

Assessing pupils' progress in mathematics at Key Stage 3

Year 9 assessment package
Shape, space and measures
Teacher pack



Year 9 Shape task: *Wrap around* and *Prismatic*

Levels (3/4/5/6)

Note that for classes consisting only of pupils at levels 3 and 4, teachers may wish to explore the material in lesson 1 more thoroughly, using interlocking cubes and creating the bands from squared paper, rather than progressing to lesson 2.

The lesson plans in this pack are set out in two columns. The left-hand column has indicative times for activities, highlights the resource sheets required and also has some examples of questions which teachers may wish to use with pupils during the activities. The right-hand column describes each activity in detail.

APP ASSESSMENT CRITERIA

These lessons may generate evidence to help inform judgements against a number of assessment criteria, including the following:

Shape, space and measure

- level 6: deduce and use formulae for the area of a triangle and parallelogram, and the volume of a cuboid; calculate volumes and surface areas of cuboids
- level 6: know and use the formulae for the circumference and area of a circle

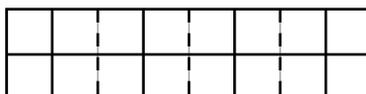
Using and applying mathematics

- level 6: solve problems and carry through substantial tasks by breaking them into smaller, more manageable tasks, using a range of efficient techniques, methods and resources, including ICT; give solutions to an appropriate degree of accuracy
- level 6: present a concise, reasoned argument, using symbols, diagrams, graphs and related explanatory texts.

LESSON 1: *WRAP AROUND*

Resources

- Eight interlocking cubes, each 1 unit in length
- A piece of squared paper, 2 units by 8 units, scored or folded along the dotted lines as shown:



- Centimetre cubes, for pupils, as needed
- Teacher resource OHT/whiteboard slides:
 - Wrap around (T11L1teacher1)*
 - Wrap around 1 solutions (T11L1teacher2)*
 - Wrap around 2 solutions (T11L1teacher3)*
 - Cross-sections of prisms (T11L1&2teacher4)*
- Each level (3/4/5) pupil, or group of pupils, needs one of the following:
 - Wrap around sheet 1 (T11L1pupil1)*Each level 5/6 pupil, or group of pupils, needs one of the following:
 - Wrap around sheet 2 (T11L1pupil2)*
- Assessment sheets for pupils:
 - Each pupil needs one or more of the following worksheets, depending on ability:
 - Level (3/4/5) pupils: *Pernickety prisms sheet 1 (T11L1assess1)*
 - Level 5/6 pupils: *Pernickety prisms sheet 2 (T11L1assess2)*
- Isometric paper for the group activity
- Centimetre squared paper and isometric paper for the assessment activity. The least able pupils may need access to cubes and scissors
- Calculators should not be needed for the activities, but may be used if necessary, as pupils are being assessed in shape, space and measures

Starter
about 10 minutes

T11L1teacher1

*Why is this shape called a prism rather than a cuboid? Is a cuboid a prism? [Yes.]
If I remove one cube from the prism, will the remaining shape still be a prism? [No.]
What if I remove 2 cubes, 3, 4, 5, ...?*

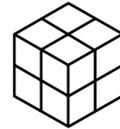
Why is the wrap around band divided into six sections? How can you tell where to start wrapping the prism?

*Why might we want to describe this solution as a 1-1-1-2-2-3 band? Would a 1-1-2-2-3-1 band be a solution? [Yes.]
Or a 1-2-2-3-1-1 band? [Yes.]
Why wouldn't a 1-2-3-1-2-1 band be a solution?*

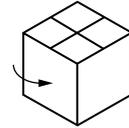
How could we describe the band for the 2-by-2-by-3 cuboid? How else could we describe the band?

What can you say about the prism that has a 3-3-3-3 band? [Assuming it is made from cubes, i.e. has right angles, the cross-section of the prism is a square.]

Show the pupils a cube made with 8 smaller cubes.

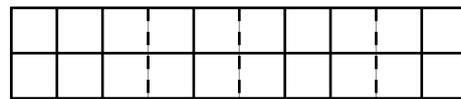
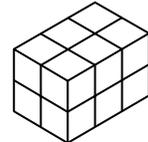


Then show how the 2-by-8 piece of paper wraps around four faces of the cube.

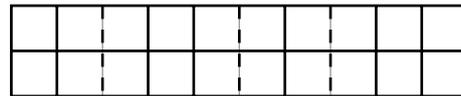


Use the top 'third' of the OHT/whiteboard slide *Wrap around (T11L1teacher1)* to show both the cube and the wrap around paper side by side.

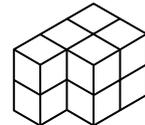
Then use the next 'third' of the same OHT/whiteboard slide to show the 2-by-2-by-3 cuboid. Ask what the wrap around paper would look like. Draw it, i.e.



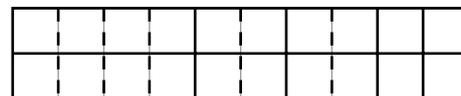
or



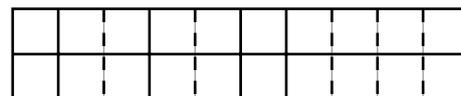
Finally, show this prism.
What would the wrap around paper look like for this prism?



Allow pupils to discuss their ideas in small groups, then together agree a solution, e.g.

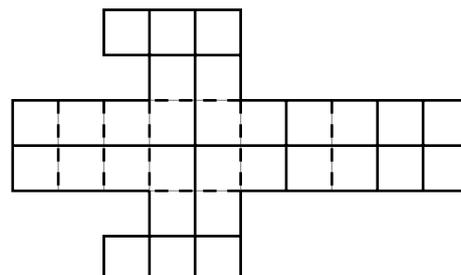


Note that the paper can start being wrapped at any edge of the prism, hence the first fold may be in a different place, e.g.



Draw a solution, ensuring that at least two, but preferably three, rows of squares above and below the 'band' (the wrap around paper) are available on the grid for further drawing.

Finally, ask the pupils how to turn the band into a net. Again, different solutions are possible depending on where the 'top' and 'bottom' faces join the band, e.g.



| | |
|---|---|
| <p>Group activity/mini-plenary about 10 minutes</p> <p>T11L1pupil1 T11L1pupil2 T11L1teacher2 T11L1teacher3</p> <p><i>Wrap around sheets 1 and 2 refer to 14 cube prisms, of height 2cm, yet the lengths of their bands are different. Can we be sure that these lengths are the minimum and maximum possible for 14 cube prisms of height 2cm? [The shape on sheet 1 is the most 'compact', so the band length is the minimum possible. The least 'compact' prism has a 1cm-by-7cm rectangular top face. This has a band length of 16cm, the same as that of the prism on sheet 2.]</i></p> <p><i>How many different 14 cube prisms could be made? [Even considering only those with a height of one cube, there are many ways of arranging the 14 cubes.]</i></p> <p><i>What is the smallest/greatest number of folds the band could have?</i></p> | <p>Give each pupil, or group of pupils, a copy of one of the following worksheets:</p> <p>Level (3/4/5 pupils): <i>Wrap around sheet 1 (T11L1pupil1)</i></p> <p>Level 5/6 pupils: <i>Wrap around sheet 2 (T11L1pupil2)</i></p> <p>Lower ability groups may need support and/or to cut out the band, which can then be placed around centimetre cubes.</p> <p>Briefly discuss the results of the group work. The teacher resource OHT/whiteboard slides: <i>Wrap around 1 solutions (T11L1teacher2)</i> <i>Wrap around 2 solutions (T11L1teacher3)</i></p> <p>show diagrammatic answers, though other views and other nets are also acceptable.</p> <p>Check that pupils use the correct units for area (cm^2) and focus especially on the fact that the surface areas of the prisms are the same as the areas of the nets.</p> |
| <p>Assessment activity about 20 minutes</p> <p>T11L1assess1 T11L1assess2</p> | <p>Give out the assessment sheets, i.e.</p> <p>Level (3/4/5 pupils): <i>Pernickety prisms sheet 1 (T11L1assess1)</i></p> <p>Level 5/6 pupils: <i>Pernickety prisms sheet 2 (T11L1assess2)</i></p> <p>All pupils will need centimetre squared paper and those doing sheet 2 will need isometric paper. The least able should also have access to centimetre cubes and scissors.</p> |
| <p>Plenary about 10 minutes</p> <p>T11L1&2teacher4</p> | <p>Say that so far we have been working with prisms made from cubes. What other types of prisms are there? [E.g. triangular.] Say that now we are going to think about cross-sections that are regular, and show the OHT/whiteboard slide: <i>Cross-sections of prisms (T11L1&2teacher4)</i></p> <p>Discuss some or all of these prisms, e.g.</p> <ul style="list-style-type: none"> • How many faces would each prism have? Is there a quick way to work this out? [Number of sides of cross-section + 2.] • What can you say about the wrap around bands for these prisms? [There will be the same number of equal sized rectangles as the number of sides of the polygon.] • What are the names of these prisms? • If the area of each cross-section is 20cm^2, and the height of each prism is 2cm, what do you know about the volume of each prism? [Each has a volume of 40cm^3.] |

LESSON 2: PRISMATIC

Resources

- Teacher resource OHT/whiteboard slides:
 - Cross-sections of prisms (T11L1&2teacher4)*
(as used in lesson 1)
 - 'Unfolded' shapes (T11L2teacher1)*
 - What do you know? (T11L2teacher2)*
- Each pupil, or group of pupils, needs one of the following worksheets:
 - Mathematical biscuits (T11L2pupil1)*Note that to save paper, two copies of the worksheet are given on each page. These need to be cut in half before the lesson
- Assessment sheets for pupils:
 - Each pupil needs one or more of the following worksheets, depending on ability:
 - Level 4/5 pupils: *Big, bigger, biggest sheet 1 (T11L2assess1)*
 - Level 5/6 pupils: *Big, bigger, biggest sheet 2 (T11L2assess2)*
- Spare paper for the assessment activity if needed
- Pupils will need calculators for the activities (or spreadsheets for the group activity and the last part of assessment sheet 1 if appropriate)

| | |
|---|---|
| <p>Starter about 10 minutes</p> <p>T11L1&2teacher4 T11L2teacher1</p> <p><i>If a regular polygon has perimeter 12cm and three sides, what sort of shape is it? [Equilateral triangle.] How long is each side? What about a square? A pentagon? How can we generalise about the length of a side of any regular polygon with perimeter 12cm?</i></p> <p><i>What is the relationship between the circumference of a circle and the radius of the circle? What other facts do you know about circles?</i></p> | <p>Start the lesson by showing again the OHT/whiteboard slide: <i>Cross-sections of prisms (T11L1&2teacher4)</i></p> <p>Say that each cross-section is of a prism of height 5cm and that each cross-section has a perimeter of 12cm. What could the net for the triangular prism look like?</p> <p>Show only the first three diagrams on the OHT/whiteboard slide: <i>'Unfolded' shapes (T11L2teacher1)</i> and ask for the dimensions of each rectangle.</p> <p>Suppose the cross-section were a regular 24-sided polygon, still of perimeter 12cm. What could the net look like?</p> <p>What about a regular 48-sided polygon? Or a regular 96-sided polygon (and so on)? Suppose the regular polygon had thousands of sides. What would the polygon look like? [Very much like a circle.] What would the rectangles look like? [Their widths would be so small that they would look as if they merged into a larger rectangle.]</p> <p>What is the special name for a solid with circular ends and a cross-section that is always the same-sized circle? [Cylinder.]</p> <p>What would a diagram of an 'unfolded' cylinder look like?</p> <p>Now show the diagram for a cylinder on the OHT/whiteboard slide: <i>'Unfolded' shapes (T11L2teacher1)</i></p> <p>Ask how to find the diameter of the circle, reminding pupils if necessary that the diameter is the circumference, i.e. 12cm, divided by pi.</p> <p>Ask how to find the radius of the circle, given the diameter. How can you find the radius if you know the circumference?</p> |
|---|---|

Group activity
about 20 minutes

T11L2pupil1

Pupils work in small groups on the worksheet:
Mathematical biscuits (T11L2pupil1)

You may wish to discuss with pupils that the context for this activity is obviously false, since in reality biscuits are not produced within the constraints given.

The amount of support pupils will need on this activity will vary. Some pupils will reason efficiently that the cylinder needs to be as high as possible OR as wide as possible to maximise the surface area. The minimum radius is 1cm, which leads to a maximum height of 19cm and a surface area of 125.6cm^2 . The minimum height is 1cm, which leads to a maximum radius of 4cm and a surface area of 125.6cm^2 . These pupils are likely to reach the solution fairly quickly, in which case they can be asked to find all integer solutions for which the volume is less than 60cm^3 but as close to 60cm^3 as possible.

Others may reason that for a given radius, only the maximum height need be calculated, leading to the following values (taking pi as 3.14):

| height | radius | volume | surface area |
|--------|--------|--------|--------------|
| 19 | 1 | 59.66 | 125.6 |
| 4 | 2 | 50.24 | 75.36 |
| 2 | 3 | 56.52 | 94.2 |
| 1 | 4 | 50.24 | 125.6 |

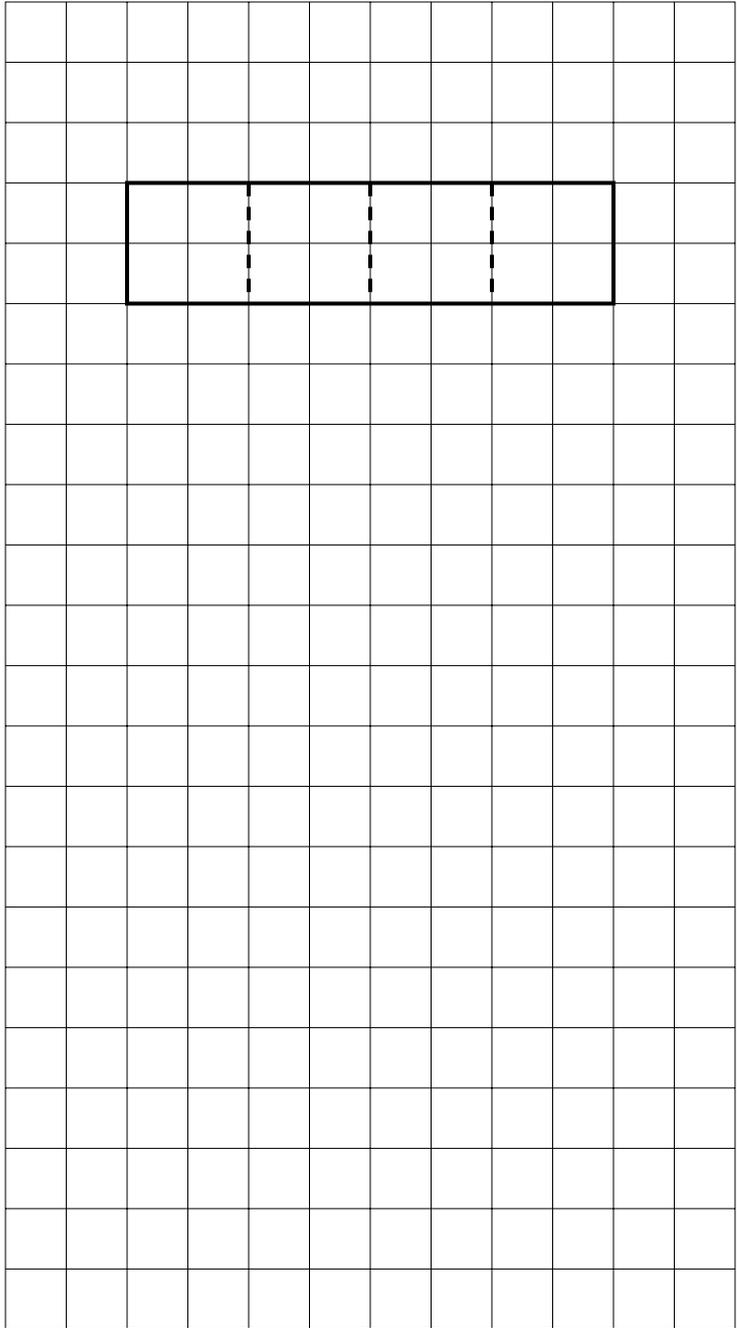
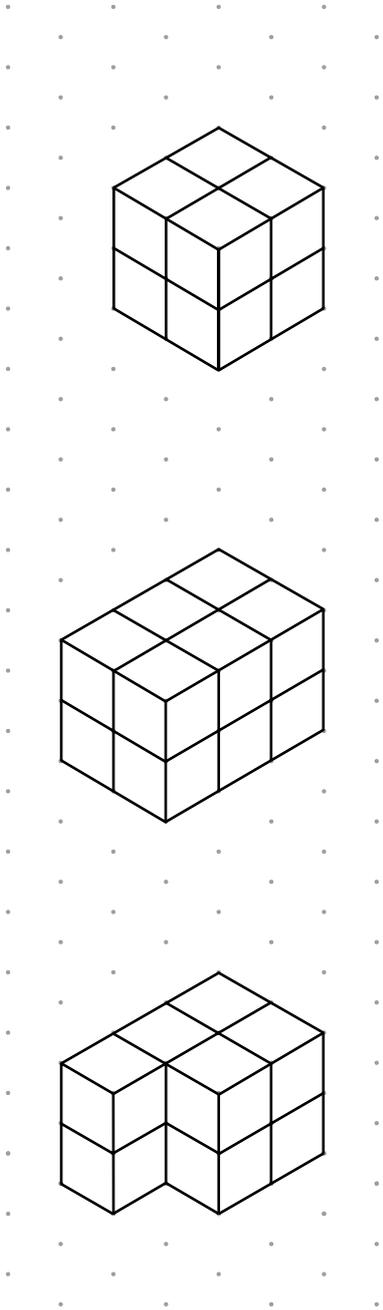
Some pupils may wish to use a spreadsheet to trial different values. The following table shows all possible solutions (taking pi as 3.14):

| height | radius | volume | surface area |
|--------|--------|--------|--------------|
| 1 | 1 | 3.14 | 12.56 |
| 1 | 2 | 12.56 | 37.68 |
| 1 | 3 | 28.26 | 75.36 |
| 1 | 4 | 50.24 | 125.6 |
| 2 | 1 | 6.28 | 18.84 |
| 2 | 2 | 25.12 | 50.24 |
| 2 | 3 | 56.52 | 94.2 |
| 3 | 1 | 9.42 | 25.12 |
| 3 | 2 | 37.68 | 62.8 |
| 4 | 1 | 12.56 | 31.4 |
| 4 | 2 | 50.24 | 75.36 |
| 5 | 1 | 15.7 | 37.68 |
| 6 | 1 | 18.84 | 43.96 |
| 7 | 1 | 21.98 | 50.24 |
| 8 | 1 | 25.12 | 56.52 |
| 9 | 1 | 28.26 | 62.8 |
| 10 | 1 | 31.4 | 69.08 |
| 11 | 1 | 34.54 | 75.36 |
| 12 | 1 | 37.68 | 81.64 |
| 13 | 1 | 40.82 | 87.92 |

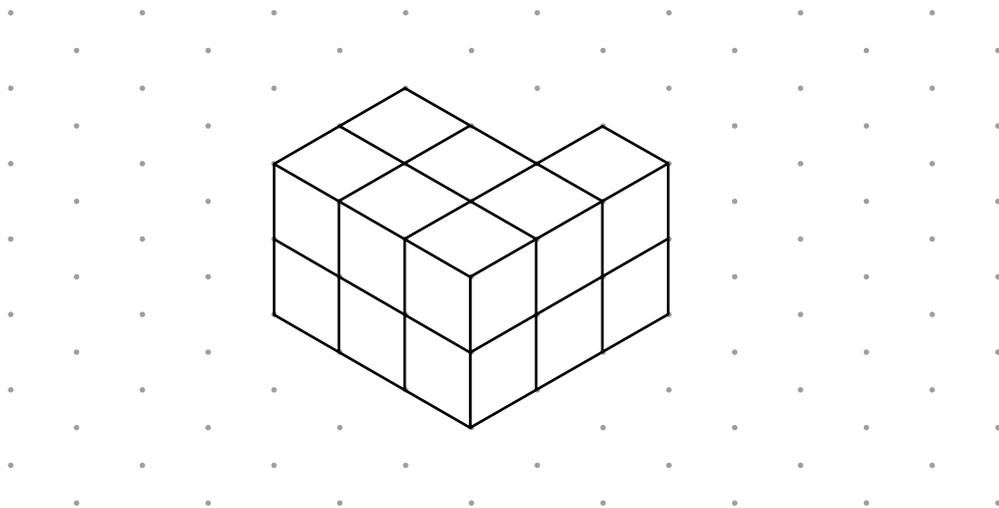
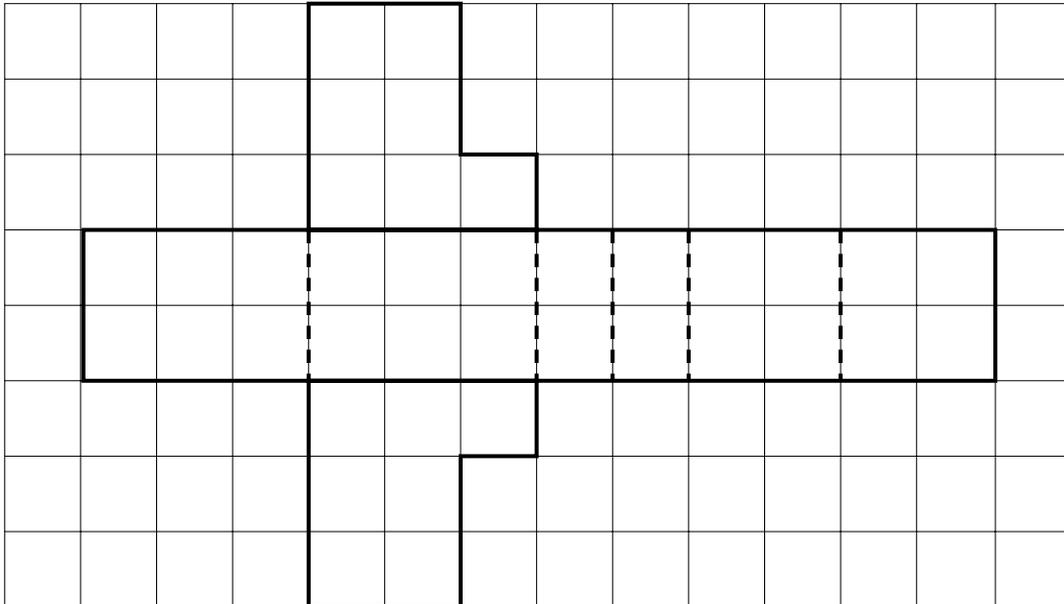
| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|-------|--------------|-------|------|----|---|------|--------|----|---|-------|--------|----|---|-------|--------|----|---|-------|--------|----|---|-------|--------------|
| | <table border="1"> <tbody> <tr> <td>14</td> <td>1</td> <td>43.96</td> <td>94.2</td> </tr> <tr> <td>15</td> <td>1</td> <td>47.1</td> <td>100.48</td> </tr> <tr> <td>16</td> <td>1</td> <td>50.24</td> <td>106.76</td> </tr> <tr> <td>17</td> <td>1</td> <td>53.38</td> <td>113.04</td> </tr> <tr> <td>18</td> <td>1</td> <td>56.52</td> <td>119.32</td> </tr> <tr> <td>19</td> <td>1</td> <td>59.66</td> <td>125.6</td> </tr> </tbody> </table> | 14 | 1 | 43.96 | 94.2 | 15 | 1 | 47.1 | 100.48 | 16 | 1 | 50.24 | 106.76 | 17 | 1 | 53.38 | 113.04 | 18 | 1 | 56.52 | 119.32 | 19 | 1 | 59.66 | 125.6 |
| 14 | 1 | 43.96 | 94.2 | | | | | | | | | | | | | | | | | | | | | | |
| 15 | 1 | 47.1 | 100.48 | | | | | | | | | | | | | | | | | | | | | | |
| 16 | 1 | 50.24 | 106.76 | | | | | | | | | | | | | | | | | | | | | | |
| 17 | 1 | 53.38 | 113.04 | | | | | | | | | | | | | | | | | | | | | | |
| 18 | 1 | 56.52 | 119.32 | | | | | | | | | | | | | | | | | | | | | | |
| 19 | 1 | 59.66 | 125.6 | | | | | | | | | | | | | | | | | | | | | | |
| | <p>After 15 minutes or so, bring the pupils together and discuss their methods. What was efficient? What was not? What would they do differently if they were starting again?</p> | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>Assessment activity about 10 minutes</p> <p>T11L2assess1 T11L2assess2</p> | <p>Give out the assessment sheets, i.e.</p> <p>Level 4/5 pupils: <i>Big, bigger, biggest sheet 1 (T11L2assess1)</i></p> <p>Level 5/6 pupils: <i>Big, bigger, biggest sheet 2 (T11L2assess2)</i></p> <p>Each pupil will need a calculator.</p> <p>Note that the final question on sheet 2 may not be attempted by all pupils as finding the surface area of the right-angled triangular prism requires the application of Pythagoras' theorem. However, an accurate scale drawing could be used if pupils have not yet met this aspect of mathematics.</p> | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>Plenary about 10 minutes</p> <p>T11L2teacher2</p> <p><i>Is the shaded cross-section a trapezium?</i> [No: it may look like one, but we do not know for certain since we do not know if any sides are parallel.]</p> | <p>Show the OHT/whiteboard slide: <i>What do you know? (T11L2teacher2)</i></p> <p>Ask pupils what else they know about the prism, e.g.</p> <ul style="list-style-type: none"> • The volume is 108cm^3 • The surface area is $2 \times 36 + 3 \times 22 = 138\text{cm}^2$ (referring back to the use of a 'wrap around' band may help explain this) | | | | | | | | | | | | | | | | | | | | | | | | |

Teacher resource sheets

Wrap around



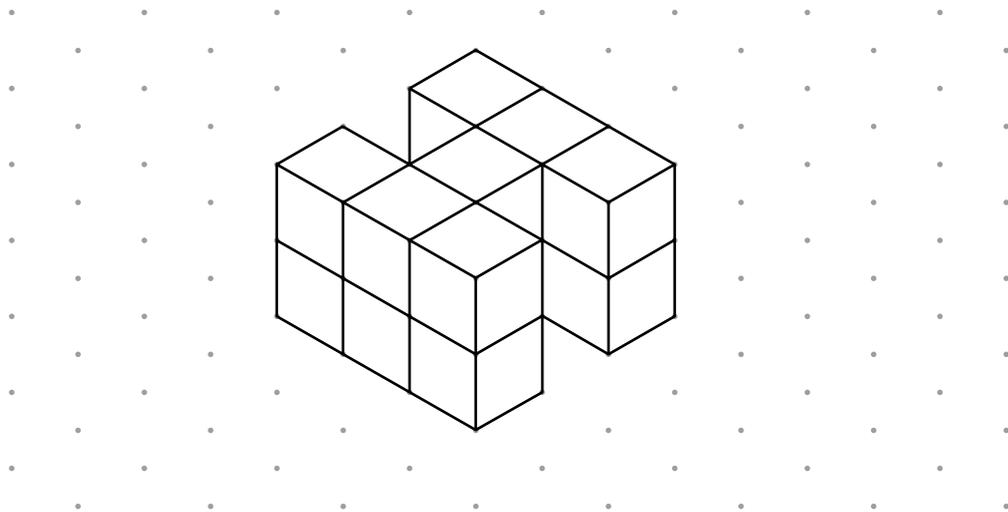
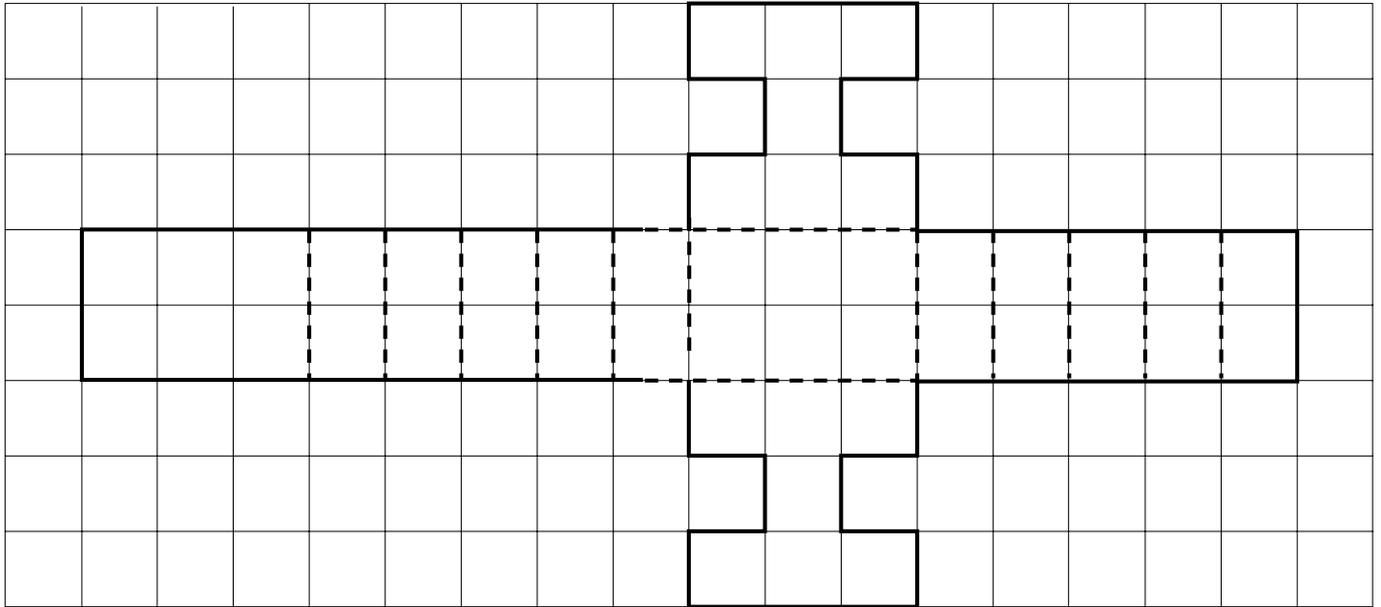
Wrap around 1 solutions



Total area of the net = **38cm²**

Surface area of the 14 cube prism = **38cm²**

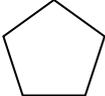
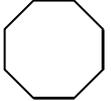
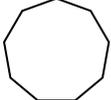
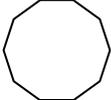
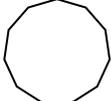
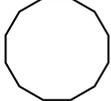
Wrap around 2 solutions



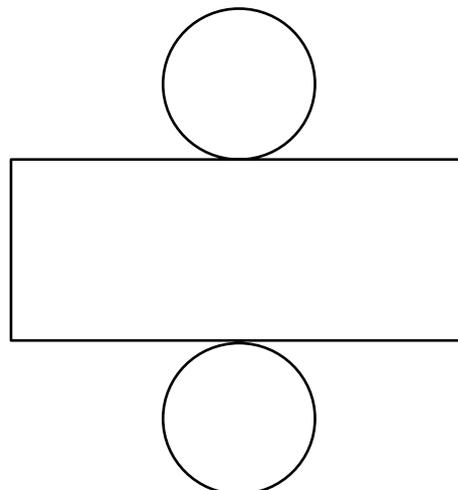
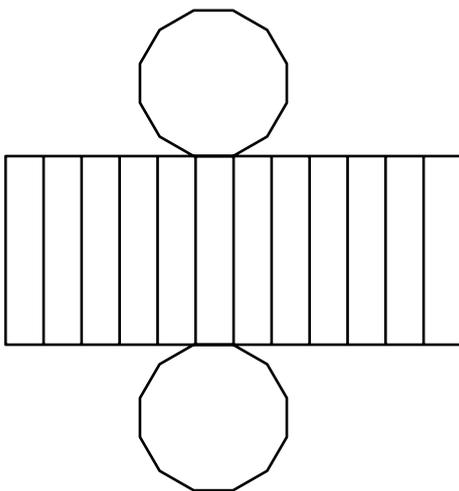
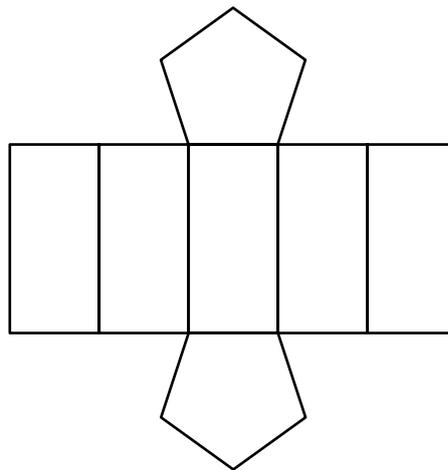
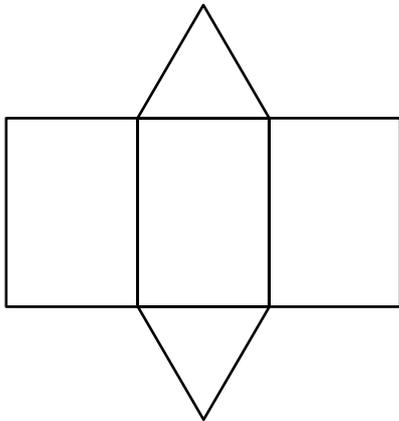
Total area of the net = 46cm^2

Surface area of the 14 cube prism = 46cm^2

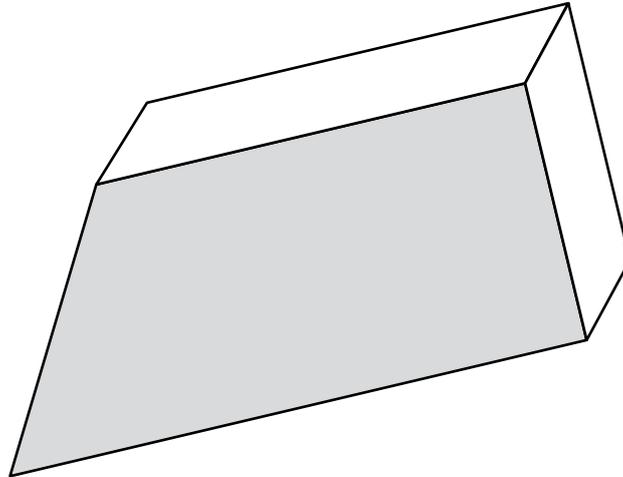
Cross-sections of prisms

| Number of sides | Cross-section of prism |
|-----------------|---|
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |
| 11 |  |
| 12 |  |

'Unfolded' shapes



What do you know?



- The area of the shaded face is 36cm^2
- The perimeter of the shaded face is 22cm
- The length of the prism is 3cm

What else do you know about this prism?

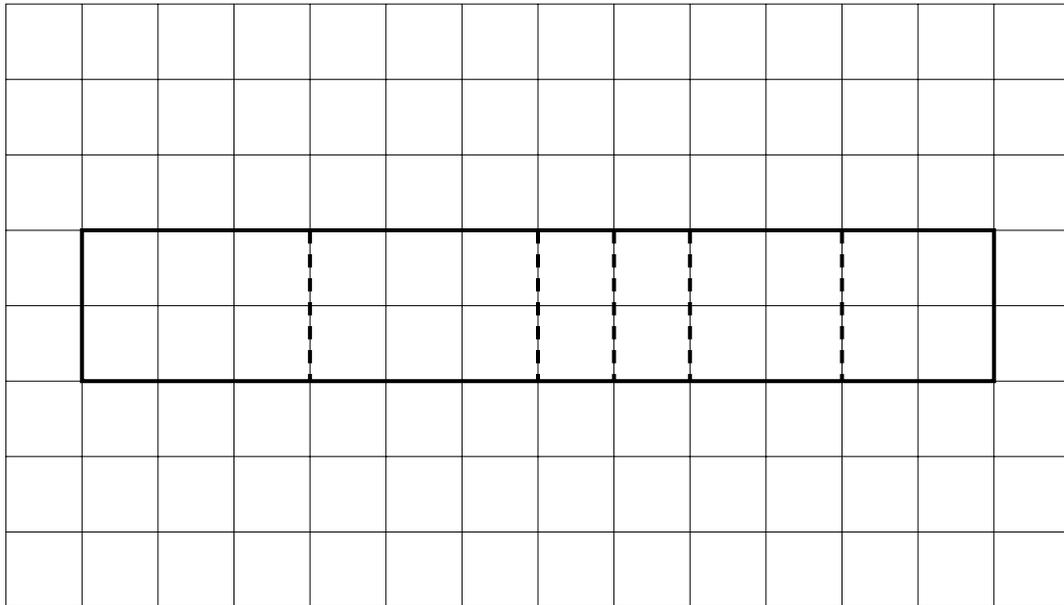
Pupil sheets

T11L1pupil1

Wrap around sheet 1

Name(s): _____

This band wraps around a 14 cube prism.



What does the 14 cube prism look like? Draw it on this isometric grid.



What could the net of the 14 cube prism look like?

Complete the net on the square grid at the top of the page.

What is the total area of the net? _____

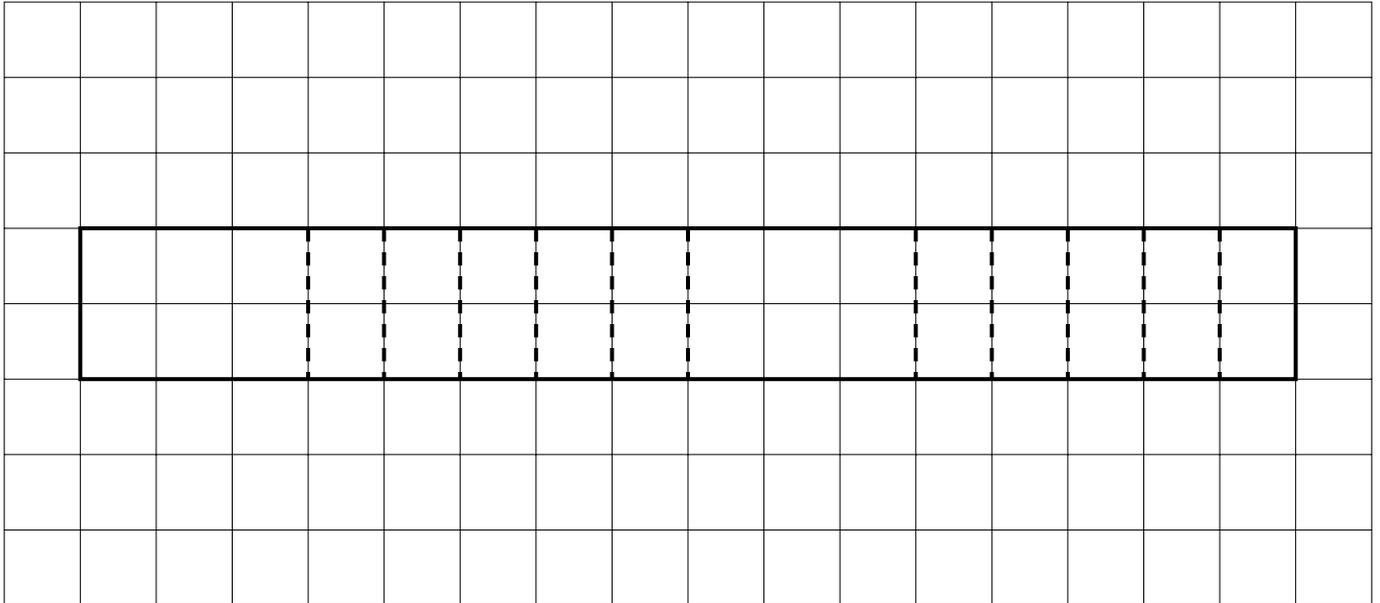
What is the surface area of the 14 cube prism? _____

T11L1pupil2

Wrap around sheet 2

Name(s): _____

This band wraps around a 14 cube prism.



What does the 14 cube prism look like? Draw it on this isometric grid.



What could the net of the 14 cube prism look like?

Complete the net on the square grid at the top of the page.

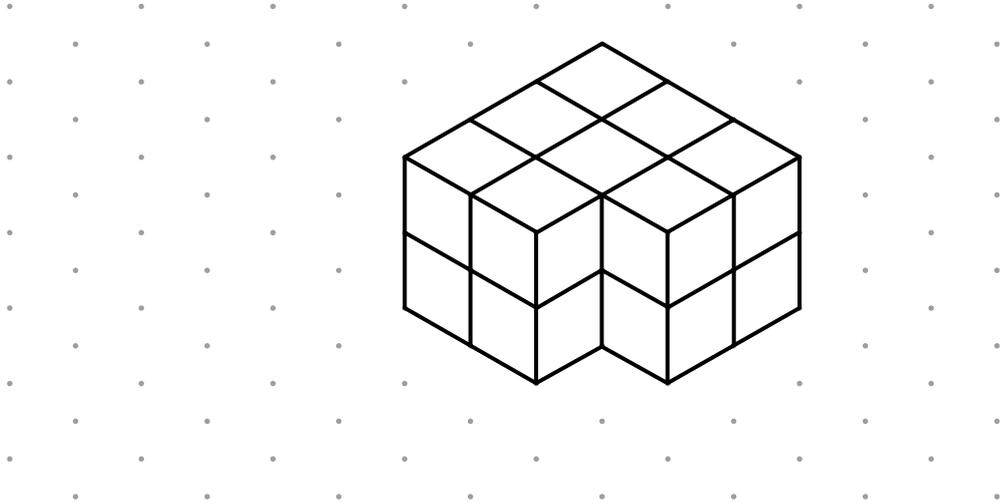
What is the total area of the net? _____

What is the surface area of the 14 cube prism? _____

Pernickety prisms sheet 1

Name: _____

Here is a drawing of a 16 cube prism.



On squared paper, draw a **net** of this 16 cube prism.

What is the **surface area** of the 16 cube prism? _____

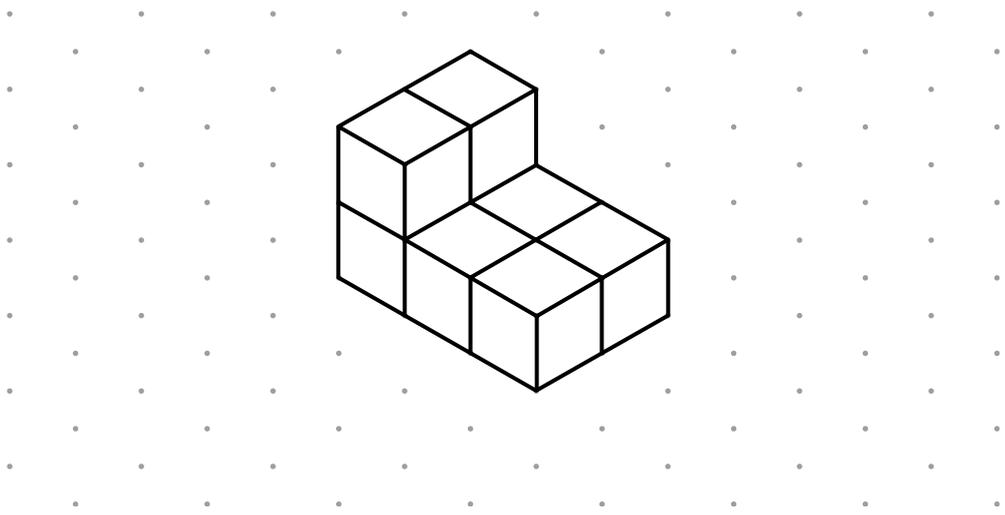
Now draw a 16 cube **cuboid**.



What is the **surface area** of the 16 cube cuboid? _____

You will need squared paper and isometric paper for your answers.

Here is a drawing of an 8 cube prism.



On squared paper, draw a **net** of this 8 cube prism.

What is the **surface area** of the 8 cube prism?

On isometric paper, draw the 8 cube prism with the **greatest possible** surface area.

What is the **surface area** of this 8 cube prism?

Explain how you know that no other 8 cube prism has a greater surface area.

What would the 8 cube prism that has the **least possible** surface area look like, and what would its surface area be?

What about a 16 cube prism? Investigate.

T11L2pupil1

Mathematical biscuits

Each biscuit must:

- be cylindrical
- have a radius that is a whole number of centimetres
- have a height that is a whole number of centimetres
- have a volume that is no more than 60cm^3



The biscuit will be covered all over in chocolate.

What is the greatest possible area that the chocolate could cover?

Mathematical biscuits

Each biscuit must:

- be cylindrical
- have a radius that is a whole number of centimetres
- have a height that is a whole number of centimetres
- have a volume that is no more than 60cm^3



The biscuit will be covered all over in chocolate.

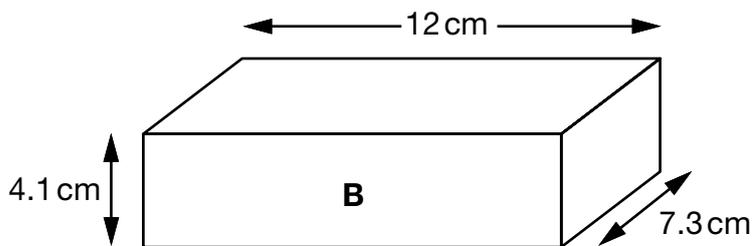
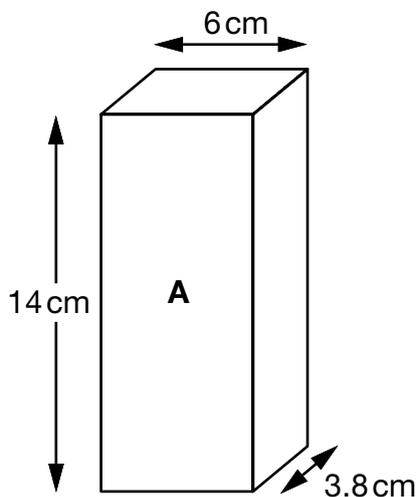
What is the greatest possible area that the chocolate could cover?

T11L2assess1

Big, bigger, biggest sheet 1

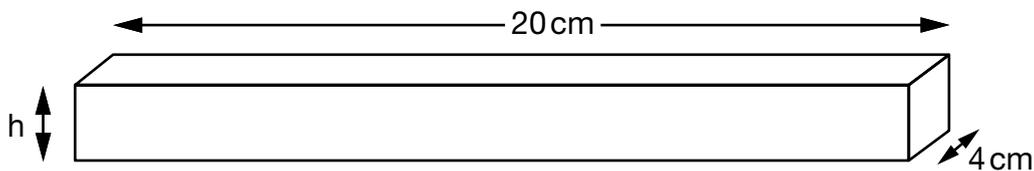
Name: _____

Here are two cuboids, A and B.



Which cuboid has the **bigger volume**? Show working to explain your answer.

The cuboid below has a **volume** of 240cm^3

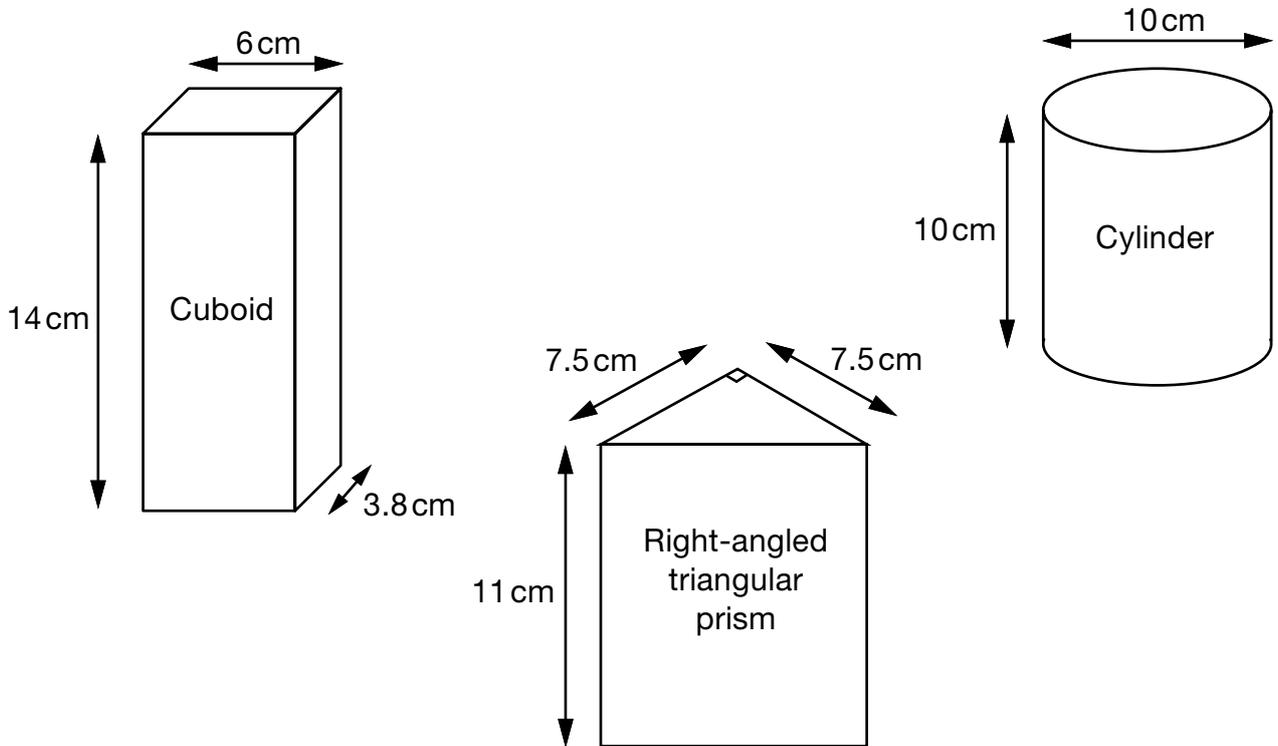


What is the height, h , of the cuboid?

What other cuboids have a volume of 240cm^3 ?

On another sheet of paper, write the lengths, widths and heights of some of them.

Here are three shapes.

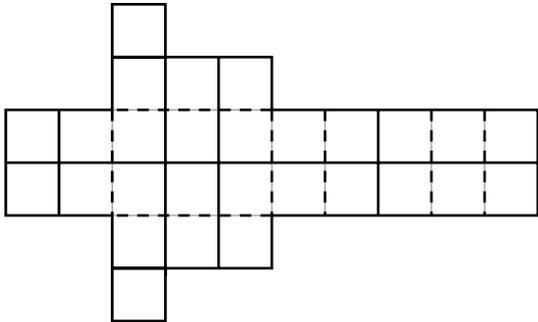
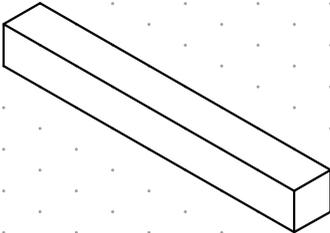
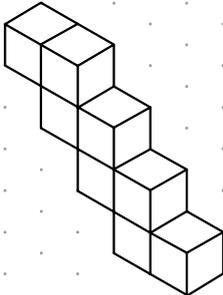


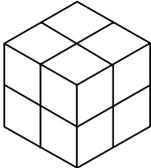
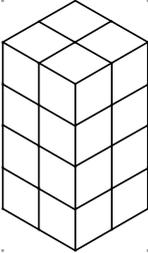
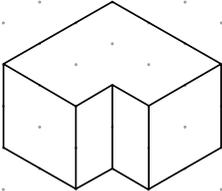
Which shape has the **biggest volume**? Show working to explain your answer.

Which shape has the **biggest surface area**? Show working to explain your answer.

Solutions and performance indicators

| Pernickety prisms sheet 1 (target level (3/4/5)) | | T11L1assess1 |
|--|--|--------------|
| Solutions | Notes | |
| <p>A correct net, with or without 'fold' lines, e.g.</p> <ul style="list-style-type: none"> | <p>Good responses show a 'wrap around band' for the prism of the correct length.</p> <p>Better responses also show the correct 'fold' lines on the band and the end faces in a correct position and orientation.</p> | |
| <p>Correct surface area or area of their net, e.g.</p> <ul style="list-style-type: none"> 40(cm²) [the correct surface area] | <p>Good responses show evidence of attempting to count or calculate the area of faces of the prism.</p> <p>Better responses use their correct net to perform an efficient calculation and give a correct unit with their area.</p> | |
| <p>A correct drawing of a 16 cube cuboid, e.g.</p> <ul style="list-style-type: none"> | <p>Good responses draw a 3-D shape using the isometric grid with minor errors or show the dimensions for a correct cuboid.</p> <p>Better responses use the isometric grid to draw a correct cuboid.</p> | |
| <p>Correct surface area for their cuboid, e.g.</p> <ul style="list-style-type: none"> 40(cm²) [2-by-2-by-4 cuboid] 48(cm²) [1-by-4-by-4 cuboid] 52(cm²) [1-by-2-by-8 cuboid] 66(cm²) [1-by-1-by-16 cuboid] | <p>Good responses show evidence of attempting to count or calculate the area of faces of their shape.</p> <p>Better responses perform an efficient calculation and give a correct unit with their area.</p> | |

| Solutions | Notes |
|--|--|
| <p>A correct net, with or without 'fold' lines, e.g.</p> <ul style="list-style-type: none">  | <p>Good responses show a 'wrap around band' for the prism of the correct length.</p> <p>Better responses also show the correct 'fold' lines on the band and the end faces in a correct position and orientation.</p> |
| <p>Correct surface area or area of their net, e.g.</p> <ul style="list-style-type: none"> 28(cm²) [the correct surface area] | <p>Good responses show evidence of attempting to count or calculate the area of faces of the prism.</p> <p>Better responses use their correct net to perform an efficient calculation and give a correct unit with their area.</p> |
| <p>A correct drawing of an 8 cube prism with the greatest possible surface area, e.g.</p> <ul style="list-style-type: none">   | <p>Good responses try different 8 cube prisms and compare surface areas.</p> <p>Better responses use the isometric grid to draw a correct prism.</p> |
| <p>Correct surface area or area of their shape, e.g.</p> <ul style="list-style-type: none"> 34(cm²) [the correct surface area] | <p>Good responses show evidence of attempting to count or calculate the area of faces of their shape.</p> <p>Better responses perform an efficient calculation and give a correct unit with their area.</p> |

| Solutions | Notes |
|---|--|
| <p>A correct explanation, e.g.</p> <ul style="list-style-type: none"> • ‘There is no other shape made of 8 cubes that is more spread out’ • ‘Other shapes you can make are shorter and fatter and this makes the surface area smaller’ • ‘Other 8 cube prisms you can make are more compact’ • ‘Joining the cubes end to end stretches the shape out as much as possible, so there are the fewest faces hidden and most are showing’ • ‘The cubes at the end have 1 side touching and all the others have 2 sides touching. If you group them more together, you’ll cover up more sides and reduce the surface area’ | <p>Good responses show evidence of trials of other 8 cube prisms.</p> <p>Better responses reason more generally about the ‘compactness’ of 3-D shapes.</p> |
| <p>Surface area = $24(\text{cm}^2)$</p> <p>A correct description or diagram of the correct cube, e.g.</p> <ul style="list-style-type: none"> • ‘A 2-by-2-by-2 cube’  | <p>Good responses show some understanding that the shape would be ‘compact’.</p> <p>Better responses show the dimensions of the cube and also give the correct surface area with a correct unit.</p> |
| <p>Within their investigation, pupils may note that the surface area of a 16 cube prism with the least possible surface area is $40(\text{cm}^2)$ and give a correct drawing, e.g.</p> <ul style="list-style-type: none"> •  •  | <p>Good responses try different 16 cube prisms and compare surface areas.</p> <p>Better responses reason more generally about the ‘compactness’ of 3-D shapes.</p> |

LESSON 1: WRAP AROUND

Performance indicators

Note that performance indicators involving an element of ‘Using and applying mathematics’ are given in **bold**.

| Worksheet | Performance indicators |
|--|---|
| <p><i>Pernickety prisms sheet 1</i> (target level (3/4/5) T11L1 assess1</p> | <p>Level 3: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • show the correct ‘wrap around band’ for a prism presented in a familiar orientation, with cubes to make models available for support. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • show the correct ‘wrap around band’ with some correct ‘fold’ lines for a prism given in a familiar orientation, with no cubes to make models; • understand that the surface area is the same as the area of their net, evaluating this area by counting squares; • show understanding of what a cuboid is by drawing a diagram; • use an isometric grid correctly to draw a 3-D shape. <p>Level 4: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • show the correct ‘wrap around band’ with some correct ‘fold’ lines for a prism given in a familiar orientation, with no cubes to make models; • understand that the surface area is the same as the area of their net, evaluating this area by counting squares; • show understanding of what a cuboid is by drawing a diagram; • use an isometric grid correctly to draw a 3-D shape. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • draw a correct net for a given prism, even if presented in an unfamiliar orientation, with ‘fold’ lines and end faces largely correct; • understand that the surface area is the same as the area of their net, evaluating this area using a more efficient method; • give correct units when finding surface areas; • show or imply the dimensions of a cuboid or other prism with a given volume; • try different prisms and compare surface areas when trying to find the greatest and/or least. |
| <p><i>Pernickety prisms sheet 2</i> (target level 5/6) T11L1 assess2</p> | <p>Level 5: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • draw a correct net for a given prism, even if presented in an unfamiliar orientation, with ‘fold’ lines and end faces largely correct; • understand that the surface area is the same as the area of their net, evaluating this area using a more efficient method; • give correct units when finding surface areas; • show or imply the dimensions of a cuboid or other prism with a given volume; • try different prisms and compare surface areas when trying to find the greatest and/or least. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • calculate surface areas of prisms without drawing nets; • identify simple prisms with the greatest and least surface areas, given the volume; • reason generally about ‘compactness’ when considering greatest and/or least surface areas. <p>(See next page for level 6 indicators)</p> |

| Worksheet | Performance indicators |
|--|--|
| <p><i>Pernickety prisms sheet 2</i> (target level 5/6) T11L1assess2</p> | <p>Level 6: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • calculate surface areas of prisms without drawing nets; • identify simple prisms with the greatest and least surface areas, given the volume; • reason generally about 'compactness' when considering greatest and/or least surface areas. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • extend ideas about 'compactness' to a prism with a volume that is not a cubic number. <p>Above level 6: At these levels, pupils are generally able to:</p> <ul style="list-style-type: none"> • give evidence for the performance indicators listed previously for pupils working at level 6, plus; • extend ideas about 'compactness' to a prism with a volume that is not a cubic number. |

| Big, bigger, biggest sheet 1 (target level 4/5) | | T11L2assess1 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|------------|--|------------|-------------|----|---|----|----|---|---|----|---|----|----|---|----|----|---|---|---|---|----|---|---|----|---|---|----|---|---|---|----|---|----|----|---|----|----|---|---|----|---|---|---|---|----|---|---|----|---|---|----|---|---|----|---|---|---|---|
| Solutions | | Notes | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>Indicates cuboid B</p> <p>A correct explanation, e.g.</p> <ul style="list-style-type: none"> $6 \times 14 \times 3.8 = 319.2\text{cm}^3$, $12 \times 4.1 \times 7.3 = 359.16\text{cm}^3$ 319(...) and 359(...) seen $6 \times 14 \times 3.8 < 12 \times 4.1 \times 7.3$ | | <p>Good responses show understanding that volume is length \times width \times height.</p> <p>Better responses give correct values for the volumes with correct units and make the correct decision.</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>$h = 3(\text{cm})$</p> | | <p>Good responses show understanding that volume is length \times width \times height.</p> <p>Better responses use division in a correct method and give the height with a correct unit.</p> | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>Dimensions of cuboids with a volume of 240cm^3, e.g.</p> <ul style="list-style-type: none"> <table border="1"> <thead> <tr> <th>length (cm)</th> <th>width (cm)</th> <th>height (cm)</th> </tr> </thead> <tbody> <tr><td>24</td><td>1</td><td>10</td></tr> <tr><td>24</td><td>2</td><td>5</td></tr> <tr><td>12</td><td>1</td><td>20</td></tr> <tr><td>12</td><td>2</td><td>10</td></tr> <tr><td>12</td><td>4</td><td>5</td></tr> <tr><td>6</td><td>1</td><td>40</td></tr> <tr><td>6</td><td>2</td><td>20</td></tr> <tr><td>6</td><td>4</td><td>10</td></tr> <tr><td>6</td><td>5</td><td>8</td></tr> <tr><td>10</td><td>1</td><td>24</td></tr> <tr><td>10</td><td>2</td><td>12</td></tr> <tr><td>10</td><td>3</td><td>8</td></tr> <tr><td>10</td><td>4</td><td>6</td></tr> <tr><td>5</td><td>1</td><td>48</td></tr> <tr><td>5</td><td>2</td><td>24</td></tr> <tr><td>5</td><td>3</td><td>16</td></tr> <tr><td>5</td><td>4</td><td>12</td></tr> <tr><td>5</td><td>6</td><td>8</td></tr> </tbody> </table> | | length (cm) | width (cm) | height (cm) | 24 | 1 | 10 | 24 | 2 | 5 | 12 | 1 | 20 | 12 | 2 | 10 | 12 | 4 | 5 | 6 | 1 | 40 | 6 | 2 | 20 | 6 | 4 | 10 | 6 | 5 | 8 | 10 | 1 | 24 | 10 | 2 | 12 | 10 | 3 | 8 | 10 | 4 | 6 | 5 | 1 | 48 | 5 | 2 | 24 | 5 | 3 | 16 | 5 | 4 | 12 | 5 | 6 | 8 | <p>Good responses show evidence of using factors of 24 and 10 to find sets of dimensions.</p> <p>Better responses use a systematic approach and possibly a spreadsheet to generate a large set of values.</p> |
| length (cm) | width (cm) | height (cm) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 24 | 1 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 24 | 2 | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 | 1 | 20 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 | 2 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 12 | 4 | 5 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 1 | 40 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 2 | 20 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 4 | 10 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 | 5 | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | 1 | 24 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | 2 | 12 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | 3 | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 10 | 4 | 6 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 1 | 48 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 2 | 24 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 3 | 16 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 4 | 12 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 | 6 | 8 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

| Big, bigger, biggest sheet 2 (target level 5/6) | | T11L2assess2 |
|---|--|--------------|
| Solutions | Notes | |
| <p>Indicates the cylinder</p> <p>A correct explanation, e.g.</p> <ul style="list-style-type: none"> $6 \times 14 \times 3.8 = 319.2\text{cm}^3$, $7.5 \times 7.5 \div 2 \times 11 = 309.375\text{cm}^3$, $\pi \times 5^2 \times 10 = 785\text{ cm}^3$ to 785.5cm^3 inclusive 319.(...), 309.(...) and 785.(...) seen $7.5 \times 7.5 \div 2 \times 11 < 6 \times 14 \times 3.8 < \pi \times 5^2 \times 10$ | <p>Good responses show understanding of how to calculate the volume of some of the shapes.</p> <p>Better responses give correct values for the volumes with correct units and make the correct decision.</p> | |
| <p>Indicates the cylinder</p> <p>A correct explanation, e.g.</p> <ul style="list-style-type: none"> $(2 \times 3.8 \times 6) + (2 \times 14 \times 6) + (2 \times 14 \times 3.8)$ = 320cm^2, $\sqrt{(7.5^2 + 7.5^2)} = 10.6066\dots$ [or by accurate drawing] $(2 \times 7.5 \times 7.5 \div 2) + (2 \times 7.5 \times 11) + (11 \times 10.6\dots)$ = 337.9(...)cm^2 or 338cm^2, $(2 \times \pi \times 5^2) + (10 \times 2 \times \pi \times 5)$ = 471cm^2 to 471.3cm^2 inclusive 320, 337.(...) (or 338) and 471.(...) seen | <p>Good responses show understanding of how to calculate the surface area of some of the shapes.</p> <p>Better responses give correct values for the surface areas with correct units and make the correct decision.</p> | |

LESSON 2: PRISMATIC

Performance indicators

Note that performance indicators involving an element of ‘Using and applying mathematics’ are given in **bold**.

| Worksheet | Performance indicators |
|---|---|
| <p><i>Big, bigger, biggest sheet 1</i> (target level 4/5) T11L2assess1</p> | <p>Level 4: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • find the volume of a cuboid; • work out one dimension of a cuboid given the other two and the volume, probably using trial and improvement or trial and error; • give dimensions for some cuboids with a given volume. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • work out one dimension of a cuboid given the other two and the volume, using a more efficient method; • give correct units for volume and length where appropriate; • use a systematic approach to generate a larger set of dimensions for cuboids with a given volume. <p>Level 5: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • work out one dimension of a cuboid given the other two and the volume, using a more efficient method; • give correct units for volume and length where appropriate; • use a systematic approach to generate a larger set of dimensions for cuboids with a given volume; • find the volume of a right-angled triangular prism; • attempt to calculate the surface area of a cuboid with some non-integer dimensions, e.g. by drawing a net. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • find the area of a circle given the diameter; • calculate the volume of a cylinder; • calculate the surface area of a cuboid with some non-integer dimensions without drawing a net; • attempt to calculate the surface area of a right-angled triangular prism, possibly using accurate drawing to find an unknown side length. |
| <p><i>Big, bigger, biggest sheet 2</i> (target level 5/6) T11L2assess2</p> | <p>Level 6: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • find the area of a circle given the diameter; • calculate the volume of a cylinder; • calculate the surface area of a cuboid with some non-integer dimensions without drawing a net; • attempt to calculate the surface area of a right-angled triangular prism, possibly using accurate drawing to find an unknown side length. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • calculate the surface area of a right-angled triangular prism, using a more efficient method to find an unknown side length; • calculate the surface area of a cylinder. <p>Above level 6: At these levels, pupils are generally able to:</p> <ul style="list-style-type: none"> • give evidence for the performance indicators listed previously for pupils working at level 6, plus; • calculate the surface area of a right-angled triangular prism, using a more efficient method to find an unknown side length; • calculate the surface area of a cylinder. |

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