Assessing pupils' progress in mathematics at Key Stage 3

Year 9 assessment package
Handling data

Teacher pack

department for
children, schools and families

QCA
Qualifications and
Curriculum Authority

Year 9 Handling data task: *Runners up!* and *Runners up! (again)*

Levels 4/5/6

Note that able pupils may benefit from a less structured approach to lesson 1: the assessment could be omitted, allowing longer for the more open assessment presented in lesson 2.

The lesson plans in this pack are set out in two columns. The left-hand column has indicative times for activities, highlights the resource sheets required and also has some examples of questions which teachers may wish to use with pupils during the activities. The right-hand column describes each activity in detail.

APP ASSESSMENT CRITERIA
These lessons may generate evidence to help inform judgements against a number of assessment criteria, including the following:

Handling data
- level 5: interpret graphs and diagrams, including pie charts, and draw conclusions
- level 6: communicate interpretations and results of a statistical survey using selected tables, graphs and diagrams in support
- level 6: select, construct and modify, on paper and using ICT:
    pie charts for categorical data
    bar charts and frequency diagrams for discrete and continuous data
    simple time graphs for time series
    scatter graphs.

LESSON 1: *RUNNERS UP!*
Resources
- Teacher resource sheets, OHT/whiteboard slides:
    Eight of the best **(T9L1teacher1)**
    Ten just for Pat **(T9L1teacher2)**
    Who's who? (answers) **(T9L1teacher3a)**
    Who's who? (answers, continued) **(T9L1teacher3b)**
    Who did better? **(T9L1teacher4)**
    Pie charts **(T9L1teacher5)**
- Each group of pupils needs one copy of each of the following pupil resource sheets:
    Who's who? **(T9L1pupil1a)**
    Who's who? (continued) **(T9L1pupil1b)**
- Assessment sheets for pupils (level 4/5/6):
    Each pupil needs the following three worksheets:
    Ali v Chris **(T9L1assess1a)**
    Ali v Chris (continued) **(T9L1assess1b)**
    Ali v Chris (continued again) **(T9L1assess1c)**
- Paper for any rough working

Starter about 10 minutes **T9L1teacher1** **T9L1teacher2** *True or false? "There was about a one-second difference between the first and last runner."* *What evidence do you have to justify your answer?* *How was the range calculated?* *The mean time for the race was 13.04 seconds. How was this calculated?* *Which four runners were faster than the mean?* *Four runners were faster than the mean, four were slower. Is this necessarily the case? How might this be different in another race?* *In what percentage of the races did Pat beat her mean race time?* *Looking at all Pat's times, why do you think she beat her mean in more than half the races?* *How consistent do you think Pat is? What evidence do you have to support your view?* *Give me five numbers where the mean is 6 and the range 8. How did you work this out? What if the median was 6 and the range 8? What if the mode was 6 and the range 8?* *Two distributions both have the same range but the first one has a median of 6 and the second has a mode of 6. Explain how these two distributions may differ.* *Make up a statement or question for this chart/graph using one of the following key words: total, range, mode (or mean).*	Explain that the pupils will be investigating the race times of 100-metre runners at a junior running club. Show the teacher resource sheet: 　　*Eight of the best* **(T9L1teacher1)** Explain that the table shows the race times for the club's best eight runners. Times are to the nearest hundredth of a second. Draw pupils' attention to the shading/labels on some of the timings. (If printed out in colour, gold = 1st, silver (shown as grey) = 2nd, bronze (shown as orange) = 3rd.) Make sure the pupils understand that the lower the value, the better the result and that the times are **decimal**, showing seconds, tenths of seconds and hundredths of seconds, **not** minutes and seconds. Check that the pupils understand the terms 'mean' and 'range' and how they were calculated for these data. Note that any mean times given over the next two lessons have been rounded to the nearest hundredth of a second, where needed. Now show Pat's race times over ten weeks: 　　*Ten just for Pat* **(T9L1teacher 2)** Discuss features of Pat's race times, e.g. how many races did this person win? Check that the pupils understand how the mean (13.03) and the range (1.59) were calculated. How does the mean time compare with Pat's best time? How can the range be used to measure consistency of performance? What would the range be if a runner always ran the race in exactly the same time?

Secondary National Strategy | Assessing pupils'
progress in mathematics at Key Stage 3

3

Group activity
about 10 minutes
 	
T9L1pupil1a
T9L1pupil1b
 	
Do you think that either Pat or Stevie have been improving over the ten weeks? What evidence is there to support your opinion?
 	
Who do you think is the better runner? What are your reasons?
 	
What information is missing from these bar charts? What do the bar charts tell us about Pat's and Stevie's ten races? What don't they tell us?
 	
Why are the points in the line graph joined by dashed lines and not solid lines or curves? Do the lines in between the week numbers have any meaning?
 	
Why is it not possible to be completely accurate when you are drawing the graph showing the race times?
	Give one copy of each of the following worksheets to each pupil or group of pupils:
	    Who's who? **(T9L1pupil1a)**
	    Who's who? (continued) **(T9L1pupil1b)**
	
	Tell the pupils the data are for two runners, Pat and Stevie. Their task is to decide which of the two race position charts is whose and complete them. They should also complete the comparative line graph on the second page to show Pat's and Stevie's race times.

Mini-plenary
about 5 minutes
 	
T9L1teacher3a
T9L1teacher3b
T9L1teacher4
	Discuss the correct answers, noting, for example, that the point on the graph for 12.47 should be closer to 12.50 than to 12.40. Show the OHT/whiteboard slides:
	    Who's who? (answers) **(T9L1teacher3a)**
	    Who's who? (answers, continued) **(T9L1teacher3b)**
	
	Ask pupils to describe Pat's and Stevie's relative performances, encouraging them to use precise and concise statements.
	
	Now show the pupils the comparative bar chart showing Pat's and Stevie's race positions, i.e. the teacher resource sheet:
	    Who did better? **(T9L1teacher4)**
	
	Ask what is missing from the chart. [The key.] Why is this information important?

Assessment activity
about 20 minutes
 	
T9L1assess1a
T9L1assess1b
T9L1assess1c
	Give each pupil the following three worksheets:
	    Level 4/5/6 pupils:
	
	    Ali v Chris **(T9L1assess1a)**
	    Ali v Chris (continued) **(T9L1assess1b)**
	    Ali v Chris (continued again) **(T9L1assess1c)**
	
	Tell pupils how long they have and remind them that on the final sheet they will need to explain their answers clearly.

Plenary about 5 minutes **T9L1teacher5** *Do these pie charts show the same information about the runners as the comparative bar charts?* *What do the pie charts tell us that the bar charts do not?* *When drawing a pie chart, what information do you need to calculate the size of the angle for each category?* *When considering a range of graphs representing the same data, which is the easiest to interpret? Why?*	Briefly review the types of graphs used and the use of mean and range. Show the pupils the teacher resource sheet: 　　*Pie charts* **(T9L1teacher5)** which shows the race positions of all four runners. Ask the pupils what the pie charts show that the bar charts do not. For example, do the pie charts give a better idea of relative proportions?

LESSON 2: *RUNNERS UP! (AGAIN)*
Resources

- Assessment sheets for pupils (level 4/5/6):
  Each pupil needs one of the following resource sheets:
  Runners up! **(T9L2assess1)**
- Paper for the assessment activity – plain, squared, graph, as required
- Each pupil should have access to a calculator

Starter about 5 minutes **T9L2assess1** *Check the results for each week. Has any runner in 1st, 2nd or 3rd place been wrongly placed? [No-one has: all entries are correct.]* *Have any runners been improving over the ten weeks? How can you tell?* *Who has the lowest mean race time for the ten weeks? Why might this be thought surprising?* *Which runner is the most 'up and down'? Which runners' times have varied by less than half a second?*	Explain that each pupil will be given a data set showing the race times for the eight runners over the last ten weeks. Their task is to use the data **(T9L2assess1)** to decide which three runners they should choose to represent the club in a major championship to be held exactly one month after the 10th week of training. Who will they choose and why? Their choice should be justified through the use of statistical calculations, charts and graphs. Briefly discuss what factors in real life would influence such a decision, e.g. motivation, preparation, etc, but state that for this activity such information is unknown, hence their decision can be based only on statistical analysis. Stress that there is no 'right' and 'wrong' selection: you are interested in their ability to analyse data, and to communicate and reason when handling the data.
Assessment activity about 40 minutes **T9L2assess1** *Explain how you would attempt this problem. How would you start?* *Explain in your own words what the problem is asking you to do.* *What information have you been given?* *What information is important for solving the problem?*	Give each pupil a copy of the assessment resource sheet: Level 4/5/6 pupils: *Runners up!* **(T9L2assess1)** Tell the pupils that plain, squared and graph paper is available and remind them to put their name on all sheets. Stress again that there is no 'right' and 'wrong' selection and that it is their ability to communicate and reason when handling data that is being assessed. Reassure them that they can use calculators and tell them how long they have for the assessment.
Plenary about 5 minutes *Which graphs/diagrams were most/ least useful and why? How might you improve your graphs/diagrams?* *What do you look for in the data when deciding on ways to represent it?*	Ask the pupils about their selections of runners for the championship and how they justified them. How much agreement is there about whom to select?

Teacher resource sheets

Eight of the best

EVENT: 100 metres sprint

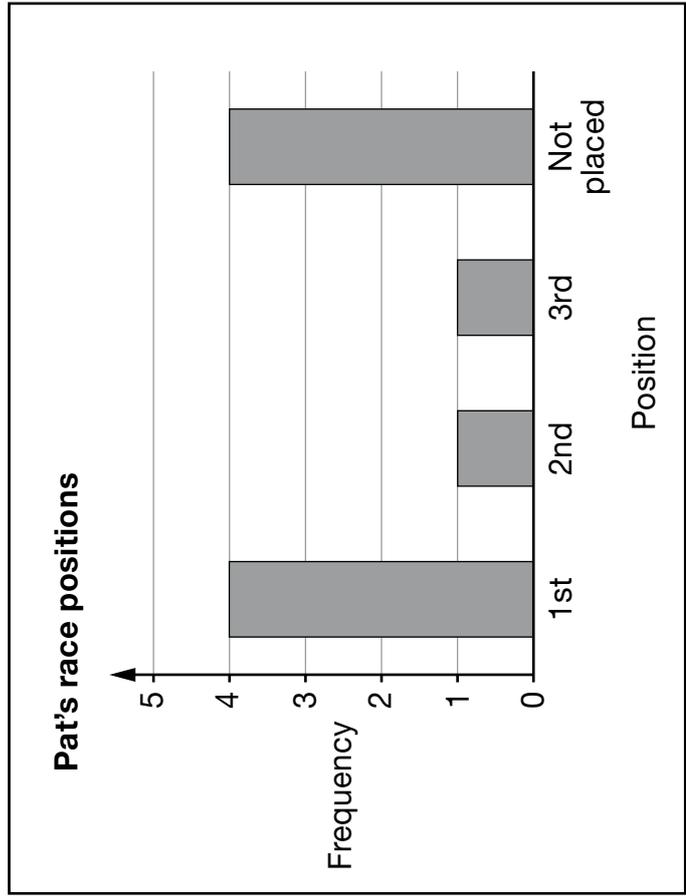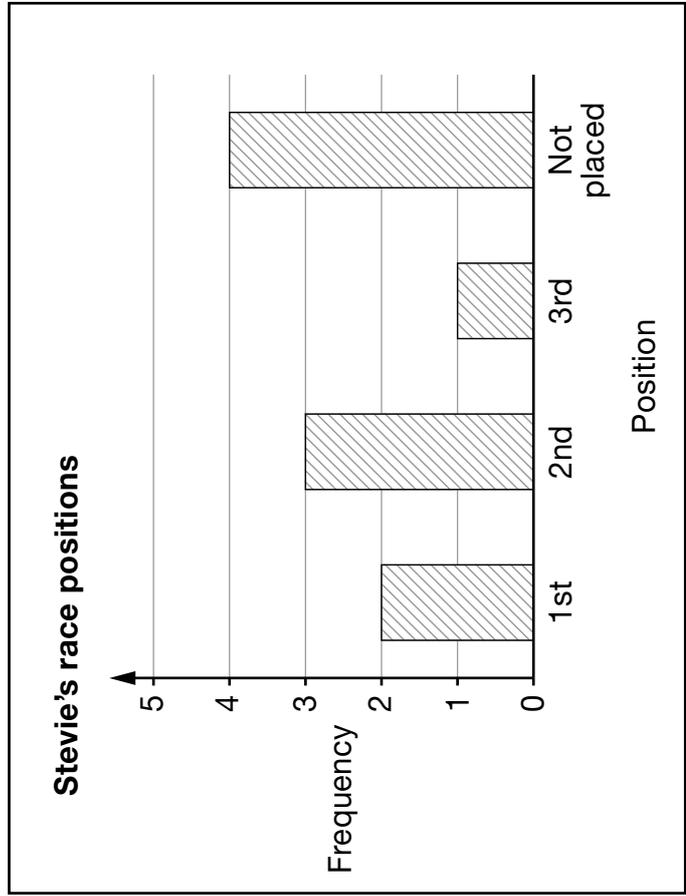| RUNNER	Week 1
Ali	*13.45*
Chris	*13.05*
Danny	*12.76* *3rd*
Jay	*13.57*
Lee	*12.80*
Mel	*13.46*
Pat	*12.53* *1st*
Stevie	*12.71* *2nd*

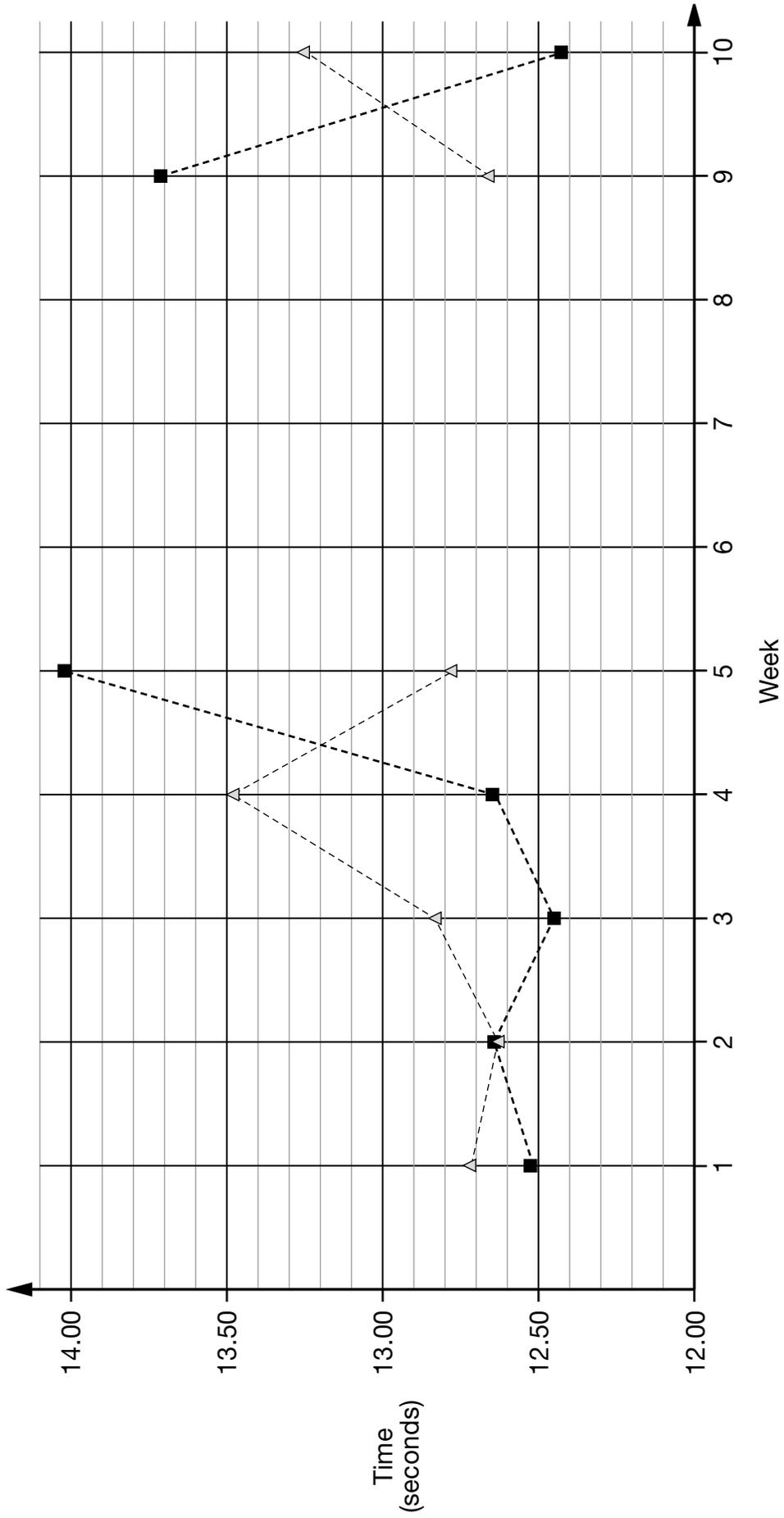|	
MEAN	*13.04*
RANGE	*1.04*

T9L1teacher2

Ten just for Pat

RUNNER	Wk 1	Wk 2	Wk 3	Wk 4	Wk 5	Wk 6	Wk 7	Wk 8	Wk 9	Wk 10	MEAN	RANGE
Pat	12.53 1st	12.64 2nd	12.45 1st	12.65 3rd	14.02	12.47 1st	13.36	14.00	13.71	12.43 1st	13.03	1.59

Who's who? (answers)

Stevie's race positions

Pat's race positions

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Who's who? (answers, continued)

Key: ■ **Pat's race times** △ **Stevie's race times**

Who did better?
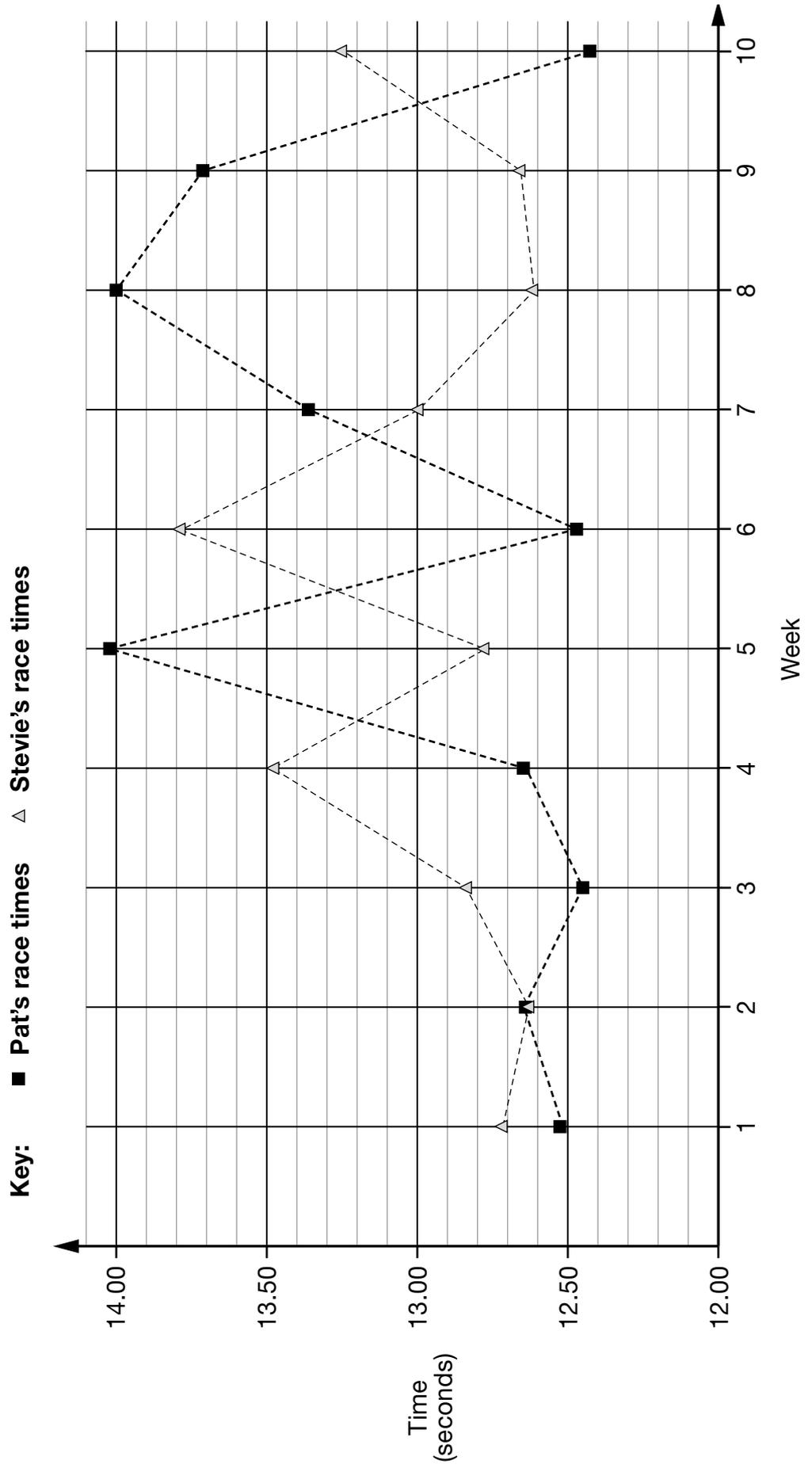
Stevie's and Pat's race positions
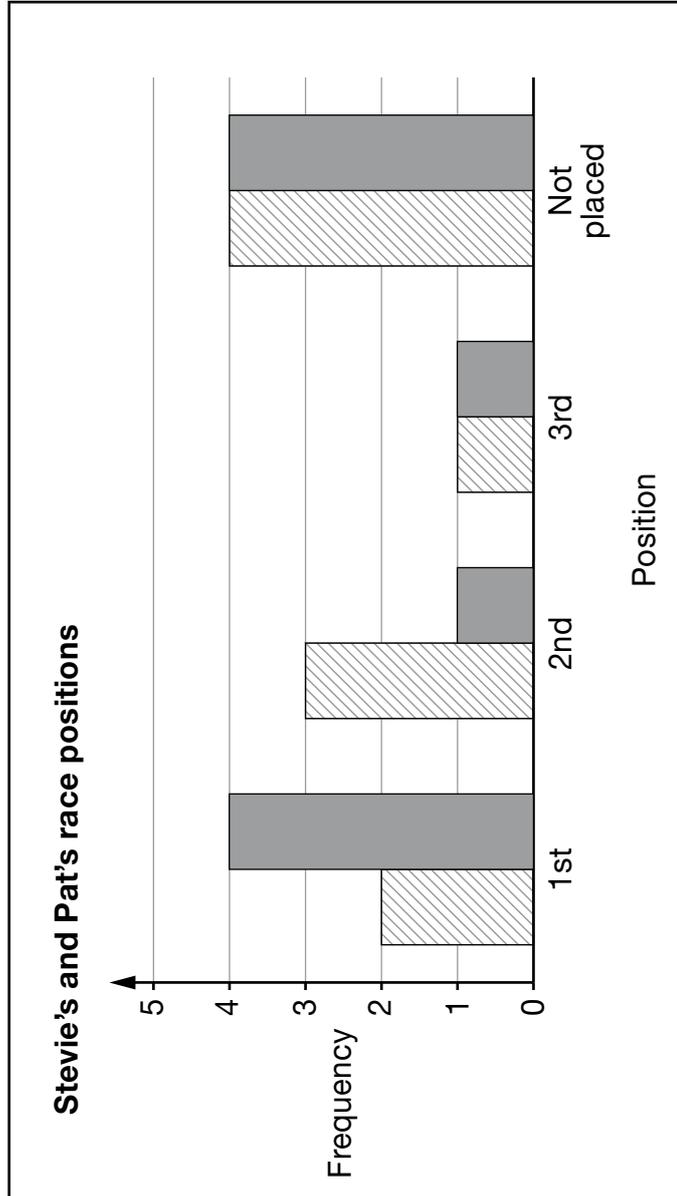
Pie charts

Pat's race positions
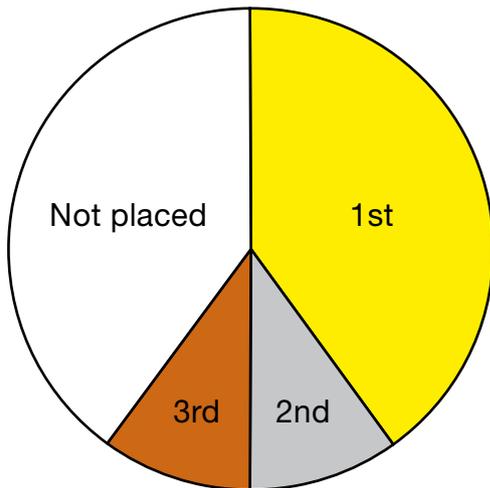
Stevie's race positions
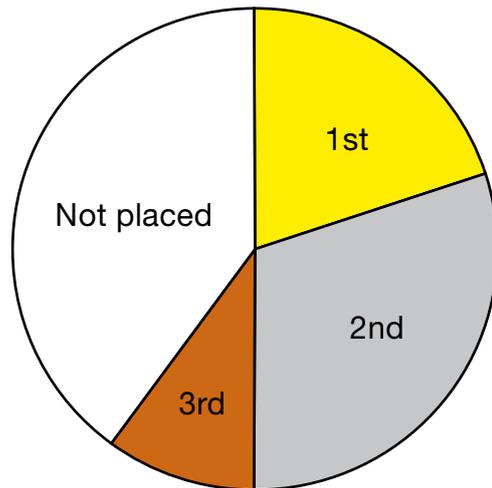
Ali's race positions
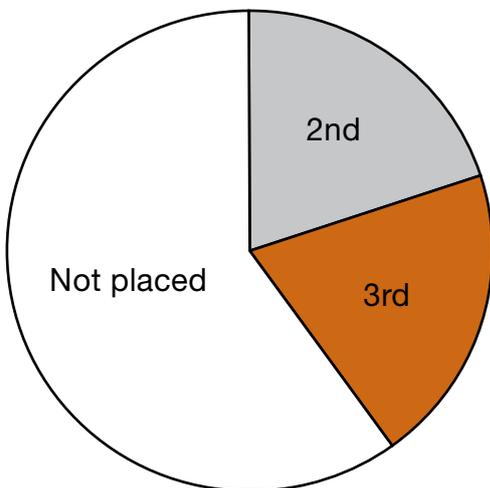
Chris's race positions
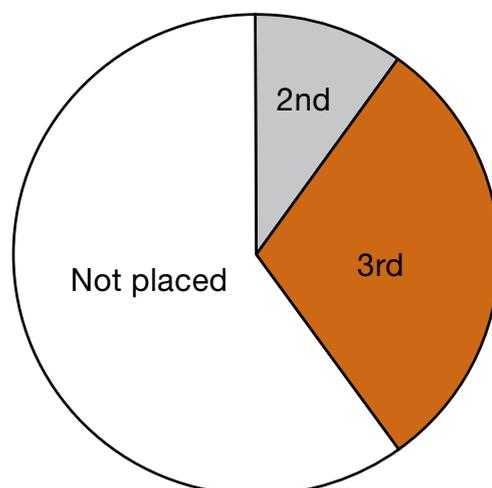
Pupil sheets

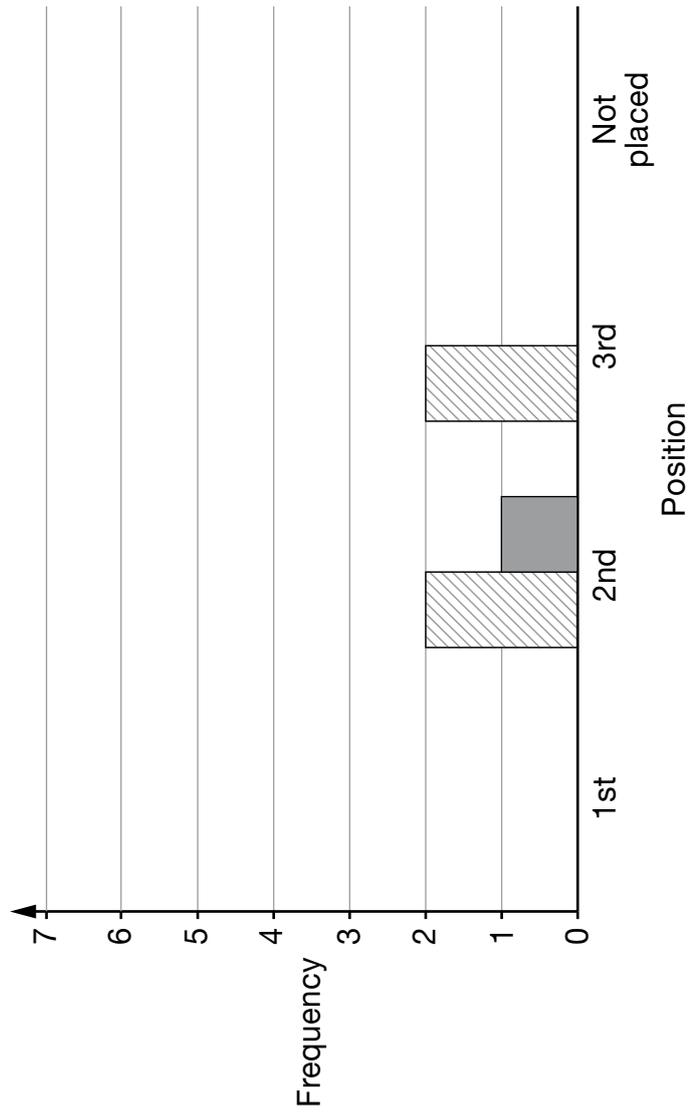*Who's who?*

Name(s): _____

RUNNER	Wk 1	Wk 2	Wk 3	Wk 4	Wk 5	Wk 6	Wk 7	Wk 8	Wk 9	Wk 10	MEAN	RANGE
Pat	12.53 1st	12.64 2nd	12.45 1st	12.65 3rd	14.02	12.47 1st	13.36	14.00	13.71	12.43 1st	13.03	1.59
Stevie	12.72 2nd	12.63 1st	12.83 2nd	13.48	12.78 2nd	13.79	13.00	12.61 1st	12.66 3rd	13.25	12.98	1.18

_____'s race positions

_____'s race positions

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Who's who? (continued)

Name(s): _____

Key:

■ ....'s race times

△ ....'s race times

Time (seconds)

Week

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T9L1assess1a

Ali v Chris

Name: _____

RUNNER	Wk 1	Wk 2	Wk 3	Wk 4	Wk 5	Wk 6	Wk 7	Wk 8	Wk 9	Wk 10	MEAN	RANGE
Ali	13.45	12.99 3rd	12.95	12.60 2nd	12.85	13.03	13.13	12.78 3rd	12.76	12.69 2nd	12.92	0.85
Chris	13.04	13.02	12.87 3rd	13.00	13.01	12.95 3rd	13.04	12.92	12.62 2nd	12.77 3rd	12.92	0.42

Complete this comparative bar chart.

Remember to add a key to your chart.

Ali's and Chris's race positions

Key:

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T9L1assess1b

Ali v Chris (continued)   Name: _____

Now use the data to complete this line graph.
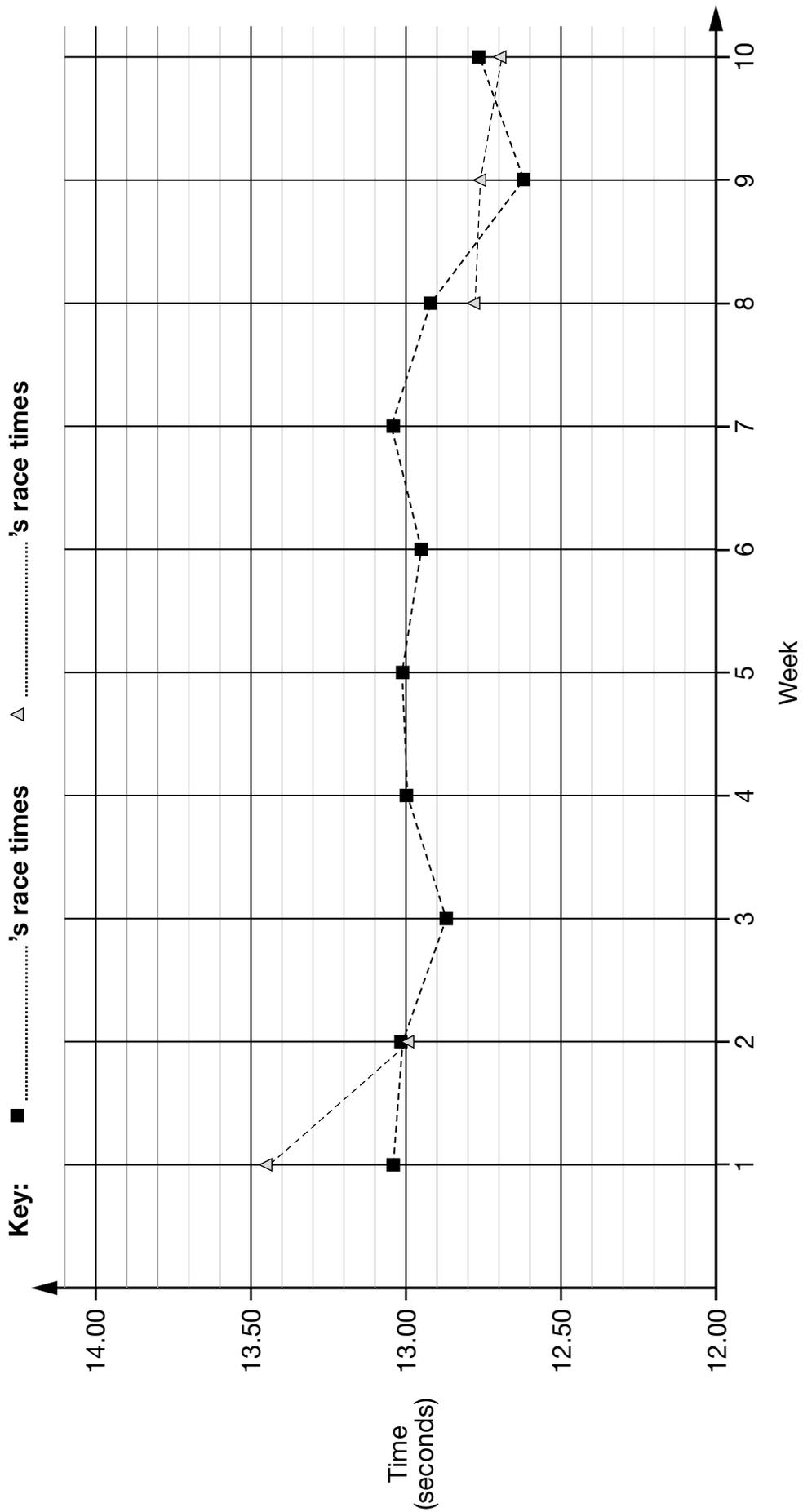
Key: ■ ....'s race times   △ ....'s race times

Ali v Chris (continued again)   Name: _____

How did Ali's and Chris's race **positions** compare?

Explain how Ali's and Chris's race **times** change over the ten weeks.

What do the mean and range tell you about each runner's performance?

Which runner do you think is better? Why?

Runners up!

Which three runners will you choose to represent the club in the championship – and why?

RUNNER	Wk 1	Wk 2	Wk 3	Wk 4	Wk 5	Wk 6	Wk 7	Wk 8	Wk 9	Wk 10	MEAN	RANGE
Pat	12.53 1st	12.64 2nd	12.45 1st	12.65 3rd	14.02	12.47 1st	13.36	14.00	13.71	12.43 1st	13.03	1.59
Stevie	12.72 2nd	12.63 1st	12.83 2nd	13.48	12.78 2nd	13.79	13.00	12.61 1st	12.66 3rd	13.25	12.98	1.18
Ali	13.45	12.99 3rd	12.95	12.60 2nd	12.85	13.03	13.13	12.78 3rd	12.76	12.69 2nd	12.92	0.85
Chris	13.04	13.02	12.87 3rd	13.00	13.01	12.95 3rd	13.04	12.92	12.62 2nd	12.77 3rd	12.92	0.42
Danny	12.76 3rd	13.54	13.41	12.54 1st	12.84 3rd	13.09	12.87 3rd	12.81	13.28	13.54	13.07	1.00
Jay	13.57	13.13	12.89	13.03	12.98	12.84 2nd	12.84 2nd	12.85	12.88	12.86	12.99	0.73
Lee	12.80	13.01	12.91	13.01	12.88	12.97	12.89	12.83	12.67	12.78	12.88	0.34
Mel	13.46	13.13	13.66	13.57	12.59 1st	12.99	12.69 1st	12.62 2nd	12.58 1st	12.79	13.01	1.08

Solutions and
performance indicators

LESSON 1: *RUNNERS UP!*   **Solutions**

Ali v Chris (target level 4/5/6)	**T9L1assess1a**
Solutions	Notes

Key correctly completed, e.g.

-

Key: ⬚ Ali

         ▥ Chris

Bar chart correctly completed, i.e.

Notes:
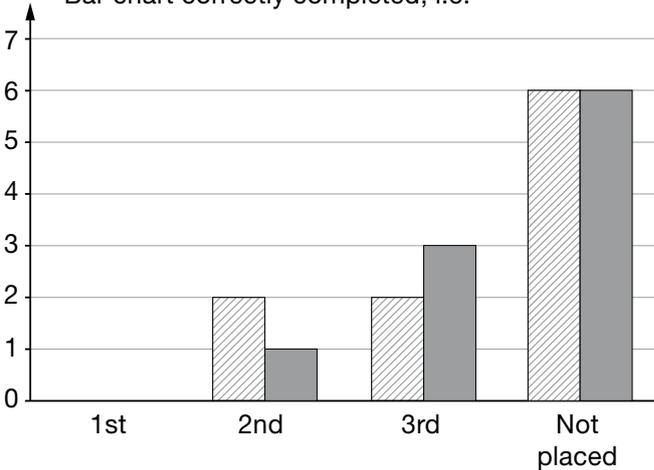
Good responses fill in some of the missing bars correctly.

Better responses give a clear key and fill in all missing bars correctly with correct shading.

Ali v Chris (continued) (target level 4/5/6)	**T9L1assess1b**
Solutions	Notes

Key correctly completed, i.e.

Key: ■ **Chris**'s race times   △ **Ali**'s race times

Line graph correctly completed with missing triangles added at (3, 12.95), (4, 12.60), (5, 12.85), (6, 13.03) and (7, 13.13), i.e.

Notes:
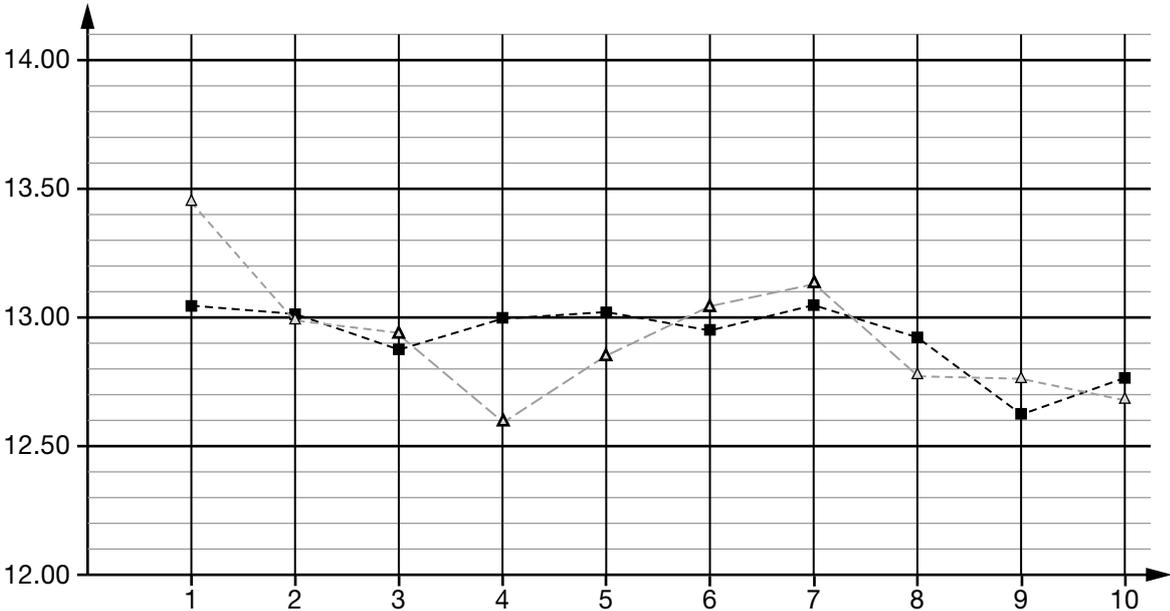
Good responses fill in some of the missing symbols accurately.

Better responses complete the key correctly and fill in all missing symbols accurately, joining them with straight, dashed or dotted lines.

Ali v Chris (continued again) (target level 4/5/6)	T9L1assess1c
Solutions	Notes
Statement(s) referring to the data comparing Ali's and Chris's race positions, e.g. • 'Ali came 2nd three times and 3rd once, but Chris only came 2nd twice and 3rd twice' • 'Neither of them came 1st' • 'They got 'not placed' the same number of times' • 'You can see from the bar chart that they both weren't placed six times but Ali is better because s/he got better positions the other four times' • 'They beat each other exactly the same number of times'	**Good** responses refer to 1st positions and the number of 'not placed' races. **Better** responses also refer to the balance of 2nd and 3rd positions.
Statement(s) referring to the data describing how Ali's and Chris's race times changed over the weeks, e.g. • 'Ali's went down, then up again, then down again' • 'Chris's times stayed fairly constant though seemed to gradually improve overall' • 'Ali started off a lot slower than Chris but got faster by week 4, then they were very close' • 'Chris didn't get much faster until week 8, then improved at the end' • 'They both got faster by the end of the ten weeks'	**Good** responses refer to high and low points in the times, though may incorrectly interpret high numbers as 'good' or 'fast' times. **Better** responses also give a general picture of trends in both runners' times.
Statement(s) explaining what the mean and range can and do show, e.g. in general terms: • 'Mean shows what the average time is overall' • 'You get the mean by adding up all the times and dividing by how many there are (or 10)' • 'A very fast runner would have a low mean' • 'Range tells you how spread out the times are' • 'Range is the highest time take away the lowest' for Ali and Chris: • 'Ali and Chris have the same mean, so they were about equal over the ten weeks' • 'Both means are 12.92, so you can't tell who is better from that' • 'Ali's range is bigger than Chris's, so Ali is less consistent' • 'Chris's range is 0.42 and Ali's range is 0.85, so Chris is always close to the average whereas Ali will sometimes be fast, but sometimes slow'	**Good** responses compare values for mean and range for Ali and Chris. **Better** responses also interpret the mean and the range in the given context.
Chooses either Ali or Chris with a justification based on the data, e.g. • 'I think Chris is better because they are the same on average, but Chris is more consistent' • 'Chris, since their means are equal but s/he was getting gradually better and was not so erratic' • 'They are the same overall, but Ali could have a good day and put in a really fast time' • 'Ali, because s/he got more 2nd places than Chris and Chris got more 3rd places' • 'I would choose Ali since s/he ran faster than the mean time more often than Chris'	**Good** responses give a reason for their decision based on one of the graphs. **Better** responses also refer to the mean and range for the runners.

LESSON 1: *RUNNERS UP!*   Performance indicators

Note that performance indicators involving an element of 'Using and applying mathematics' are given in **bold**.

Worksheet	Performance indicators
Ali v Chris (target level 4/5/6) **T9L1assess1a** *Ali v Chris (continued)* (target level 4/5/6) **T9L1assess1b** *Ali v Chris (continued again)* (target level 4/5/6) **T9L1assess1c**	**Level 4:** At this level, pupils are generally able to: • complete a bar chart correctly using a table of data, writing a correct key for the chart; • interpret a scale involving decimals to plot some of the points on a line graph correctly, joining them with straight lines; • make some observations about the positions of two runners over ten races; • note that the given means of two runners are the same; • **give logical reasons when considering which of two runners is better**, probably based on their positions in past races. However, they are less likely to be able to: • interpret a scale involving decimals consistently correctly; • appreciate the importance, in the given context, of joining the points of a line graph with dashed or dotted lines; • make correct observations about the trends in two runners' times over ten races, understanding that lower times are faster/improved times; • **recognise that a small difference in range is significant in the given context**; • **show understanding of mean or range in the given context**. **Level 5:** At this level, pupils are generally able to: • interpret a scale involving decimals consistently correctly to complete a line graph, joining the points with dashed or dotted lines; • make detailed observations about the positions of two runners over ten races; • make correct observations about the trends in two runners' times over ten races, understanding that lower times are faster/improved times; • show some understanding of either mean or range, even if this is not related to the given context; • **give logical reasons when considering which of two runners is better**, possibly considering range. However, they are less likely to be able to: • **interpret both mean and range correctly within the given context**; • **present a detailed argument when considering which of two runners is better** that takes both mean and range into account. **Level 6 and above:** At this level, pupils are generally able to: • give evidence for the performance indicators listed previously for pupils working at level 5, plus; • **interpret both mean and range correctly within the given context**; • **present a detailed argument when considering which of two runners is better, taking account of a range of evidence**.

LESSON 2: *RUNNERS UP! (AGAIN)*   **Solutions**

Runners up! (target level 4/5/6)	**T9L2assess1**
Solutions	Notes
Chooses three runners with justifications based on the data and supporting charts they have produced, e.g. for Pat: • 'Pat has come 1st more than anyone else, 2nd once and 3rd once' • 'Pat hasn't got a good mean and is not consistent, but s/he could run very fast on the day' for Stevie: • 'Stevie doesn't run well every time, but does get a medal most of the time' for Ali: • 'Ali has got the second best mean and the fourth best range, so is quite consistent' for Chris: • 'Chris has quite a fast mean and is very reliable because s/he has the second best range' for Danny: • 'Danny is a good bet as he has quite a low mean and range but comes in the top three quite often' for Jay: • 'Jay has come 2nd twice, was near 13 seconds every time and was usually under 13 seconds' for Lee: • 'Lee hasn't come in the top three, but is by far the most consistent and overall the fastest runner' • 'Lee has got the best mean and will always put in a good performance' for Mel: • 'Mel has got the second-most gold medals and one silver, so might be good for one big race'	**Good** responses choose runners based on their positions in past races, with a supporting chart. **Better** responses take account of a range of evidence, produce supporting charts and discuss which considerations are most important within the context.

LESSON 2: *RUNNERS UP! (again)*   Performance indicators

Note that performance indicators involving an element of 'Using and applying mathematics' are given in **bold**.

Worksheet	Performance indicators
Runners up! (target level 4/5/6) **T9L2assess1**	**Level 4:** At this level, pupils are generally able to: • **select an appropriate chart** for one aspect of their three runners' performances, e.g. a bar chart showing 1st, 2nd, 3rd and lower places; • **present a simple argument** for their three runners, based on one aspect of performance, e.g. positions in past races. However, they are less likely to be able to: • show <u>all</u> information in their chart(s) accurately and clearly; • plot an accurate line graph to show trends in the runners' times; • **refer to statistics such as mean and range within their argument**. **Level 5:** At this level, pupils are generally able to: • **select appropriate chart(s)** for aspect(s) of their runners' performances; • complete selected chart(s) with a good degree of accuracy and clarity; • **present a logical argument** for their three runners based on different aspects of performance, possibly referring to mean and/or range. However, they are less likely to be able to: • use a range of evidence to support each decision, possibly discussing reasons for not selecting certain runners; • **interpret both mean and range correctly within the given context**; • **comment on the wider context**, e.g. considering which characteristics might be beneficial in different situations. **Level 6:** At this level, pupils are generally able to: • **select appropriate chart(s)** for aspect(s) of their runners' performances; • complete selected chart(s) with a good degree of accuracy and clarity; • **present a logical argument** for their three runners based on different aspects of performance, including **correct interpretation of mean and range in the given context**; • use a range of evidence to support each decision, possibly discussing reasons for not selecting certain runners; • **make some comment on the wider context**, e.g. noting that a single race or overall performance in a series of races might produce different decisions. However, they are less likely to be able to: • use a wide range of evidence to support each decision, considering all runners before making a final decision; • **comment in detail on the wider context**. **Above level 6:** At this level, pupils are generally able to: • give evidence for the performance indicators listed previously for pupils working at level 6, plus; • **present a sophisticated argument** for their three runners based on many different aspects of performance; • use a wide range of evidence to support each decision, considering all runners before making a final decision; • **comment in detail on the wider context**.