

Assessing pupils' progress in mathematics at Key Stage 3

Year 8 assessment package
Algebra
Teacher pack



Year 8 Algebra task: *Mind readers* and *What's the trick?*

Levels (3)/4/5/6

Note that for classes consisting of pupils at levels 3 and 4, you may wish to explore the material in lesson 1 more thoroughly, rather than progressing to lesson 2.

The lesson plans in this pack are set out in two columns. The left-hand column has indicative times for activities, highlights the resource sheets required and also has some examples of questions which teachers may wish to use with pupils during the activities. The right-hand column describes each activity in detail.

APP ASSESSMENT CRITERIA

These lessons may generate evidence to help inform judgements against a number of assessment criteria, including the following:

Algebra

- level 4: begin to use simple formulae expressed in words
- level 5: construct, express in symbolic form, and use simple formulae involving one or two operations

Calculating

- level 4: use a range of mental methods of computation with all operations
- level 5: use known facts, place value, knowledge of operations and brackets to calculate including using all four operations with decimals to two places

Using and applying mathematics

- level 5: check results, considering whether these are reasonable
- level 5: show understanding of situations by describing them mathematically using symbols, words and diagrams.

LESSON 1: *MIND READERS*

Resources

- One large piece of paper and a marker pen, or other means for a pupil to show a number to the class, for the starter activity
- Each pupil, or each group of pupils, needs a copy of the worksheet:
Mind readers (T7L1pupil1)
- A large-screen computer or an interactive whiteboard that can show the Excel spreadsheet entitled *Year 8 algebra mind readers*
- Computers for groups of pupils to work on the spreadsheet activity
- Each pupil needs the following worksheet:
Level 3/4/5 pupils: *Puzzling it out (T7L1assess1)*
- Paper for any rough working
- Note that calculators should not be used. However, you should use your judgement as to whether lower ability pupils would benefit from the use of a calculator and/or spreadsheet: this should be taken into account when assessing the work

<p>Starter about 5 minutes</p>	<p>Ask for a volunteer, but choose a pupil who is able to perform mental calculations accurately and quickly. Lower ability groups may need the support of a calculator.</p> <p>Tell the volunteer to think of a number between 1 and 10 without telling you what it is. The pupil then writes the number on some paper and shows it to the class (but not you) then hides/destroys it.</p> <p>Tell the pupils that you are going to read the volunteer's mind and tell them the number being thought of. To make it more difficult you are going to give some calculations to be done on the number first.</p> <p>As the rest of the class know what the number was, encourage them to do the calculations, but remind them not to say answers out loud.</p> <p>Tell the pupil to:</p> <ul style="list-style-type: none"> • think of his/her number • multiply it by 9 • subtract 3 • divide by 3 • add 1 • multiply by 3 • finally, if their answer has more than one digit they should add the digits of their number <p>Then tell the class that the number they are now thinking of is 9. (Note that the trick is explained during the plenary, so should not be explained here.)</p>
<p>Group activity about 5 minutes</p> <p>T7L1pupil1</p> <p><i>Suppose the first calculation of the first puzzle was 'add 10'. What would need to change in the puzzle to make the answer be the same as the number you started with? Why?</i> [Note that this idea is further developed in sheets 4 and 5 of the spreadsheet.]</p>	<p>Give each pupil, or each group of pupils, a copy of the worksheet: <i>Mind readers (T7L1pupil1)</i></p> <p>Ask the pupils to choose three numbers for each puzzle, and to fill in each cell to check that the puzzles give the answers claimed. Say that the puzzles work for all numbers, so they do not need to choose numbers between 1 and 10.</p> <p>Encourage more able pupils to try 'harder' numbers such as very large numbers, negatives, non-integers, etc.</p>
<p>Mini-plenary/group activity about 20 minutes</p> <p>Excel spreadsheet Year 8 algebra mind readers</p> <p><i>How many examples prove that a hypothesis is false? [One]</i> <i>How many examples prove that a hypothesis is true? [You cannot prove it by putting in numbers because there are always more numbers to test.]</i></p> <p><i>What if we start with a negative number? Or a decimal? What about a negative decimal? Is the start number still the same as the final number?</i></p>	<p>Discuss with the pupils whether they can be certain the puzzles will always give the answers claimed. Even if everyone in the class tried with a different number, how could they be sure that there isn't some other number that it won't work for?</p> <p>Explain that being certain is very important in mathematics, and that the next lesson will be about how they can be 100% certain.</p> <p>Using a large screen computer or an interactive whiteboard, show sheet 1 of the spreadsheet. (Note that this is the same puzzle as the first puzzle shown on the worksheet <i>Mind readers</i>.)</p> <p>Ask a few pupils for one of their start values, and type them one at a time in cell C2 (pressing return after typing each value). If necessary, choose some negative and decimal values yourself. Confirm that in each case the final number is the same as the start number.</p>

Look at the formulae in cells C3 to C6. Each formula refers to another cell. Which cell?

Here is a simple puzzle:

Think of a number

Multiply it by 2

Halve your answer

What is the final number?

How can you be sure that this simple puzzle always gives a final number that is the same as the start number?

What about this simple puzzle? Will it always give a final number that is the same as the start number?

Think of a number

Subtract 7

Add 3

Now add 4

How could you 'undo' the instruction 'divide by 10'?

How could you 'undo' the instruction $\times 3, + 4$? How many different ways can you find?

Look together at the formulae entered in cells C3 to C6 and check that the pupils understand how to enter these simple formulae. Pupils need to know:

- that you type '=' to let the computer know you are about to enter an instruction (a formula)
- that each cell has its own label, e.g. C4, and how the labelling system works
- that the operations \times and \div are typed as * and / respectively

Now show **sheet 2** of the spreadsheet, and ask the pupils what the formulae for this puzzle should be. (Note that this is the same puzzle as the second puzzle shown on the worksheet *Mind readers*.) Work together to insert the formulae, i.e.

Think of a number	(Insert number)
Add 6	= C2+6 (or 6+C2)
Double your answer	= C3*2 (or 2*C3)
Subtract 8	= C4-8
Halve your answer	= C5/2
Subtract the number you started with	= C6-C2

Then change the starting value inserted in cell C2 and confirm that for all these values the puzzle shows 2 as the final number.

Now ask the pupils to work in small groups on their own computers, using the puzzle shown on **sheet 3** of the spreadsheet. (Note that this is the same puzzle as the third puzzle shown on the worksheet *Mind readers*.) Groups could possibly start the work by writing on print-outs from the spreadsheet, testing their formulae on a computer when one is available. Answers are:

Think of a number	(Insert number)
Multiply it by 5	= C2*5 (or 5*C2)
Subtract 10	= C3-10
Double your answer	= C4*2 (or 2*C4)
Divide your answer by 10	= C5/10
Subtract the number you started with	= C6-C2

Note that since the final number should be -2, this is a self-checking exercise.

When the pupils are ready, they can progress to **sheet 4** and/or **sheet 5** of the spreadsheet where they are asked to find missing values (shown by '?' marks). Sheet 5 is more demanding than sheet 4.

Answers:

Sheet 4: The second missing number must be half the first missing number.

Sheet 5: The second missing number must be double the first missing number.

<p>Assessment activity about 10 minutes</p> <p>T7L1assess1</p>	<p>Each pupil will need one of the following worksheets: Level 3/4/5 pupils: <i>Puzzling it out (T7L1assess1)</i></p> <p>Note that as this activity is assessing calculating with negative numbers as well as understanding formulae, calculators should not be available. However, less able pupils may benefit from their use, though this should be taken into account when assessing their work.</p>														
<p>Plenary about 10 minutes</p> <p>Excel spreadsheet Year 8 algebra mind readers</p> <p><i>The puzzle has 'Multiply it by 9' as one of its instructions. Is there anything special about the 9 times table that makes this puzzle work?</i></p> <p><i>Would the puzzle work if we changed 'Multiply it by 9' to 'Multiply it by 3'? What else would need to change? Could we keep the last instruction 'Add the digits...?'</i></p> <p><i>Are there other numbers we could have multiplied by at the start?</i></p>	<p>Remind the pupils of the puzzle at the beginning of the lesson where you 'read someone's mind' and knew that the final number was 9. Say that you are going to investigate this puzzle on a spreadsheet. Remind the pupils that the start number had to be between 1 and 10.</p> <p>Show the pupils sheet 6 of the spreadsheet.</p> <p>Focus initially on column B. Ask which formulae need to be inserted. Note that the instruction 'Add the digits of the answer' should be left blank (a correct formula would be too advanced here).</p> <table border="1" data-bbox="592 913 1422 1182"> <tr> <td>Think of a number</td> <td>(This is cell B2)</td> </tr> <tr> <td>Multiply it by 9</td> <td>= B2*9 (or 9*B2)</td> </tr> <tr> <td>Subtract 3</td> <td>= B3-3</td> </tr> <tr> <td>Divide by 3</td> <td>= B4/3</td> </tr> <tr> <td>Add 1</td> <td>= B5+1 (or 1+B5)</td> </tr> <tr> <td>Multiply by 3</td> <td>= B6*3 (or 3*B6)</td> </tr> <tr> <td>Add the digits of the answer</td> <td></td> </tr> </table> <p>Copy the formulae from cells B3 to B7 into the relevant cells in columns C to K (see below for a quick way to copy formulae).</p> <p>Now look together at the numbers in the row labelled 'Multiply by 3'. What do they notice? Why does the fact that all the numbers are in the 9 times table mean that the final number must always be 9?</p> <p><i>Hint for copying cells: Click in cell B3. Then, holding the mouse button down, drag the cursor as far as cell B7. Cells B3 to B7 will now be highlighted (black). Position the cursor at the bottom right of this block of cells; you will see a small cross appear. Holding the mouse button down, drag this cross as far as column K. All formulae will be copied across.</i></p> <p><i>Note also that the worksheets are protected so that pupils have access only to certain cells. If you wish to amend the spreadsheet in any way, you will need to unprotect the relevant sheet first. Click on Tools, select Protection, then click on Unprotect Sheet. Make the amendments, then click on Tools, select Protection, click on Protect Sheet and click on OK (do not check any of the boxes other than the one already checked). If you wish to impose further security you can enter a password of your choice.</i></p>	Think of a number	(This is cell B2)	Multiply it by 9	= B2*9 (or 9*B2)	Subtract 3	= B3-3	Divide by 3	= B4/3	Add 1	= B5+1 (or 1+B5)	Multiply by 3	= B6*3 (or 3*B6)	Add the digits of the answer	
Think of a number	(This is cell B2)														
Multiply it by 9	= B2*9 (or 9*B2)														
Subtract 3	= B3-3														
Divide by 3	= B4/3														
Add 1	= B5+1 (or 1+B5)														
Multiply by 3	= B6*3 (or 3*B6)														
Add the digits of the answer															

LESSON 2: WHAT'S THE TRICK?

Resources

- A large screen computer or an interactive whiteboard that can show the Excel spreadsheet entitled *Year 8 algebra mind readers*
- Computers for groups of pupils to work on the spreadsheet activity
- Each pupil needs one of the following worksheets, depending on ability:
 - Level 4/5 pupils: *The n version sheet 1 (T7L2assess1)*
 - Level 5/6 pupils: *The n version sheet 2 (T7L2assess2)*
- Paper for any rough working
- Note that calculators are not needed for any of the activities

Starter
about 10 minutes

Excel spreadsheet *Year 8 algebra mind readers*

*The spreadsheet formula for 'double your answer' is = 2 * C3. Could it have been = C3 * 2?*

Is $2 \times n$ the same expression as $n \times 2$? What is another way to write $2 \times n$? [$2n$] Is n^2 a correct way of writing the same expression? [As it is unconventional it is to be discouraged.]

The expression $2n + 8$ is not necessarily easier to use than $2(n + 4)$ but why is it more helpful than $2(n + 4)$ here?

What in the table convinces you that you start and end with the same value? What if we referred to our start number as x or p ? How would the bottom right cell change? Why wouldn't it be helpful to use the letter 'o'?

How can we check that our algebra works? [Substitution]

Using a large screen computer or an interactive whiteboard, show **sheet 7** of the spreadsheet. Remind them that they have worked with this puzzle before (spreadsheet 1 in lesson 1). Say that as we do not know the start number we will call it n .

The formula for the instruction 'Add 4' is shown, i.e. = C2 + 4

Explain that this time we are going to write 'the n version' of this formula, using n rather than C2. As the n version is not a spreadsheet we do not need the equals sign, so we can write the formula as $n + 4$

Enter $n + 4$ in the relevant cell, i.e.

Think of a number		n
Add 4	= C2 + 4	$n + 4$

What should the formula be for the next row?

Discuss the expression $2(n + 4)$ and ask how to write it without brackets. Remind the pupils how to multiply out brackets and enter $2n + 8$ in the relevant cell, i.e.

Think of a number		n
Add 4	= C2 + 4	$n + 4$
Double your answer	= 2 * C3	$2n + 8$

Work through each row, simplifying as you go, i.e.

Think of a number		n
Add 4	= C2 + 4	$n + 4$
Double your answer	= 2 * C3	$2n + 8$
Subtract 8	= C4 - 8	$2n$
Halve your answer	= C5 / 2	n

How does the algebra show that the puzzle will always work for every possible start number? (Note that this is not a trivial step as research shows that even when pupils have produced a formal algebraic proof they can sometimes fail to understand the underlying meaning.)

<p>Group activity/mini-plenary about 10 minutes</p> <p>Excel spreadsheet Year 8 algebra mind readers</p> <p><i>Why don't we write $n5$ for $5n$? Can $5n$ ever mean $5 + n$? [Note, this is a common misconception.]</i></p> <p><i>Write $10n - 20$ using brackets [$10(n - 2)$] What is $10(n - 2)$ divided by 10? [$n - 2$]</i></p> <p><i>What instructions would give: n $n - 2$ $10(n - 2)$?</i></p> <p><i>Does it matter whether the puzzle starts like this: Multiply it by 5 Subtract 10 Double your answer or like this: Subtract 2 Multiply by 10?</i></p>	<p>Show the pupils sheet 8 of the spreadsheet, again reminding them that they have seen this puzzle before (spreadsheet 3 in lesson 1).</p> <p>Ask them to work in groups to complete the n versions of the formulae, simplifying as they go. Remind pupils that the spreadsheet is not working out answers for them, but is acting as a word processor to record their answers. Pupils could work on print-outs from the spreadsheet if it is more convenient.</p> <p>Note that weaker groups are likely to need teacher support on this activity, especially in the row requiring subtraction of n.</p> <p>Discuss answers, i.e.</p> <table border="1" data-bbox="579 636 1436 864"> <tr> <td>Think of a number</td> <td></td> <td>n</td> </tr> <tr> <td>Multiply it by 5</td> <td>$= C2 * 5$</td> <td>$5n$</td> </tr> <tr> <td>Subtract 10</td> <td>$= C3 - 10$</td> <td>$5n - 10$</td> </tr> <tr> <td>Double your answer</td> <td>$= 2 * C4$</td> <td>$10n - 20$</td> </tr> <tr> <td>Divide your answer by 10</td> <td>$= C5 / 10$</td> <td>$n - 2$</td> </tr> <tr> <td>Subtract the number you started with</td> <td>$= C6 - C2$</td> <td>$- 2$</td> </tr> </table> <p>How does the algebra show that the puzzle will always work for every possible start number?</p>	Think of a number		n	Multiply it by 5	$= C2 * 5$	$5n$	Subtract 10	$= C3 - 10$	$5n - 10$	Double your answer	$= 2 * C4$	$10n - 20$	Divide your answer by 10	$= C5 / 10$	$n - 2$	Subtract the number you started with	$= C6 - C2$	$- 2$
Think of a number		n																	
Multiply it by 5	$= C2 * 5$	$5n$																	
Subtract 10	$= C3 - 10$	$5n - 10$																	
Double your answer	$= 2 * C4$	$10n - 20$																	
Divide your answer by 10	$= C5 / 10$	$n - 2$																	
Subtract the number you started with	$= C6 - C2$	$- 2$																	
<p>Assessment activity about 15 minutes</p> <p>T7L2assess1 T7L2assess2</p>	<p>Each pupil will need one of the following worksheets:</p> <p>Level 4/5 pupils: <i>The n version sheet 1 (T7L2assess1)</i></p> <p>Level 5/6 pupils: <i>The n version sheet 2 (T7L2assess2)</i></p>																		
<p>Group activity about 5 minutes</p> <p>Excel spreadsheet Year 8 algebra mind readers</p>	<p>Pupils work in small groups on sheet 9 of the spreadsheet activity. Pupils enter a number of their choice in cell D2; the spreadsheet then uses this start number for both puzzles.</p> <p>Pupils are asked to find what start number makes the final number the same for both puzzles. Encourage them to work systematically – they should note that when using positive numbers, the larger the number entered as the start number, the greater the difference between the final numbers.</p> <p>Once the solution of -8 has been found, pupils can progress to sheet 10 to try to find the solution of -7.</p>																		

Plenary
about 10 minutes

**Excel spreadsheet Year 8
algebra mind readers**

How could we use both sides of the equation to check that $n = -8$?

[Substitution on both sides]

Look at both the equations with solutions $n = -8$ and $n = -7$? What single change has taken place?

[$2n + 6$ becomes $2n + 9$]

How could you predict the solution to $2n + 12 = 5(n + 6)$? How can you tell? Is there a pattern to the changes and their effects?

Which similar equation has solution $n = 0$? Which similar equation gives the smallest positive whole number solution bigger than 0? Are decimal solutions possible?

What about $2n = 5(n + 6)$

[i.e. $2n + 0 = 5(n + 6)$]?]

[When exploring this 'sequence' of equations with systematic changes to the value being added to/subtracted from $2n$, it might be helpful to set them out in rows on the board.]

Show **sheet 9** again, and ask for the algebra (the n version) for each puzzle. Write the expressions $2n + 6$ and $5(n + 6)$ and ask why $2n + 6 = 5(n + 6)$.

Does this mean that $2n + 6 = 5n + 30$?

Ask for the solution of this equation, and revisit solving techniques learnt earlier in the term to confirm that the solution is indeed -8 .

Now ask for an equation for the puzzles on **sheet 10**, i.e. $2n + 9 = 5(n + 6)$, leading to the solution $n = -7$.

What is the same about the two equations? Can the pupils write an equation with n on both sides that has the solution $n = -6$?

Pupil sheets

T7L1pupil1

Mind readers

Name(s): _____

Think of a number:			
• Add 4			
• Double your answer			
• Subtract 8			
• Halve your answer			

The final number is the same as the start number 😊

Think of a number:			
• Add 6			
• Double your answer			
• Subtract 8			
• Halve your answer			
• Subtract the number you started with			

The final number is 2 😊

Think of a number:			
• Multiply it by 5			
• Subtract 10			
• Double your answer			
• Divide your answer by 10			
• Subtract the number you started with			

The final number is -2 😊

T7L1assess1

Puzzling it out

Name: _____

Here are the instructions for a puzzle. Fill in the missing numbers.

Think of a number:	60	14	-4	Start number
• Add 14	74			
• Halve your answer				
• Subtract 28				
• Multiply your answer by 4				
• Add 84				
• Halve your answer				

Complete this sentence:

The final number in this puzzle will always be _____

Now complete the instructions for this different puzzle.

The first two rows are done for you.

Think of a number:	<i>This is cell C2</i>
• Add 4	= C2 + 4
•	= C3 * 8
•	= C4 / 2
•	= C5 - 16
•	= C6 / 4
•	= C7 - C2

Complete this sentence:

The final number in this puzzle will always be _____

Explain how you know.

T7L2assess1

The n version sheet 1

Name: _____

Use algebra to show how this puzzle works.

	The n version	
Think of a number:	n	Start number
• Double it		
• Add 12		
• Halve your answer		
• Subtract the number you started with		Final number

What will the final number of the puzzle always be? _____

Here is a different puzzle. The algebra shows how the puzzle works. Complete the instructions.

	The n version	
Think of a number:	n	Start number
•	$n + 9$	
•	$2n + 18$	
•	$2n$	
•	n	Final number

The algebra shows that the final number of this puzzle will always be n . Use words to explain what that means.

T7L2assess2

The n version sheet 2

Name: _____

Use algebra to show how this puzzle works.

	The n version	
Think of a number:	n	Start number
• Add 6		
• Double your answer		
• Add 4		
• Halve your answer		Final number
• Subtract the number you started with		

What will the final number of the puzzle always be? _____

Here is a different puzzle. The algebra shows how the puzzle works. Complete the instructions.

	The n version	
Think of a number:	n	Start number
•	$\frac{n}{5}$	
•	$\frac{n}{5} + 1$	
•	$2n + 10$	
•	$2n$	Final number

The algebra shows that the final number of this puzzle will always be $2n$. Use words to explain what that means.

Solutions and performance indicators

LESSON 1: MIND READERS Solutions

Puzzling it out (target level 3/4/5)		T7L1assess1																													
Solutions		Notes																													
<p>Table completed correctly, i.e.</p> <table border="1"> <tr> <td>Think of a number:</td> <td>60</td> <td>14</td> <td>-4</td> </tr> <tr> <td>• Add 14</td> <td>74</td> <td>28</td> <td>10</td> </tr> <tr> <td>• Halve your answer</td> <td>37</td> <td>14</td> <td>5</td> </tr> <tr> <td>• Subtract 28</td> <td>9</td> <td>-14</td> <td>-23</td> </tr> <tr> <td>• Multiply your answer by 4</td> <td>36</td> <td>-56</td> <td>-92</td> </tr> <tr> <td>• Add 84</td> <td>120</td> <td>28</td> <td>-8</td> </tr> <tr> <td>• Halve your answer</td> <td>60</td> <td>14</td> <td>-4</td> </tr> </table>		Think of a number:	60	14	-4	• Add 14	74	28	10	• Halve your answer	37	14	5	• Subtract 28	9	-14	-23	• Multiply your answer by 4	36	-56	-92	• Add 84	120	28	-8	• Halve your answer	60	14	-4	<p>Good responses perform calculations involving positive numbers accurately, following through from any errors.</p> <p>Better responses perform calculations involving positive and negative numbers accurately.</p>	
Think of a number:	60	14	-4																												
• Add 14	74	28	10																												
• Halve your answer	37	14	5																												
• Subtract 28	9	-14	-23																												
• Multiply your answer by 4	36	-56	-92																												
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• Halve your answer	60	14	-4																												
<p>Sentence completed correctly, e.g.</p> <ul style="list-style-type: none"> • 'The final number in this puzzle will always be equal to the start number' • 'The final number in this puzzle will always be the number you started with' 		<p>Good responses use their values to make a sensible statement about the puzzle.</p> <p>Better responses deduce the correct rule and use this to check for errors.</p>																													
<p>Table completed correctly, e.g.</p> <table border="1"> <tr> <td>Think of a number:</td> <td><i>This is cell C2</i></td> </tr> <tr> <td>• Add 4</td> <td>= C2 + 4</td> </tr> <tr> <td>• Multiply by 8</td> <td>= C3 * 8</td> </tr> <tr> <td>• Divide by 2 (or halve)</td> <td>= C4 / 2</td> </tr> <tr> <td>• Subtract 16</td> <td>= C5 - 16</td> </tr> <tr> <td>• Divide by 4</td> <td>= C6 / 4</td> </tr> <tr> <td>• Subtract the start number</td> <td>= C7 - C2</td> </tr> </table>		Think of a number:	<i>This is cell C2</i>	• Add 4	= C2 + 4	• Multiply by 8	= C3 * 8	• Divide by 2 (or halve)	= C4 / 2	• Subtract 16	= C5 - 16	• Divide by 4	= C6 / 4	• Subtract the start number	= C7 - C2	<p>Good responses complete some of the instructions correctly.</p> <p>Better responses complete the instructions clearly using correct words.</p>															
Think of a number:	<i>This is cell C2</i>																														
• Add 4	= C2 + 4																														
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• Subtract 16	= C5 - 16																														
• Divide by 4	= C6 / 4																														
• Subtract the start number	= C7 - C2																														
<p>Sentence completed correctly, e.g.</p> <ul style="list-style-type: none"> • 'The final number in this puzzle will always be 0' • 'The final number in this puzzle will always be zero' • 'The final number in this puzzle will always be nothing' 		<p>Good responses use their instructions to make a sensible statement about the puzzle.</p> <p>Better responses deduce the correct rule.</p>																													
<p>A correct explanation, e.g.</p> <ul style="list-style-type: none"> • 'I tried it with a few numbers, e.g. $1 + 4 = 5$, $5 \times 8 = 40$, $40 \div 2 = 20$, $20 - 16 = 4$, $4 \div 4 = 1$, $1 - 1 = 0$ $10 + 4 = 14$, $14 \times 8 = 112$, $112 \div 2 = 56$, $56 - 16 = 40$, $40 \div 4 = 10$, $10 - 10 = 0$' • $n + 4 \rightarrow 8n + 32 \rightarrow 4n + 16 \rightarrow 4n \rightarrow n \rightarrow 0$ • <table border="1"> <tr> <td>= C2 + 4</td> <td>= C2 + 4</td> </tr> <tr> <td>= C3 * 8</td> <td>= 8 * C2 + 32</td> </tr> <tr> <td>= C4 / 2</td> <td>= 4 * C2 + 16</td> </tr> <tr> <td>= C5 - 16</td> <td>= 4 * C2</td> </tr> <tr> <td>= C6 / 4</td> <td>= C2</td> </tr> <tr> <td>= C7 - C2</td> <td>= 0</td> </tr> </table> 		= C2 + 4	= C2 + 4	= C3 * 8	= 8 * C2 + 32	= C4 / 2	= 4 * C2 + 16	= C5 - 16	= 4 * C2	= C6 / 4	= C2	= C7 - C2	= 0	<p>Good responses apply the instructions correctly to at least one value.</p> <p>Better responses use more general reasoning, possibly involving algebra.</p>																	
= C2 + 4	= C2 + 4																														
= C3 * 8	= 8 * C2 + 32																														
= C4 / 2	= 4 * C2 + 16																														
= C5 - 16	= 4 * C2																														
= C6 / 4	= C2																														
= C7 - C2	= 0																														

LESSON 1: MIND READERS Performance indicators

Note that performance indicators involving an element of ‘Using and applying mathematics’ are given in **bold**.

Worksheet	Performance indicators
<p><i>Puzzling it out</i> (target level 3/4/5) T7L1assess1</p>	<p>Level 3: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • complete a series of calculations correctly involving positive integers, given instructions in words; • complete some calculations correctly involving negative integers, given instructions in words; • interpret some instructions expressed as simple spreadsheet formulae. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • avoid errors in calculations involving negative integers; • recognise the rule linking start numbers with final numbers after a series of calculations using specific values; • interpret a series of instructions expressed as simple spreadsheet formulae, writing them in words. <p>Level 4: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • complete a series of calculations correctly involving positive and negative integers, given instructions in words; • recognise the rule linking start numbers with final numbers after a series of calculations using specific values; • interpret a series of instructions expressed as simple spreadsheet formulae, writing them in words. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • check calculations carefully when looking for a pattern or rule; • avoid using specific values from numerical examples when trying to use an unknown value within a series of instructions; • think of a method for finding the rule linking start numbers with final numbers, when given only a series of instructions. <p>Level 5: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> • check calculations carefully when looking for a pattern or rule; • avoid using specific values from numerical examples when trying to use an unknown value within a series of instructions; • find the rule linking start numbers with final numbers, when given only a series of instructions; • give a reason for a rule based on specific numerical examples. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> • give a more general reason for a rule, for example using algebra. <p>Level 6 and above: At these levels, pupils are generally able to:</p> <ul style="list-style-type: none"> • give evidence for the performance indicators listed previously for pupils working at level 5; plus • give a more general reason for a rule, for example using algebra.

The n version sheet 1 (target level 4/5)		T7L2assess1																				
Solutions		Notes																				
<p>Table completed with correct expressions, e.g.</p> <ul style="list-style-type: none"> <table border="1"> <tr><td>Think of a number:</td><td>n</td></tr> <tr><td>• Double it</td><td>$2n$</td></tr> <tr><td>• Add 12</td><td>$2n + 12$</td></tr> <tr><td>• Halve your answer</td><td>$n + 6$</td></tr> <tr><td>• Subtract the number you started with</td><td>6</td></tr> </table> <table border="1"> <tr><td>Think of a number:</td><td>n</td></tr> <tr><td>• Double it</td><td>$n \times 2$</td></tr> <tr><td>• Add 12</td><td>$n \times 2 + 12$</td></tr> <tr><td>• Halve your answer</td><td>$(n \times 2 + 12) \div 2$</td></tr> <tr><td>• Subtract the number you started with</td><td>$(n \times 2 + 12) \div 2 - n$</td></tr> </table> 		Think of a number:	n	• Double it	$2n$	• Add 12	$2n + 12$	• Halve your answer	$n + 6$	• Subtract the number you started with	6	Think of a number:	n	• Double it	$n \times 2$	• Add 12	$n \times 2 + 12$	• Halve your answer	$(n \times 2 + 12) \div 2$	• Subtract the number you started with	$(n \times 2 + 12) \div 2 - n$	<p>Good responses show some correct expressions.</p> <p>Better responses show correct expressions, simplifying where appropriate.</p>
Think of a number:	n																					
• Double it	$2n$																					
• Add 12	$2n + 12$																					
• Halve your answer	$n + 6$																					
• Subtract the number you started with	6																					
Think of a number:	n																					
• Double it	$n \times 2$																					
• Add 12	$n \times 2 + 12$																					
• Halve your answer	$(n \times 2 + 12) \div 2$																					
• Subtract the number you started with	$(n \times 2 + 12) \div 2 - n$																					
6		<p>Good responses use their expressions to write their final number.</p> <p>Better responses deduce the correct final number.</p>																				
<p>Table completed with correct instructions, e.g.</p> <ul style="list-style-type: none"> <table border="1"> <tr><td>Think of a number:</td><td>n</td></tr> <tr><td>• Add 9</td><td>$n + 9$</td></tr> <tr><td>• Double your answer</td><td>$2n + 18$</td></tr> <tr><td>• Subtract 18</td><td>$2n$</td></tr> <tr><td>• Subtract the number you started with</td><td>n</td></tr> </table> <table border="1"> <tr><td>Think of a number:</td><td>n</td></tr> <tr><td>• + 9</td><td>$n + 9$</td></tr> <tr><td>• $\times 2$</td><td>$2n + 18$</td></tr> <tr><td>• - 18</td><td>$2n$</td></tr> <tr><td>• $\div 2$</td><td>n</td></tr> </table> 		Think of a number:	n	• Add 9	$n + 9$	• Double your answer	$2n + 18$	• Subtract 18	$2n$	• Subtract the number you started with	n	Think of a number:	n	• + 9	$n + 9$	• $\times 2$	$2n + 18$	• - 18	$2n$	• $\div 2$	n	<p>Good responses complete some of the instructions correctly.</p> <p>Better responses complete the instructions clearly using correct words.</p>
Think of a number:	n																					
• Add 9	$n + 9$																					
• Double your answer	$2n + 18$																					
• Subtract 18	$2n$																					
• Subtract the number you started with	n																					
Think of a number:	n																					
• + 9	$n + 9$																					
• $\times 2$	$2n + 18$																					
• - 18	$2n$																					
• $\div 2$	n																					
<p>Correct explanation, e.g.</p> <ul style="list-style-type: none"> 'The final number in this puzzle will always be the number you started with' 'The final number in this puzzle will always be equal to the start number' 		<p>Good responses make a sensible statement about the puzzle.</p> <p>Better responses correctly explain the meaning of ending with n.</p>																				

The n version sheet 2 (target level 5/6)	T7L2assess2																								
Solutions	Notes																								
<p>Table completed with correct expressions, e.g.</p> <ul style="list-style-type: none"> <table border="1" data-bbox="240 331 963 636"> <tr><td>Think of a number:</td><td>n</td></tr> <tr><td>• Add 6</td><td>$n + 6$</td></tr> <tr><td>• Double your answer</td><td>$2n + 12$</td></tr> <tr><td>• Add 4</td><td>$2n + 16$</td></tr> <tr><td>• Halve your answer</td><td>$n + 8$</td></tr> <tr><td>• Subtract the number you started with</td><td>8</td></tr> </table> <table border="1" data-bbox="240 669 963 974"> <tr><td>Think of a number:</td><td>n</td></tr> <tr><td>• Add 6</td><td>$n + 6$</td></tr> <tr><td>• Double your answer</td><td>$2 \times (n + 6)$</td></tr> <tr><td>• Add 4</td><td>$2 \times (n + 6) + 4$</td></tr> <tr><td>• Halve your answer</td><td>$(2 \times (n + 6) + 4) \div 2$</td></tr> <tr><td>• Subtract the number you started with</td><td>$(2 \times (n + 6) + 4) \div 2 - n$</td></tr> </table> 	Think of a number:	n	• Add 6	$n + 6$	• Double your answer	$2n + 12$	• Add 4	$2n + 16$	• Halve your answer	$n + 8$	• Subtract the number you started with	8	Think of a number:	n	• Add 6	$n + 6$	• Double your answer	$2 \times (n + 6)$	• Add 4	$2 \times (n + 6) + 4$	• Halve your answer	$(2 \times (n + 6) + 4) \div 2$	• Subtract the number you started with	$(2 \times (n + 6) + 4) \div 2 - n$	<p>Good responses show some correct expressions.</p> <p>Better responses show correct expressions, simplifying where appropriate.</p>
Think of a number:	n																								
• Add 6	$n + 6$																								
• Double your answer	$2n + 12$																								
• Add 4	$2n + 16$																								
• Halve your answer	$n + 8$																								
• Subtract the number you started with	8																								
Think of a number:	n																								
• Add 6	$n + 6$																								
• Double your answer	$2 \times (n + 6)$																								
• Add 4	$2 \times (n + 6) + 4$																								
• Halve your answer	$(2 \times (n + 6) + 4) \div 2$																								
• Subtract the number you started with	$(2 \times (n + 6) + 4) \div 2 - n$																								
<p>8</p>	<p>Good responses use their expressions to write their final number.</p> <p>Better responses deduce the correct final number.</p>																								
<p>Table completed with correct instructions, e.g.</p> <ul style="list-style-type: none"> <table border="1" data-bbox="240 1296 963 1527"> <tr><td>Think of a number:</td><td>n</td></tr> <tr><td>• Divide by 5</td><td>$\frac{n}{5}$</td></tr> <tr><td>• Add 1</td><td>$\frac{n}{5} + 1$</td></tr> <tr><td>• Multiply by 10</td><td>$2n + 10$</td></tr> <tr><td>• Subtract 10</td><td>$2n$</td></tr> </table> <table border="1" data-bbox="240 1552 963 1783"> <tr><td>Think of a number:</td><td>n</td></tr> <tr><td>• $\div 5$</td><td>$\frac{n}{5}$</td></tr> <tr><td>• $+ 1$</td><td>$\frac{n}{5} + 1$</td></tr> <tr><td>• $\times 5$ then double</td><td>$2n + 10$</td></tr> <tr><td>• $- 10$</td><td>$2n$</td></tr> </table> 	Think of a number:	n	• Divide by 5	$\frac{n}{5}$	• Add 1	$\frac{n}{5} + 1$	• Multiply by 10	$2n + 10$	• Subtract 10	$2n$	Think of a number:	n	• $\div 5$	$\frac{n}{5}$	• $+ 1$	$\frac{n}{5} + 1$	• $\times 5$ then double	$2n + 10$	• $- 10$	$2n$	<p>Good responses complete some of the instructions correctly.</p> <p>Better responses complete the instructions clearly using correct words.</p>				
Think of a number:	n																								
• Divide by 5	$\frac{n}{5}$																								
• Add 1	$\frac{n}{5} + 1$																								
• Multiply by 10	$2n + 10$																								
• Subtract 10	$2n$																								
Think of a number:	n																								
• $\div 5$	$\frac{n}{5}$																								
• $+ 1$	$\frac{n}{5} + 1$																								
• $\times 5$ then double	$2n + 10$																								
• $- 10$	$2n$																								
<p>Correct explanation, e.g.</p> <ul style="list-style-type: none"> 'The final number in this puzzle will always be double the start number' 'The final number in this puzzle will always be twice the number you started with' 	<p>Good responses make a sensible statement about the puzzle.</p> <p>Better responses correctly explain the meaning of ending with $2n$.</p>																								

LESSON 2: WHAT'S THE TRICK?

Performance indicators

Note that performance indicators involving an element of 'Using and applying mathematics' are given in **bold**.

Worksheet	Performance indicators
<p><i>The n version sheet 1</i> (target level 4/5) T7L2assess1</p>	<p>Level 3: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> choose arbitrary values and complete a series of calculations correctly, given instructions in words; understand some aspects of the meaning of simple algebraic expressions. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> write some simple algebraic expressions, given instructions in words; interpret a series of instructions expressed as simple algebraic expressions, writing them in words. <p>Level 4: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> write some simple algebraic expressions, given instructions in words; interpret a series of instructions expressed as simple algebraic expressions, writing them in words. <p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> use conventional notation for algebra, e.g. $2n$ for $n \times 2$; understand the correct order of operations when manipulating algebraic expressions, e.g. dividing <u>both</u> terms in an expression by 2; interpret a series of instructions expressed as more complex algebraic expressions, e.g. including algebraic fractions, writing them in words; use a series of algebraic expressions to deduce the rule linking start numbers with final numbers; explain in words the meaning of a rule linking the start number, n, with the final number, n or $2n$, after a series of instructions. <p>Level 5: At this level, pupils are generally able to:</p> <ul style="list-style-type: none"> use conventional notation for algebra, e.g. $2n$ for $n \times 2$; understand the correct order of operations when manipulating algebraic expressions, e.g. dividing <u>both</u> terms in an expression by 2; interpret a series of instructions expressed as more complex algebraic expressions, e.g. including algebraic fractions, writing them in words; use a series of algebraic expressions to deduce the rule linking start numbers with final numbers; explain in words the meaning of a rule linking the start number, n, with the final number, n, after a series of instructions.
<p><i>The n version sheet 2</i> (target level 5/6) T7L2assess2</p>	<p>However, they are less likely to be able to:</p> <ul style="list-style-type: none"> check algebra carefully when looking for a pattern or rule; explain in words the meaning of a rule linking the start number, n, with the final number, $2n$, after a series of instructions. <p>Level 6 and above: At these levels, pupils are generally able to:</p> <ul style="list-style-type: none"> give evidence for the performance indicators listed previously for pupils working at level 5; plus check algebra carefully when looking for a pattern or rule; explain in words the meaning of a rule linking the start number, n, with the final number, $2n$, after a series of instructions.

Audience: Teachers
Date of issue: 06-2009
Ref: **00588-2009PDF-EN-07**

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