the worksheet *The first hundred* **(T8L2pupil1)**. Explain that their first task is to present the data from this frequency table on a graph so that someone else can see the information quickly and clearly. Pupils will need to decide for themselves what type of graph to draw and what sort of paper to use. If necessary, briefly state the types of graphs that they could choose from. Remind them that they should label their graph clearly and that you want them to explain why they chose their type of graph. Task two should also be explained, though you should encourage pupils to ask if they need clarification once they start this part of the assessment. Emphasise that pupils will need to decide for themselves what mathematical tools to use. However, note that some pupils (particularly any working at level 3) may need teacher support, for example being prompted to use the mean or mode, and may benefit from being given a simple writing frame.

Plenary about 10 minutes **T8L2pupil1** [The probing questions given within the lesson plan reinforce and extend those used in lesson 1.]	Ask the pupils to look again at their sheet: *The first hundred* **(T8L2pupil1)** Ask pupils for examples of numbers with only two factors. What name is given to this type of number? [Prime] Repeat for numbers with three factors. Is there anything special about these numbers? [They are all square numbers.] Do all square numbers have exactly three factors? Why not? Is there a way of knowing whether a square number will have three factors or not, without finding the factors? [Prime numbers squared have exactly three factors.] Discuss the number 14 (which has four factors). Draw attention to the fact that 14 is double 7, so 14 is double a prime. How does that explain why 14 must have four factors? [The factors of 7 are 1 and 7; the factors of 14 are 1, 7, 2 and 14 and there are no others. The fact that 2 is also a prime explains why there are no more than four factors.] Are all numbers that have exactly four factors double a prime? [No, they are the product of any two primes: neither needs to be 2.]

Teacher resource sheet

T8L1teacher1

The first twenty

1	1																				1
2	1	2																			2
3	1		3																		2
4	1	2		4																	3
5	1				5																2
6	1	2	3			6															4
7	1						7														2
8	1	2		4				8													4
9	1		3						9												3
10	1	2			5					10											4
11	1										11										2
12	1	2	3	4		6						12									6
13	1												13								2
14	1	2					7							14							4
15	1		3		5										15						4
16	1	2		4				8								16					5
17	1																17				2
18	1	2	3			6			9									18			6
19	1																		19		2
20	1	2		4	5					10										20	6

Pupil sheets

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T8L1pupil1

How many factors? sheet 1 Name(s): _____

Work in groups to find the missing numbers in this table.

Number	1	2	4	8	16
How many factors?					

What number patterns can you see? Can you predict how the table would continue?

How many factors? sheet 1 Name(s): _____

Work in groups to find the missing numbers in this table.

Number	1	2	4	8	16
How many factors?					

What number patterns can you see? Can you predict how the table would continue?

T8L1pupil2

How many factors? sheet 2 Name(s): _____

Work in groups to find the missing numbers in this table.

Number	1	3	9	27	81
How many factors?					

What number patterns can you see? Can you predict how the table would continue?

How many factors? sheet 2 Name(s): _____

Work in groups to find the missing numbers in this table.

Number	1	3	9	27	81
How many factors?					

What number patterns can you see? Can you predict how the table would continue?

T8L1pupil3

How many factors? sheet 3 Name(s): _____

Predict what you think the missing numbers in this table will be.
Then work in groups to see if you were correct.

Number	1	4	16	64	256
How many factors? prediction					
How many factors?					

What number patterns can you see? Can you predict how the table would continue?

How many factors? sheet 3 Name(s): _____

Predict what you think the missing numbers in this table will be.
Then work in groups to see if you were correct.

Number	1	4	16	64	256
How many factors? prediction					
How many factors?					

What number patterns can you see? Can you predict how the table would continue?

T8L1assess1

Powerful stuff sheet 1 Name: _____

Here are the numbers from *How many factors? sheet 1* again, with more shown.

Number	How many factors?
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1 024	11
2 048	12
4 096	13
8 192	14
16 384	15
32 768	16
65 536	17
131 072	18
262 144	19
524 288	20
1 048 576	21
2 097 152	22
4 194 304	23
8 388 608	24

Which number has **18 factors**?
Write the number in words.

Find the first number in the table that is bigger than 1 million.
How many factors does it have?

The number 8 388 608 has 24 factors.
What number has 25 factors?

The number 64 has 7 factors.
What are they?

The number 8 has 4 factors.
Can you find another number (not in this table) that also has exactly 4 factors?

T8L1assess2a

Powerful stuff sheet 2 Name: _____

Here are the tables for powers of 2, 3 and 4.

powers of 2 →

Number	1	2	4	8	16
How many factors?	1	2	3	4	5

powers of 3 →

Number	1	3	9	27	81
How many factors?	1	2	3	4	5

powers of 4 →

Number	1	4	16	64	256
How many factors?	1	3	5	7	9

Write all the factors of 1, 5, 25, 125 and 625 to show that the table below is correct.

powers of 5 →

Number	1	5	25	125	625
How many factors?	1	2	3	4	5

Look at all the tables.
What is the same for powers of 2, powers of 3 and powers of 5?
What is different for powers of 4?

Why might this be?
Think about how the number 4 is different from the numbers 2, 3, and 5

T8L1assess2b

Powerful stuff sheet 2 (continued) Name: _____

Which type of table would you predict for **powers of 9**? Why?
(Try to answer this without working out the numbers of factors.)

Why would you expect the table for powers of 6 **not** to be like the tables we have seen so far?

Continue investigating…

T8L2pupil1

The first hundred

Number	Number of factors
1	1
2	2
3	2
4	3
5	2
6	4
7	2
8	4
9	3
10	4
11	2
12	6
13	2
14	4
15	4
16	5
17	2
18	6
19	2
20	6
21	4
22	4
23	2
24	8
25	3

Number	Number of factors
26	4
27	4
28	6
29	2
30	8
31	2
32	6
33	4
34	4
35	4
36	9
37	2
38	4
39	4
40	8
41	2
42	8
43	2
44	6
45	6
46	4
47	2
48	10
49	3
50	6

Number	Number of factors
51	4
52	6
53	2
54	8
55	4
56	8
57	4
58	4
59	2
60	12
61	2
62	4
63	6
64	7
65	4
66	8
67	2
68	6
69	4
70	8
71	2
72	12
73	2
74	4
75	6

Number	Number of factors
76	6
77	4
78	8
79	2
80	10
81	5
82	4
83	2
84	12
85	4
86	4
87	4
88	8
89	2
90	12
91	4
92	6
93	4
94	4
95	4
96	12
97	2
98	6
99	6
100	9

T8L2assess1

Task one ***The first hundred: a frequency table***

Number of factors	Frequency
1	1
2	25
3	4
4	32
5	2
6	16
7	1
8	10
9	2
10	2
11	0
12	5
Total	100

Use the data from this frequency table to draw a graph so that someone else can see the information quickly and clearly.

You can decide what type of graph to draw and what sort of paper to use.
Remember to label your graph clearly.

Now say **why** you chose that type of graph.

Task two ***The first thirty: a hypothesis***

Is this statement true?

'Even numbers have more factors than odd numbers.'

Use the data from the worksheet *The first hundred* to investigate.
Work only with the numbers **1 to 30**

Show all your working and justify your conclusion in a brief report.

Solutions and performance indicators

LESSON 1: *POWERFUL STUFF* **Solutions**

Powerful stuff sheet 1 (target level (3/)4)	**T8L1assess1**
Solutions	Notes
The number 131 072 expressed using words so that the place value is correctly interpreted, e.g. • 'One hundred and thirty-one thousand and seventy-two' • '1 hundred 31 thousand 72'	**Good** responses show understanding of the place value within their number. **Better** responses use correct spellings to write the correct value in words.
21	**Good** responses indicate the first value in the table greater than 1 million, i.e. 1 048 576. **Better** responses indicate the correct number of factors.
A number with 25 factors, e.g. • 16 777 216 • 282 429 536 481	**Good** responses show understanding that the last value in the table must be doubled. **Better** responses give a correct value.
1, 2, 4, 8, 16, 32 and 64, in any order	**Good** responses show some of the correct factors. **Better** responses show the correct factors with no errors.
A number with exactly 4 factors, apart from 8 Note that numbers between 1 and 100 with 4 factors are: 6, (8,) 10, 14, 15, 21, 22, 26, 27, 33, 34, 35, 38, 39, 46, 51, 55, 57, 58, 62, 65, 69, 74, 77, 82, 85, 86, 87, 91, 93, 94, 95	**Good** responses give a value with exactly 4 factors. **Better** responses may use their previous understanding of powers of 3 or 5, e.g. giving the value 27 or 125.

Powerful stuff sheet 2 (target level 5/6)	**T8L1assess2a**
Solutions	Notes
Factors of all five values in the table shown, e.g. • <table><tr><td>Number</td><td>Factors</td></tr><tr><td>1</td><td>1</td></tr><tr><td>5</td><td>1, 5</td></tr><tr><td>25</td><td>1, 5, 25</td></tr><tr><td>125</td><td>1, 5, 25, 125</td></tr><tr><td>625</td><td>1, 5, 25, 125, 625</td></tr></table> • 1 × 1 1 × 5 1 × 25, 5 × 5 1 × 125, 5 × 25 1 × 625, 5 × 125, 25 × 25	**Good** responses show some correct factors. **Better** responses show consistently correct factors for all five values.
Correct statement about what is the same for powers of 2, 3 and 5, e.g. • 'For powers of 2, 3 and 5 the number of factors goes up in steps of 1 starting from 1' • 'The numbers of factors for 2, 3 and 5 powers go in the sequence 1, 2, 3, 4, etc' • '2, 3 and 5 give the sequence of counting numbers' Correct statement about what is different for powers of 4, e.g. • 'For powers of 4 the number of factors goes up in steps of 2 starting from 1' • 'The numbers of factors for 4 powers go in the sequence 1, 3, 5, 7, etc' • '4 gives the sequence of odd numbers'	**Good** responses show understanding of the similarities and differences for the tables of factors. **Better** responses explain the similarities and differences clearly, referring to the step sizes within the sequences.
Relevant fact(s) about the differences between the numbers 2, 3 and 5 and the number 4, e.g. • '2, 3 and 5 are prime numbers, but 4 is not' • '4 is the only one with more than 2 factors' • '4 has 3 factors' • '4 is a square number' • 'If the number has only 2 factors, then the only new factor each time is the new number itself' • 'Since 4 has the factors 1, 2 and 4, when you multiply by 4 each time, you get two new factors, e.g. 2 × 4 and 4 × 4'	**Good** responses refer to the fact that 4 is not a prime number. **Better** responses show understanding of the effect of the extra factor of 4.

Solutions	Notes
Correct prediction about the number of factors for powers of 9, e.g. • 'It will be like the powers of 4 table' • 'The same as the third table' • <table><tr><td>Number</td><td>1</td><td>9</td><td>81</td><td>729</td><td>6561</td></tr><tr><td>How many factors?</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr></table> Correct statement about the reason for this, e.g. • '9 is a square number' • '9 also has 3 factors'	**Good** responses show understanding that the factors do not increase in steps of 1. **Better** responses specify that the pattern of factors will be the same as for powers of 4.
Correct explanation for why the table for powers of 6 will be different, e.g. • '6 has 4 factors, but all the previous numbers have only 2 or 3' • '6 has more factors so the numbers will go up in bigger steps than before' • <table><tr><td>Number</td><td>1</td><td>6</td><td>36</td><td>216</td><td>1296</td></tr><tr><td>How many factors?</td><td>1</td><td>4</td><td>9</td><td>16</td><td>25</td></tr></table> • 'When you multiply by all the factors of 6 each time, you will get more new factors each time'	**Good** responses show understanding that 6 has more factors than previous numbers. **Better** responses link the number of factors to the size of the increases in numbers of factors.
Further investigation into other powers The following information may be helpful: <table><tr><th>Type of number</th><th>E.g.</th><th>Pattern in factors</th><th>Number of factors for number^n</th></tr><tr><td>1</td><td>1</td><td>1, 1, 1, 1, etc</td><td>1</td></tr><tr><td>Prime</td><td>2, 3, 5, 7</td><td>1, 2, 3, 4, etc</td><td>$n + 1$</td></tr><tr><td>Numbers with 3 factors (primes squared)</td><td>4, 9, 25</td><td>1, 3, 5, 7, etc</td><td>$2n + 1$</td></tr><tr><td>Numbers with 4 factors: 1, itself and two PRIME factors</td><td>6, 10, 14, 15</td><td>1, 4, 9, 16, etc</td><td>$(n + 1)^2$</td></tr><tr><td>Numbers with 4 factors: 1, itself, one PRIME and one NON-PRIME (primes cubed)</td><td>8, 27, 125</td><td>1, 4, 7, 10, etc</td><td>$3n + 1$</td></tr><tr><td>Numbers with 5 factors: 1, itself, one PRIME and two NON-PRIME</td><td>16, 81</td><td>1, 5, 9, 13, etc</td><td>$4n + 1$</td></tr><tr><td>Numbers with 6 factors: 1, itself, two PRIME and two NON-PRIME</td><td>12, 18</td><td>1, 6, 15, 28, etc</td><td>$2n^2 + 3n + 1$</td></tr></table>	**Good** responses calculate numbers of factors for some other numbers and start to collect results in a systematic way. **Better** responses may use a spreadsheet in order to generate more results and start reasoning in more general terms.

LESSON 1: *POWERFUL STUFF* Performance indicators

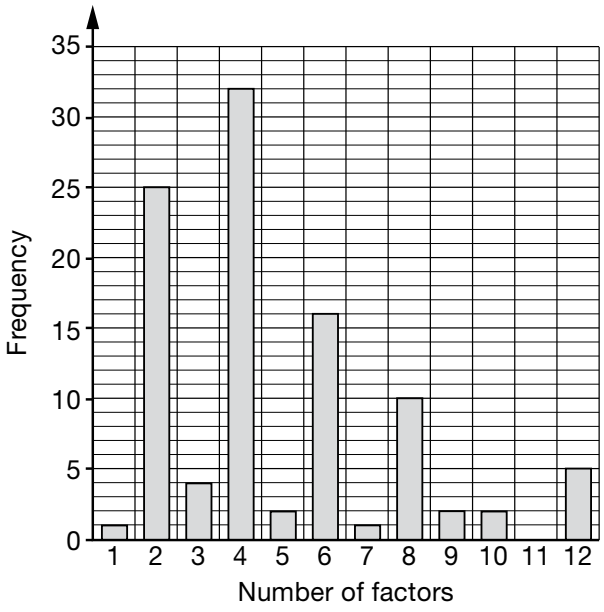
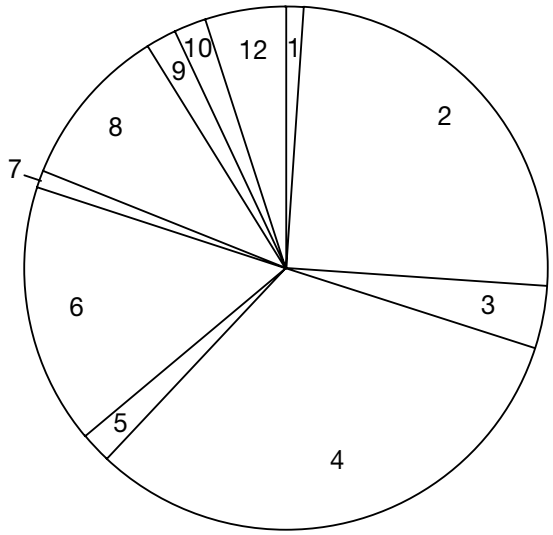
Note that performance indicators involving an element of 'Using and applying mathematics' are given in **bold**.

<table>
<tr><th>Worksheet</th><th>Performance indicators</th></tr>
<tr><td>Powerful stuff sheet 1
(target level (3/)4)
T8L1assess1</td><td rowspan="3">Level 3:
At this level, pupils are generally able to:
• identify the correct value from a table;
• identify the first number greater than 1 million in a list and read off the related number of factors from a table;
• recognise the pattern of numbers in a table of factors and use it to find the next value.
However, they are less likely to be able to:
• write a six-digit number in words, correctly interpreting place value;
• find the factors of values, given how many there are;
• write a number with 4 factors that is different from one given.

Level 4:
At this level, pupils are generally able to:
• write a six-digit number in words, correctly interpreting place value;
• find the factors of values, given how many there are;
• write a number with 4 factors that is different from one given;
• describe similarities and differences in simple number patterns.
However, they are less likely to be able to:
• recognise the importance of different numbers of factors to differences in factor patterns for powers;
• use correct mathematical language for special numbers such as prime or square.

Level 5:
At this level, pupils are generally able to:
• recognise the importance of different numbers of factors to differences in factor patterns for powers;
• use correct mathematical language for special numbers such as prime or square;
• make predictions about factor patterns for powers based on previous results, recognising the importance of different numbers of factors.
However, they are less likely to be able to:
• reason about why a greater number of factors results in bigger increases in factor patterns for powers;
• research systematically further factor patterns for values with varying numbers of factors.

Level 6 and above:
At these levels, pupils are generally able to:
• give evidence for the performance indicators listed previously for pupils working at level 5, plus;
• reason about why a greater number of factors results in bigger increases in factor patterns for powers;
• research systematically further factor patterns for values with varying numbers of factors.</td></tr>
<tr><td>Powerful stuff sheet 2
(target level 5/6)
T8L1assess2a</td></tr>
<tr><td>Powerful stuff sheet 2 (continued)
(extension activity)
T8L1assess2b</td></tr>
</table>

Task one – The first hundred: a frequency table (target level 3/4/5/6)	**T8L2assess1**
Solutions	Notes
An appropriate frequency chart summarising the data, e.g. • Bar chart to show numbers of factors for the numbers from 1 to 100  • Pie chart to show numbers of factors for the numbers from 1 to 100  	**Good** responses show a simple chart. **Better** responses show an accurate chart with correct heading and labels.
A sensible reason for choosing their type of chart, e.g. • 'A bar chart is easy to draw and read' • 'I chose a bar chart because it is simple to read and you can compare values easily' • 'You can see quickly from the bar chart which numbers of factors were most or least common' • 'A line graph can show you any patterns. You can see it goes up and down, with even numbers of factors more common than odd numbers' • 'A pie chart makes it easy to see the big slices and the small slices'	**Good** responses focus on the clear and easy-to-read nature of their chart. **Better** responses give more detail about the types of information that can be easily seen from their chart.

Task two – The first thirty: a hypothesis (target level 3/4/5/6)	**T8L2assess1**
Solutions	Notes

(cont) A report summarising their conclusion, supported by an appropriate diagram and/or argument, e.g.

-

Even		Odd	
Number	Factors	Number	Factors
2	2	1	1
4	3	3	2
6	4	5	2
8	4	7	2
10	4	9	3
12	6	11	2
14	4	13	2
16	5	15	4
18	6	17	2
20	6	19	2
22	4	21	4
24	8	23	2
26	4	25	3
28	6	27	4
30	8	29	2
Total:	74	Total:	37

There is the same number of even and odd numbers, but the even numbers have a much bigger total, so I think the statement is true.

-

Even		Odd	
Factors	Frequency	Factors	Frequency
1	0	1	1
2	1	2	9
3	1	3	2
4	6	4	3
5	1	5	0
6	4	6	0
7	0	7	0
8	2	8	0
Total:	15	Total:	15

The modal (most common) number of factors is 4 for even numbers, but only 2 for odd numbers. The median (middle) number of factors is also 4 for even numbers and 2 for odd numbers.
The mean (average) number of factors is $74 \div 15 = 4.9$ for even numbers and $37 \div 15 = 2.5$ for odd numbers.
So the statement seems to be true when you look at the first 30 numbers.

The statement is true because for the first 30 numbers, there are 74 factors for the 15 even numbers and 34 factors for the 15 odd numbers.
The only even number with 2 factors is 2, but all other prime numbers will be odd. Even numbers can have both odd and even factors, but odd numbers can only have odd factors.

Good responses collect useful data about the factors of odd and even numbers between 1 and 30.

Better responses also make a logical decision about the given hypothesis and suggest reasons for the pattern of factors.

LESSON 2: *THE FIRST HUNDRED* Performance indicators

Note that performance indicators involving an element of 'Using and applying mathematics' are given in **bold**.

<table>
<tr><th>Worksheet</th><th>Performance indicators</th></tr>
<tr><td>Task one –
The first hundred:
a frequency table
(target level 3/4/5/6)
T8L2assess1

Task two –
The first thirty:
a hypothesis
(target level 3/4/5/6)
T8L2assess1</td><td>
Level 3:
At this level, pupils are generally able to:
• draw a bar chart, pictogram or other simple chart from a set of data.
However, they are less likely to be able to:
• avoid errors in the scale or body of their chart;
• ensure their chart has appropriate heading, labels and/or key;
• give a sensible reason for their choice of chart;
• correctly interpret 'even and odd numbers' as referring to the numbers between 1 and 30 rather than the numbers of factors.

Level 4:
At this level, pupils are generally able to:
• draw accurately a bar chart, pictogram, line graph or other simple chart from a set of data;
• avoid errors in the scale or body of their chart;
• ensure their chart has appropriate heading, labels and/or key;
• give a sensible reason, based on its simplicity to read and/or draw, for their choice of chart;
• correctly interpret 'even and odd numbers' as referring to the numbers between 1 and 30 rather than the numbers of factors;
• collect useful data about the factors of odd and even numbers between 1 and 30 in an effective way;
• use their data to make a logical decision about the given hypothesis.
However, they are less likely to be able to:
• appreciate the importance of details within the presentation of some charts, e.g. gaps between bars or dotted line graphs for discrete data;
• give more detailed reasons about the types of information that can be easily seen from their choice of chart;
• draw a useful chart or charts using their data on the factors of odd and even numbers between 1 and 30;
• reason about the pattern of factors for odd and even numbers.

Level 5:
At this level, pupils are generally able to:
• appreciate the importance of details within the presentation of some charts, e.g. gaps between bars or dotted line graphs for discrete data;
• give more detailed reasons about the types of information that can be easily seen from their choice of chart;
• draw a useful chart or charts using their data on the factors of odd and even numbers between 1 and 30;
• reason about the pattern of factors for odd and even numbers.
However, they are less likely to be able to:
• draw a pie chart from a set of data;
• give some explanation using more general reasoning for the pattern of factors for odd and even numbers.

Level 6 and above:
At these levels, pupils are generally able to:
• give evidence for the performance indicators listed previously for pupils working at level 5, plus;
• draw a pie chart from a set of data;
• give some explanation using more general reasoning for the pattern of factors for odd and even numbers.
</td></tr>
</table>

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