

# Assessing pupils' progress in mathematics at Key Stage 3

Year 9 assessment package  
Number  
Teacher pack



## **Year 9 Number task: *Thinking proportionately* and *Secret sequences***

### **Levels 4/5/6**

Note that for classes consisting only of pupils at level 4, teachers may wish to explore the material in lesson 1 more thoroughly, rather than progress to lesson 2.

The lesson plans in this pack are set out in two columns. The left-hand column has indicative times for activities, highlights the resource sheets required and also has some examples of questions which teachers may wish to use with pupils during the activities. The right-hand column describes each activity in detail.

### **APP ASSESSMENT CRITERIA**

These lessons may generate evidence to help inform judgements against a number of assessment criteria, including the following:

#### **Algebra**

- level 5: construct, express in symbolic form, and use simple formulae involving one or two operations
- level 6: generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT; write an expression to describe the  $n$ th term of an arithmetic sequence

#### **Numbers and the number system**

- level 4: recognise and describe number relationships including multiple, factor and square
- level 4: recognise approximate proportions of a whole and use simple fractions and percentages to describe these
- level 5: recognise and use number patterns and relationships
- level 5: use equivalence between fractions and order fractions and decimals
- level 5: reduce a fraction to its simplest form by cancelling common factors
- level 6: use the equivalence of fractions, decimals and percentages to compare proportions

#### **Calculating**

- level 6: add and subtract fractions by writing them with a common denominator, calculate fractions of quantities (fraction answers), multiply and divide an integer by a fraction

#### **Using and applying mathematics**

- level 4: present information and results in a clear and organised way
- level 5: show understanding of situations by describing them mathematically using symbols, words and diagrams
- level 6: solve problems and carry through substantial tasks by breaking them into smaller, more manageable tasks, using a range of efficient techniques, methods and resources, including ICT; give solutions to an appropriate degree of accuracy.

### **LESSON 1: *THINKING PROPORTIONATELY***

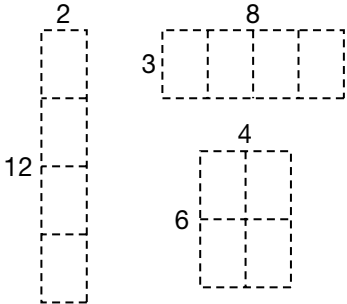
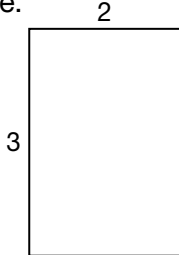
#### **Resources**

- Three teacher resource sheets:  
*Proportion cards, set 1 (T10L1resource1)*  
*Proportion cards, set 2 (T10L1resource2)*  
*Proportion cards, set 3 (T10L1resource3)*

Preparation: Before the lesson, print or stick onto card sufficient copies of the resource sheets to enable each group to have a complete set for the group work. Cut the cards, and shuffle to lose the ordering

- OHTs/whiteboard slides of the three teacher resource sheets to show the correct groups of cards

- Assessment sheets for pupils:  
Each pupil needs one of the following worksheets, depending on ability:  
Level 4/5 pupils: *Using proportions, sheet 1 (T10L1assess1)*  
Level 5/6 pupils: *Using proportions, sheet 2 (T10L1assess2)*
- Squared paper for pupils to work on throughout the lesson, including the assessment

<p>Starter about 10 minutes</p> <p>Solutions:</p>  <p><i>Is it harder to find a possible larger rectangle when the original area is <math>\frac{2}{3}</math> of the larger rather than <math>\frac{1}{4}</math> of it?</i></p> <p><i>What efficient methods are there to solve each problem?</i></p> <p><i>What if the proportion had been <math>\frac{1}{3}</math> rather than <math>\frac{2}{3}</math>?</i></p> <p><i>Is a numerator of 1 helpful?</i></p>	<p>Show the pupils a 2 by 3 rectangle in the orientation shown here (no internal lines shown at this stage). Explain that you have shown them only part of a larger rectangle.</p>  <p>Firstly, tell them that this smaller rectangle is <math>\frac{1}{4}</math> of the larger rectangle.</p> <p>What could the dimensions of the larger rectangle be? Ask a pupil to draw it and then explain their reasoning.</p> <p>What else could the dimensions be? And what else? Are there any other solutions? How do you know? (Use of factors shows there are only three solutions if the original 2 by 3 rectangle is not cut. If it is cut, there is an infinite number of solutions.)</p> <p>Start again with the 2 by 3 rectangle. Now explain that its area is <math>\frac{2}{3}</math> of a larger rectangle. Ask a pupil to draw the larger rectangle and then explain their reasoning.</p> <p>What else could the dimensions be? (Pupils are most likely to find the 3 by 3 and the 2 by 4.5 solutions, although there is an infinite number of rectangles with area 9.)</p> <p>Finally, starting again with the 2 by 3 rectangle, say it is now <math>\frac{3}{10}</math> of a larger rectangle. Again ask pupils to draw and explain.</p> <p>What was it about the numerators and denominators of the fractions <math>\frac{1}{4}</math>, <math>\frac{2}{3}</math> and <math>\frac{3}{10}</math> that made the dimensions of the larger rectangle easier to work out?</p>
<p>Group activity about 10 minutes</p> <p><b>T10L1resource1</b> <b>T10L1resource2</b> <b>T10L1resource3</b></p>	<p>Briefly introduce the activity. In groups, pupils should match cards of part-shapes into sets with the whole shape that has the correct area. There are three sets that should be cut out and shuffled (<b>T10L1resource1, 2 and 3</b>).</p> <p>Discuss the pupils' sets and show the correct sets on an OHT/whiteboard. Briefly discuss pupils' methods.</p>
<p>Assessment activity about 20 minutes</p> <p><b>T10L1assess1</b> <b>T10L1assess2</b></p>	<p>Give out the assessment sheets as follows: Level 4/5 pupils: <i>Using proportions, sheet 1 (T10L1assess1)</i> Level 5/6 pupils: <i>Using proportions, sheet 2 (T10L1assess2)</i></p> <p>Ensure pupils have squared paper to work on, and remind them to write their names on their work.</p>

Plenary  
about 10 minutes

*If the first rectangle measures 2 by 3, the next 3 by 4, the third 4 by 5 and so on, what are the measurements of the 12th rectangle? What about the nth rectangle?*

*What is the area of the nth rectangle?*

*What fraction of the area of the nth rectangle is the area of the first?*

*How would you explain to someone that  $\frac{6}{20}$  simplifies to  $\frac{3}{10}$  and  $\frac{6}{30}$  to  $\frac{1}{5}$ , etc?*

*Will the fraction always be between 0 and 1? How can we be certain?*

Start with the 2 by 3 rectangle again. Explain it is  $\frac{1}{2}$  of a larger rectangle and ask the pupils to draw it. What else could the dimensions be?

Thinking about the 3 by 4 rectangle in particular, discuss the sequence of a 2 by 3 rectangle as a fraction of a 3 by 4, then of a 4 by 5, then of a 5 by 6 and so on, i.e.

$$\frac{6}{12}, \frac{6}{20}, \frac{6}{30}, \frac{6}{42} \text{ etc.}$$

Draw diagrams of the rectangles. Briefly discuss simplifying the fractions and note that this will be looked at further in the next lesson.

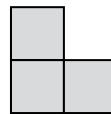
## LESSON 2: SECRET SEQUENCES

### Resources

- Assessment sheets for pupils:  
Each pupil needs one of the following worksheets, depending on ability:  
Level 4/5 pupils: *Secret sequences, sheet 1 (T10L2assess1)*  
Level 5/6 pupils: *Secret sequences, sheet 2 (T10L2assess2)*
- Squared paper for the assessment for level 4/5 pupils
- Paper for any rough working
- Calculators (for group activity in particular)

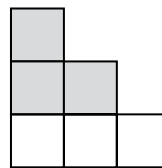
Starter  
about 10 minutes

Draw the following shape with area 3 and colour in red  
(or another bright colour):



Explain its area is half the area of another shape. Ask pupils to draw what the shape could be. Prompt pupils to give examples where the two halves are not congruent.

Now focus on one possibility, i.e.



(Use colour/shading to distinguish the original shape.)

What fraction of the whole shape is the red part? How else could the fraction be written? (Ensure  $\frac{3}{6}$  is one example.)

Add another row of four squares along the bottom. Now what fraction of the whole shape is the red part? Say that this is the first part of a sequence and write  $\frac{3}{6}$  and  $\frac{3}{10}$  as the first two terms.

How might the sequence continue? Generate the first four terms, by adding rows (one square longer each time) to the diagram.

Put the results into a table (see next page), asking pupils for the decimal and percentage equivalents.

Briefly discuss how the decimal and percentage columns are useful to see that the size of the terms is decreasing, but then ask which column of the table is the most useful to work out what the next term of the sequence will be.

Generate the next three or four terms as fractions and simplify (no need for decimal/percentage equivalents).

*How do the diagrams continue? In the next (third) diagram, how many squares are there in the longest row? How many squares are there in the longest row of the nth diagram?*

*Look at the sequence of unsimplified fractions. How can we find the next denominator in the sequence?*

*Which sets of equivalent fractions, decimals and percentages do you know?*

*How would you find the decimal and percentage equivalents for  $\frac{5}{12}$ ? Or ...?*

*Give me a fraction between  $\frac{1}{3}$  and  $\frac{1}{2}$ . How did you find it? How could you use decimals?*

From one set that you know (e.g.  $\frac{1}{10} \equiv 0.1 \equiv 10\%$ ), which others can you deduce?

In the table, the decimal and percentage for  $\frac{1}{7}$  has been rounded. How can we tell that  $\frac{1}{7}$  is a recurring decimal? How could we write it?

As the sequence progresses, can we spot other recurring decimals from their fractional equivalents? How can we tell?

Predict the next recurring decimal in the sequence.

Which of the following is closest to  $\frac{1}{3}$ ?

$\frac{10}{31}, \frac{20}{61}, \frac{30}{91}, \frac{50}{151}$

Look at the fraction/decimal/percentage columns. What is a useful strategy for ordering fractions?

Shape number	Fraction		Decimal	Percentage
	Not simplified	Simplified		
1	$\frac{3}{6}$	$\frac{1}{2}$	0.5	50%
2	$\frac{3}{10}$	$\frac{3}{10}$	0.3	30%
3	$\frac{3}{15}$	$\frac{1}{5}$	0.2	20%
4	$\frac{3}{21}$	$\frac{1}{7}$	0.143...	14.3...%

Discuss how the column of simplified fractions 'hides' the number pattern and say that a sequence like this is going to be called a 'secret sequence' in this lesson.

### Group activity about 5 minutes

How does looking separately at the numerators and denominators help to solve the secret sequence?

How might we tell whether the fractions are getting closer to 1 or closer to 0 as the sequence continues?

Explain why the  $n$ th fraction in the sequence is  $\frac{n+3}{2n+3}$

Can you tell from the  $n$ th term that the denominator is always greater than the numerator?

Quickly think of an expression for the  $n$ th fraction where (a) the numerator 'catches up' with and overtakes the denominator (b) the fractions are always greater than 1

How can we use the expression for the  $n$ th fraction to find, say, the 10th fraction?

How might you go about making up a secret sequence of your own? How could you be sure it would work?

Show the pupils the fraction sequence  $\frac{4}{5}, \frac{5}{7}, \frac{2}{3}, \frac{7}{11}, \frac{8}{13}$ .

Ask them to work in groups to find out how the sequence works and predict the next fraction.

After a few minutes, even if not all groups have finished, discuss how the sequence is formed (seen by converting  $\frac{2}{3}$  to  $\frac{6}{9}$ ), agree on the rule and therefore that the next term is  $\frac{9}{15}$ , which simplifies to  $\frac{3}{5}$ .

The rule described is likely to be of the form:

numerator = previous numerator + 1, and

denominator = previous denominator + 2

Discuss also how this could be written as  $\frac{n+3}{2n+3}$  where  $n$  is the term/shape number.

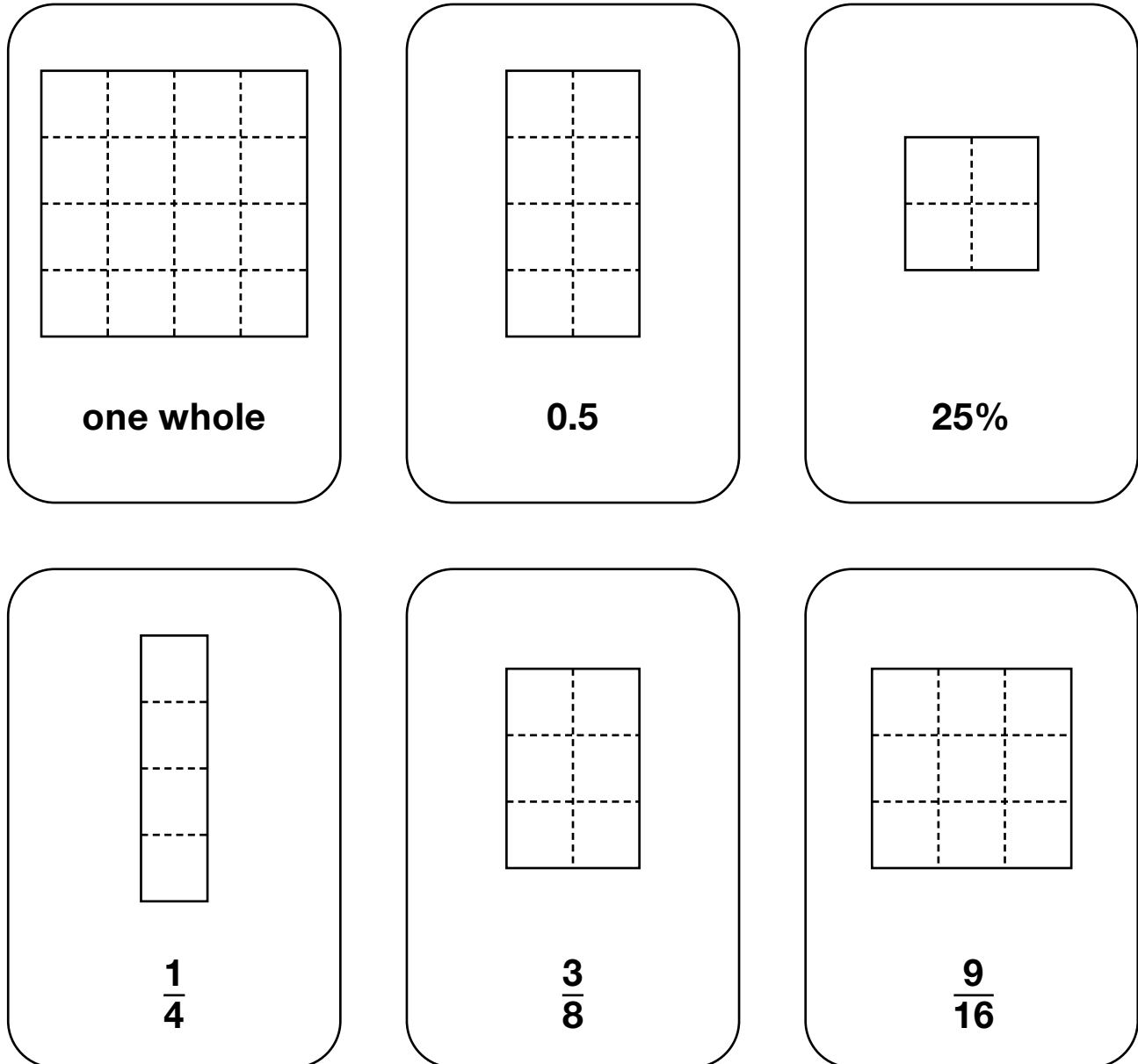
<p>Assessment activity about 30 minutes</p> <p><b>T10L2assess1</b> <b>T10L2assess2</b></p>	<p>Give out the assessment sheets as follows: Level 4/5 pupils: <i>Secret sequences, sheet 1 (T10L2assess1)</i> Level 5/6 pupils who are confident can also attempt: <i>Secret sequences, sheet 2 (T10L2assess1)</i></p>
<p>Plenary about 5 minutes</p> <p><i>In secret sequences we need to use unsimplified fractions to continue them. Why are fractions in the sequence of the form <math>\frac{2n}{3n}</math> easy to predict when already simplified?</i></p> <p><i>Quickly think of an example of a general secret sequence rule best predicted when the fractions are</i> (a) <i>simplified</i> (b) <i>unsimplified.</i></p> <p><i>What sort of fractional sequence rule would give terminating decimals or percentages?</i></p> <p><i>Which is greater: <math>\frac{3}{4}</math> of 24 or <math>\frac{2}{3}</math> of 21?</i></p> <p><i>What strategies can we use to find a common denominator? [Here this could extend to percentage denominators, in the absence of a calculator, as well as scaling to find the LCM – see below.]</i></p> <p><i>If you scored 93 out of 150 in one test and 70% in another test, how could you quickly work out which test score is better?</i></p>	<p>Discuss values given in fraction (simplified or unsimplified), decimal and percentage forms and how different forms are more useful in different situations.</p> <p>Recap the ‘secret sequence’ situation in which unsimplified fractions were most useful. Briefly show other examples in which other forms would be more helpful.</p> <p>Examples could include the sequence with the <math>n</math>th term expression <math>\frac{2n}{3n}</math> (simplified fractions show all terms are <math>\frac{2}{3}</math>). Also include ordering/comparing fractions where unsimplified fractions or decimals/percentages may be most useful, e.g. showing the size of the terms was decreasing in the sequence in the group activity or comparing <math>\frac{93}{150}</math> and 70% scores in a test.</p>

# Teacher resource sheets



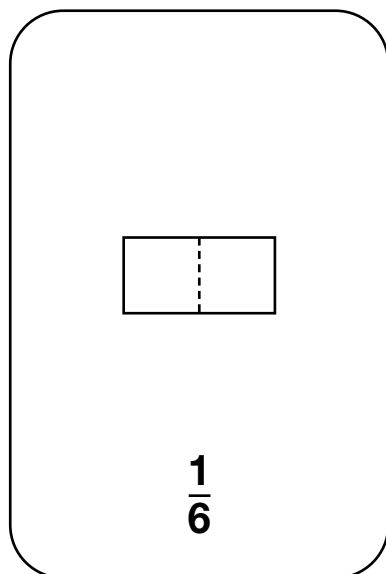
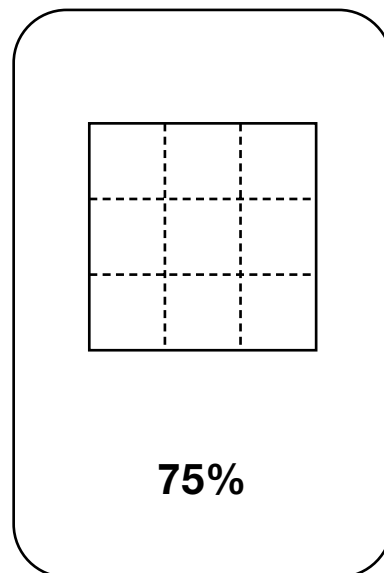
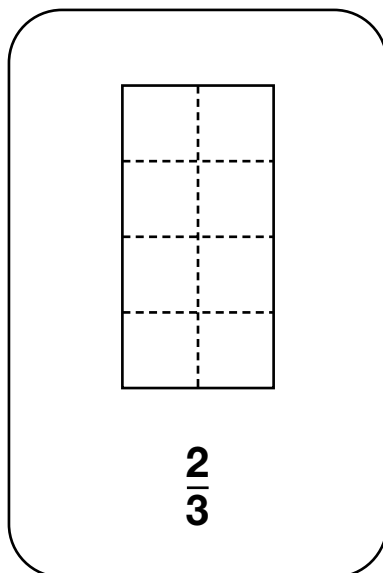
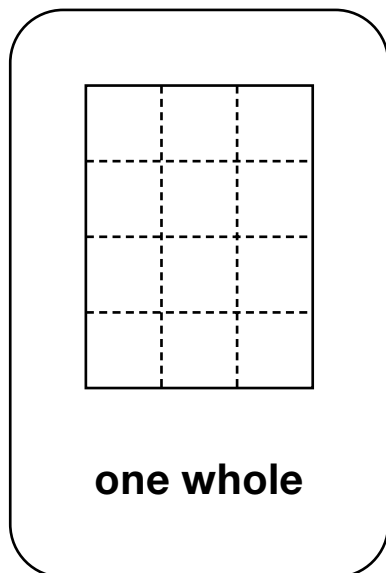
**Proportion cards, set 1**

First matched set (either or both of the last two could be omitted for lower ability):



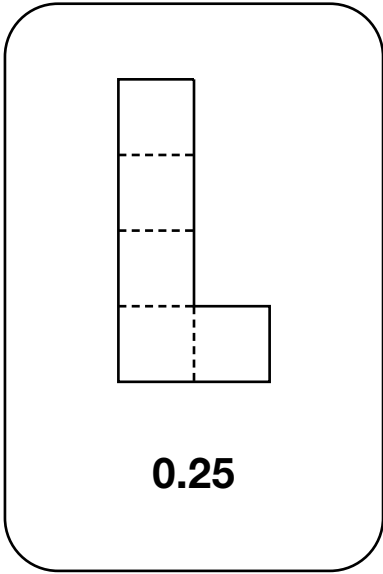
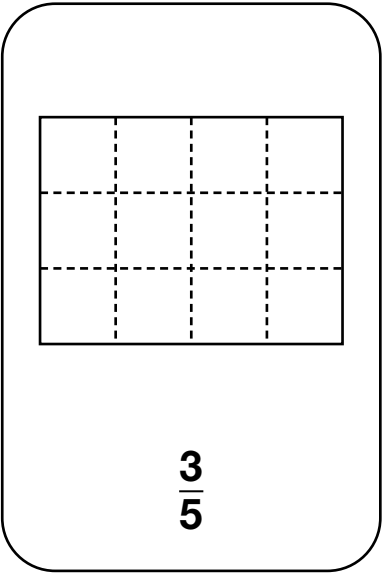
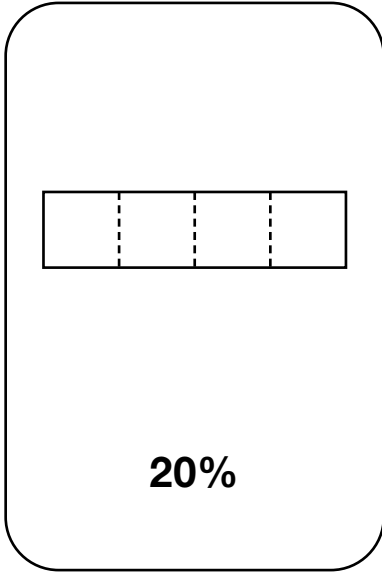
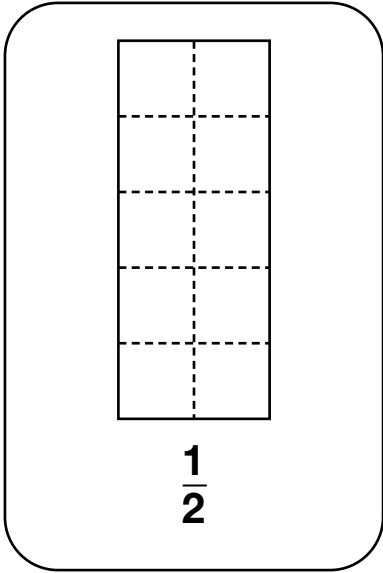
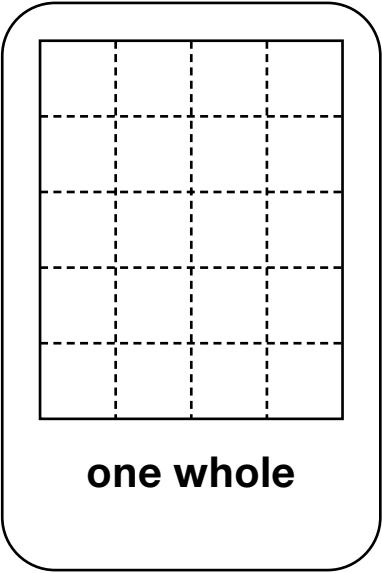
**Proportion cards, set 2**

Second matched set:



*Proportion cards, set 3*

Third matched set:



# Pupil sheets

**Using proportions, sheet 1**

Complete this task on squared paper. Write your name at the top of the paper.

*Part 1*

Look at this information.

Rectangle A is  $\frac{1}{3}$  of rectangle B.  
Rectangle B is  $\frac{1}{3}$  of rectangle C.

Draw three rectangles to show what size rectangles A, B and C could be.

Label them A, B and C.

Near your drawings, copy this sentence and fill in the missing **fraction**.

Rectangle A is \_\_\_\_\_ of rectangle C.

Now draw a different set of rectangles to show what size rectangles A, B and C could be.

Label them A, B and C.

*Part 2*

Now look at this information.

Shape D is  $\frac{3}{5}$  of shape E.

Draw two shapes to show what shapes D and E could be.

Label them D and E.

**Using proportions, sheet 2**

Complete this task on squared paper. Write your name at the top of the paper.

*Part 1*

Look at this information.

Shape P is  $\frac{3}{5}$  of shape Q.  
Shape Q is  $\frac{3}{5}$  of shape R.

Draw three shapes to show what shapes P, Q and R could be.

You can use rectangles, or other shapes.

Label them P, Q and R.

Near your drawings, copy this sentence and fill in the missing **fraction**.

Shape P is \_\_\_\_\_ of shape R.

Now copy this sentence and fill in the missing **percentage**.

Shape P is \_\_\_\_\_ % of shape R.

*Part 2*

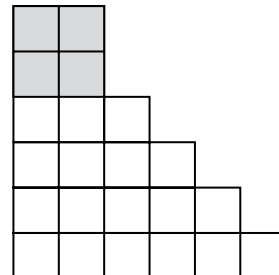
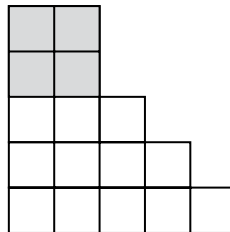
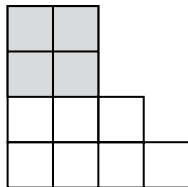
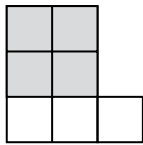
Can you draw a different set of shapes for P, Q and R?

**Secret sequences, sheet 1**

Name: \_\_\_\_\_

*Part 1*

(a) For the sequence of shapes below, complete the fraction shaded for each shape.



Fraction shaded:

$$\frac{4}{7}$$

$$\frac{4}{12}$$

$$\frac{4}{15}$$

$$\frac{4}{20}$$

(b) Now simplify the fractions as much as possible to make the secret sequence.

—

—

—

—

(c) What are the next three fractions in the secret sequence?

*Part 2*

Here are the first four fractions of a different secret sequence.

$$\frac{2}{3}$$

$$\frac{1}{2}$$

$$\frac{2}{5}$$

$$\frac{1}{3}$$

(a) For each one, write an equivalent fraction that has a numerator of 4

$$\frac{4}{6}$$

$$\frac{4}{4}$$

$$\frac{4}{10}$$

$$\frac{4}{12}$$

(b) What could the sequence of shapes that makes this secret sequence look like?

Draw it on squared paper.

**Secret sequences, sheet 2**

Name: \_\_\_\_\_

*Part 1*

A secret sequence can be made using the expression  $\frac{n+2}{30}$

$n$  is the term number, so the first term is when  $n = 1$

Therefore, the first term is  $\frac{1+2}{30} = \frac{3}{30}$

Use the expression  $\frac{n+2}{30}$  to find the first seven terms of the sequence.

1st	2nd	3rd	4th	5th	6th	7th
$\frac{3}{30}$	$\frac{\quad}{30}$	$\frac{\quad}{30}$	$\frac{\quad}{30}$	$\frac{\quad}{30}$	$\frac{\quad}{30}$	$\frac{\quad}{30}$

Now simplify the fractions as much as possible to make the secret sequence.

1st	2nd	3rd	4th	5th	6th	7th
$\frac{1}{10}$						

*Part 2*

Here are the first seven terms of a different secret sequence:

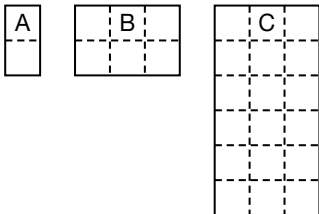

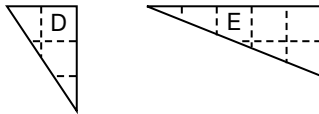
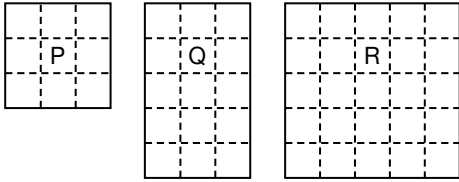
1st	2nd	3rd	4th	5th	6th	7th
$\frac{3}{4}$	$\frac{1}{2}$	$\frac{5}{12}$	$\frac{3}{8}$	$\frac{7}{20}$	$\frac{1}{3}$	$\frac{9}{28}$

What expression was used to make the secret sequence?



# **Solutions and performance indicators**

## LESSON 1: THINKING PROPORTIONATELY Solutions

Using proportions, sheet 1 (target level 4/5)		T10L1assess1
Solutions		Notes
<p>A set of three rectangles with areas in the ratio 1 : 3 : 9, labelled A, B and C respectively, e.g.</p> <ul style="list-style-type: none"> <li>  </li> </ul>		<p><b>Good</b> responses give shapes with areas in the correct ratio.</p> <p><b>Better</b> responses also label the shapes correctly, and may use diagrams with dimensions labelled rather than accurate drawing.</p>
<p>A correct fraction, e.g.</p> <ul style="list-style-type: none"> <li>'Rectangle A is <math>\frac{1}{9}</math> of rectangle C'</li> <li>'Rectangle A is <math>\frac{2}{18}</math> of rectangle C'</li> </ul>		<p><b>Good</b> responses show understanding of the correct proportion.</p> <p><b>Better</b> responses give a correct, simplified fraction.</p>
<p>A different set of three rectangles with areas in the ratio 1 : 3 : 9, labelled A, B and C respectively</p>		<p><b>Good</b> responses give shapes with areas in the correct ratio.</p> <p><b>Better</b> responses also label the shapes correctly, and may use diagrams with dimensions labelled rather than accurate drawing.</p>
<p>Two shapes with areas in the ratio 3 : 5, labelled D and E respectively, e.g.</p> <ul style="list-style-type: none"> <li>  </li> <li>  </li> </ul>		
Using proportions, sheet 2 (target level 5/6)		T10L1assess2
Solutions		Notes
<p>A set of three shapes with areas in the ratio 9 : 15 : 25, labelled P, Q and R respectively, e.g.</p> <ul style="list-style-type: none"> <li>  </li> </ul>		<p><b>Good</b> responses give shapes with areas in the correct ratio.</p> <p><b>Better</b> responses also label the shapes correctly, and may use diagrams of shapes other than rectangles with dimensions labelled rather than accurate drawing.</p>
<p>A correct fraction, e.g.</p> <ul style="list-style-type: none"> <li>'Shape P is <math>\frac{9}{25}</math> of shape R'</li> </ul>		<p><b>Good</b> responses show understanding of the correct proportion.</p> <p><b>Better</b> responses give a correct, simplified fraction.</p>
<p>A correct percentage, i.e.</p> <p>'Shape P is 36% of shape R'</p>		<p><b>Good</b> responses write their previous proportion correctly as a percentage.</p> <p><b>Better</b> responses give the correct percentage.</p>
<p>A different set of three shapes with areas in the ratio 9 : 15 : 25, labelled P, Q and R respectively</p>		

## LESSON 1: *THINKING PROPORTIONATELY* Performance indicators

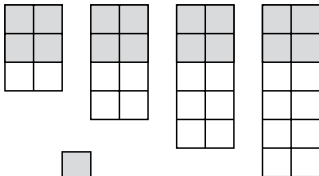
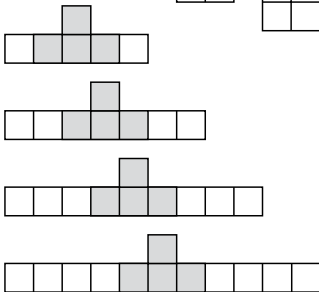
Note that performance indicators involving an element of ‘Using and applying mathematics’ are given in **bold**.

Worksheet	Performance indicators
<p><i>Using proportions, sheet 1</i> (target level 4/5) <b>T10L1assess1</b></p>	<p><b>Level 4:</b> At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> <li>draw rectangles satisfying given area conditions involving the fraction <math>\frac{1}{3}</math>;</li> <li><b>preserve the correct size order of rectangles given within area conditions</b> involving the fraction <math>\frac{1}{3}</math> by labelling correctly;</li> <li><b>give a different set of three rectangles satisfying the same conditions</b> involving the fraction <math>\frac{1}{3}</math>.</li> </ul> <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> <li><b>give the smallest area as a correct fraction of the largest</b> where each is <math>\frac{1}{3}</math> of the next;</li> <li>avoid the misconception that since each rectangle is <math>\frac{1}{3}</math> of the size of another, the final fraction must be <math>\frac{1}{6}</math>;</li> <li>draw shapes satisfying given area conditions involving the fraction <math>\frac{3}{5}</math>.</li> </ul> <p><b>Level 5:</b> At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> <li><b>give the smallest area as a correct fraction of the largest</b> where each is <math>\frac{1}{3}</math> of the next;</li> <li>draw at least two shapes, probably rectangles, satisfying given area conditions involving the fraction <math>\frac{3}{5}</math>;</li> <li>write a simple, even if incorrect, proportion as a percentage, e.g. <math>\frac{3}{5}</math> as 60%.</li> </ul> <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> <li>give a simplified fraction where applicable;</li> <li>draw three shapes aiming to satisfy given area conditions involving the fraction <math>\frac{3}{5}</math>, where the smallest area is a multiple of 9, the middle area is a multiple of 15 or the largest area is a multiple of 25;</li> <li><b>give the smallest area as a correct fraction of the largest</b> where each is <math>\frac{3}{5}</math> of the next;</li> <li>write a more complex proportion as a percentage.</li> </ul> <p>(See next page for level 6 and above indicators)</p>
<p><i>Using proportions, sheet 2</i> (target level 5/6) <b>T10L1assess2</b></p>	

Worksheet	Performance indicators
<p><i>Using proportions, sheet 2</i> (target level 5/6) <b>T10L1assess2</b></p>	<p><b>Level 6:</b> At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> <li>draw three shapes, probably rectangles, satisfying given area conditions involving the fraction <math>\frac{3}{5}</math>;</li> <li><b>give their smallest area as a correct fraction of their largest</b>, giving a simplified fraction where applicable;</li> <li>write a more complex, even if incorrect, proportion as a percentage.</li> </ul> <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> <li>draw a different set of three shapes satisfying given area conditions involving the fraction <math>\frac{3}{5}</math>, with no errors;</li> <li>give the smallest area as a correct fraction <u>and</u> percentage of the largest where each is <math>\frac{3}{5}</math> of the next.</li> </ul> <p><b>Above level 6:</b> At these levels, pupils are generally able to:</p> <ul style="list-style-type: none"> <li><b>draw two different sets</b> of three shapes, probably rectangles, satisfying given area conditions involving the fraction <math>\frac{3}{5}</math>;</li> <li><b>use a strategy that ensures all sets of shapes can have integer side lengths where possible</b>;</li> <li>draw sets of three shapes other than rectangles satisfying given area conditions involving the fraction <math>\frac{3}{5}</math>;</li> <li>give the smallest area as a correct fraction <u>and</u> percentage of the largest where each is <math>\frac{3}{5}</math> of the next.</li> </ul>

## LESSON 2: SECRET SEQUENCES

## Solutions

Secret sequences, sheet 1 (target level 4/5)		T10L2assess1
Solutions		Notes
Correct denominators, i.e. $\frac{4}{11}, \frac{4}{16}, \frac{4}{22}$		
Correct simplified fractions, i.e. $\frac{4}{7}, \frac{4}{11}, \frac{1}{4}, \frac{2}{11}$		<p><b>Good</b> responses show some of their fractions simplified, if only partially.</p> <p><b>Better</b> responses show all the correct fractions fully simplified.</p>
<p>Correct fractions, e.g.</p> <ul style="list-style-type: none"> <li><math>\frac{4}{29}, \frac{4}{37}, \frac{4}{46}</math></li> <li><math>\frac{4}{29}, \frac{4}{37}, \frac{2}{23}</math></li> </ul>		<p><b>Good</b> responses show some understanding of how the next fractions are generated.</p> <p><b>Better</b> responses show all three correct fractions fully simplified.</p>
Correct denominators, i.e. $\frac{4}{6}, \frac{4}{8}, \frac{4}{10}, \frac{4}{12}$		<p><b>Good</b> responses show some correct denominators.</p> <p><b>Better</b> responses are more consistent.</p>
<p>Possible patterns for the sequence, e.g.</p> <ul style="list-style-type: none"> <li>  </li> <li>  </li> </ul>		<p><b>Good</b> responses show some understanding of how one or two of the terms could be illustrated geometrically.</p> <p><b>Better</b> responses show the correct proportions shaded within a pattern that shows a clear geometric rule for continuing.</p>
Secret sequences, sheet 2 (target level 5/6)		T10L2assess2
Solutions		Notes
Correct numerators, i.e. $\frac{4}{30}, \frac{5}{30}, \frac{6}{30}, \frac{7}{30}, \frac{8}{30}, \frac{9}{30}$		
Correct simplified fractions, i.e. $\frac{2}{15}, \frac{1}{6}, \frac{1}{5}, \frac{7}{30}, \frac{4}{15}, \frac{3}{10}$		<p><b>Good</b> responses show some of their fractions simplified, if only partially.</p> <p><b>Better</b> responses show all the correct fractions fully simplified.</p>
<p>Fractions <math>\frac{3}{4}, \frac{4}{8}, \frac{5}{12}, \frac{6}{16}, \frac{7}{20}, \frac{8}{24}, \frac{9}{28}</math></p> <p>Correct <math>n</math>th term expression <math>\frac{n+2}{4n}</math> or equivalent</p>		<p><b>Good</b> responses show some understanding that equivalent fractions may help uncover a pattern.</p> <p><b>Better</b> responses show some generalisation of the pattern.</p>

## LESSON 2: SECRET SEQUENCES

## Performance indicators

Note that performance indicators involving an element of 'Using and applying mathematics' are given in **bold**.

Worksheet	Performance indicators
<p><i>Secret sequences, sheet 1</i> (target level 4/5) <b>T10L2assess1</b></p>	<p><b>Level 4:</b> At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> <li>• use a diagrammatic representation of a number pattern to attempt to complete the denominators of fractions;</li> <li>• simplify fractions with even numerators and denominators, even if only partially;</li> <li>• write fractions with numerators of 2 as equivalent fractions with numerators of 4;</li> <li>• draw one or two shapes with proportions shaded that correspond to given fractions.</li> </ul> <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> <li>• ensure their denominators represent the total number of squares rather than the number of squares unshaded;</li> <li>• simplify fractions with even numerators and denominators fully;</li> <li>• recognise the pattern within both numerators and denominators of an unsimplified fraction sequence and generate more terms;</li> <li>• write fractions with numerators of 1 as equivalent fractions with numerators of 4;</li> <li>• create a sequence of shapes with proportions shaded that correspond to a given sequence of fractions.</li> </ul> <p><b>Level 5:</b> At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> <li>• use a diagrammatic representation of a number pattern to complete the denominators of fractions correctly;</li> <li>• simplify fractions with even numerators and denominators fully;</li> <li>• recognise the pattern within both numerators and denominators of an unsimplified fraction sequence and generate more terms;</li> <li>• write fractions with numerators of 1 or 2 as equivalent fractions with numerators of 4;</li> <li>• create a sequence of shapes with proportions shaded that correspond to a given sequence of fractions;</li> </ul>
<p><i>Secret sequences, sheet 2</i> (target level 5/6) <b>T10L2assess2</b></p>	<ul style="list-style-type: none"> <li>• show an understanding of how to substitute values into an <math>n</math>th term expression to generate terms;</li> <li>• simplify some fractions that involve both odd and even numerators and denominators.</li> </ul> <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> <li>• <b>ensure that their sequence of shapes shows a clear geometric pattern or rule for continuing indefinitely;</b></li> <li>• simplify accurately and consistently fractions that involve both odd and even numerators and denominators;</li> <li>• <b>show an understanding that generating equivalent fractions for some terms in a fraction sequence may help to identify the pattern;</b></li> <li>• identify how the numerators and/or denominators are increasing with each new term, possibly attempting to show this using algebra.</li> </ul> <p>(See next page for level 6 and above indicators)</p>

Worksheet	Performance indicators
<p><i>Secret sequences, sheet 2</i> (target level 5/6) <b>T10L2assess2</b></p>	<p><b>Level 6:</b> At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> <li>• simplify accurately and consistently fractions that involve both odd and even numerators and denominators;</li> <li>• <b>show an understanding that generating equivalent fractions for some terms in a fraction sequence may help to identify the pattern;</b></li> <li>• identify how the numerators and/or denominators are increasing with each new term, possibly attempting to show this using algebra.</li> </ul> <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> <li>• express the change in either numerator or denominator as a correct <math>n</math>th term expression;</li> <li>• work out the <math>n</math>th term expression for the sequence as a whole.</li> </ul> <p><b>Above level 6:</b> At these levels, pupils are generally able to:</p> <ul style="list-style-type: none"> <li>• <b>generate equivalent fractions for some terms in a fraction sequence to identify the pattern;</b></li> <li>• express the change in either numerator or denominator in correct algebraic terms;</li> <li>• work out the <math>n</math>th term expression for the sequence as a whole.</li> </ul>

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