The National Strategies
Secondary

Assessing pupils' progress in mathematics at Key Stage 3: Standards File

Pupil N

department for
children, schools and families

QCA Qualifications and Curriculum Authority

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Pupil N – Year 9 – Low level 8

Assessing pupils' progress in mathematics at Key Stage 3

Assessment summary

Pupil N's teacher judges that her attainment in mathematics overall is best described as low level 8. Her performance is strongest in algebra and geometry and measures. Her performance in using and applying data is more characteristic of level 7 with some aspects of reasoning, analysing and communicating at level 8.

Using the Standards Files

- The current Standards Files are based on work planned and assessed in relation to the 1999 National Curriculum programme of study. A new set of Standards Files based on the 2008 National Curriculum are currently in production, but the current set will provide useful guidance on making APP assessments against national standards in the transition period as the new programmes of study are introduced.
- The commentaries in the Standards Files are provided for guidance and reference, and are much more extensive than any teacher would be expected to make when carrying out APP assessments. It is also important to remember that APP encourages and enables a broader overview of current learning, and that there is no need to collect special portfolios of pupils' work. Evidence from pupils' written and oral work, backed up by brief teacher's notes where necessary, is all that is required.
- The evidence base presented in each Standards File is necessarily partial, as it would of course be impractical to reproduce all of each pupil's work. Examples of each pupil's work have been selected to provide evidence to support judgements against APP criteria. This evidence should be considered in conjunction with the teacher's notes, which will provide a broader context and further justification for the assessments that are made.

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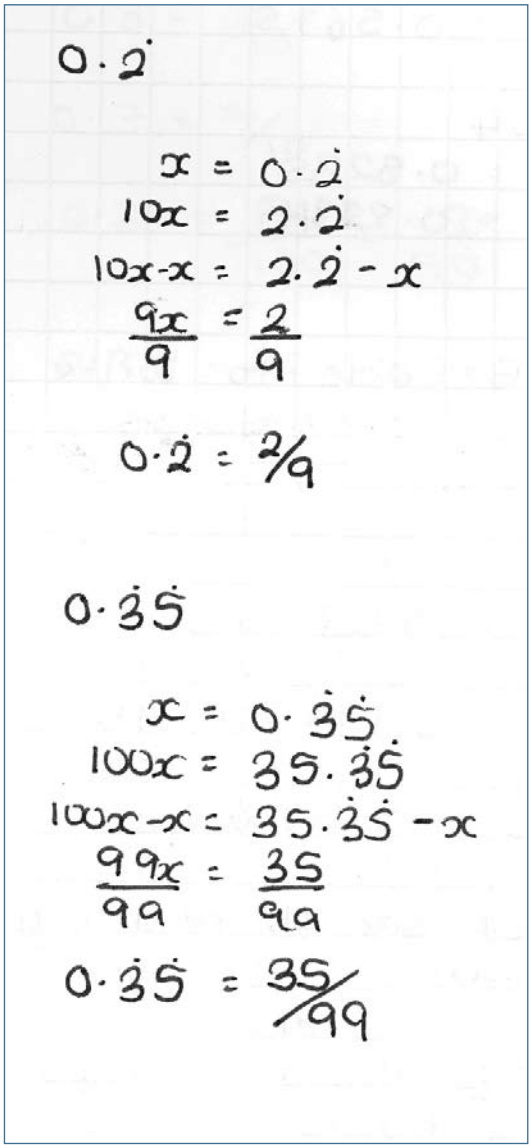
Assessment focus

Numbers and the number system; Calculating

Context

Classwork: In a unit on numbers and the number system, pupils used a variety of methods to convert decimals to fractions and vice versa.



Teacher's notes

- uses an algebraic method to convert a recurring decimal to a fraction

Next steps

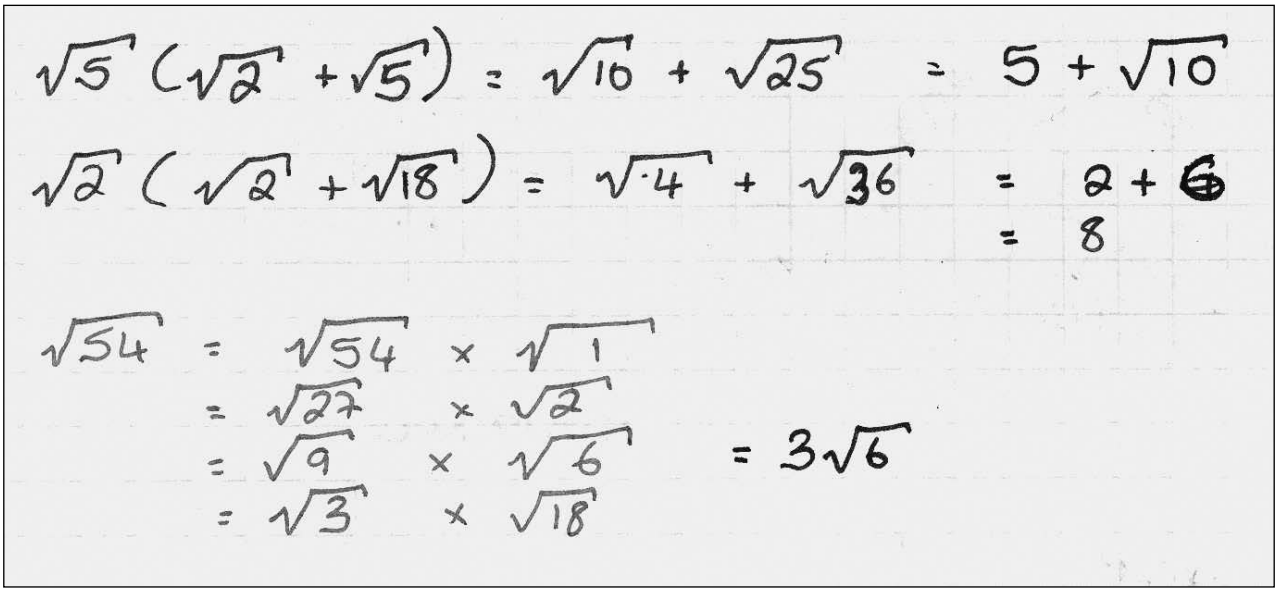
- explore patterns arising from converting rational numbers to decimal form, for example, by division
- reason about the results
- demonstrate and justify a non-calculator method for converting a recurring decimal to a fraction

Assessment focus

Calculating

Context

Classwork: Towards the beginning of a unit of work on representations of numbers, pupils evaluated calculations involving surds.



Teacher's notes

- identifies square numbers and factor pairs
- manipulates surds competently and simplifies sums and products
- uses accurate notation

Next steps

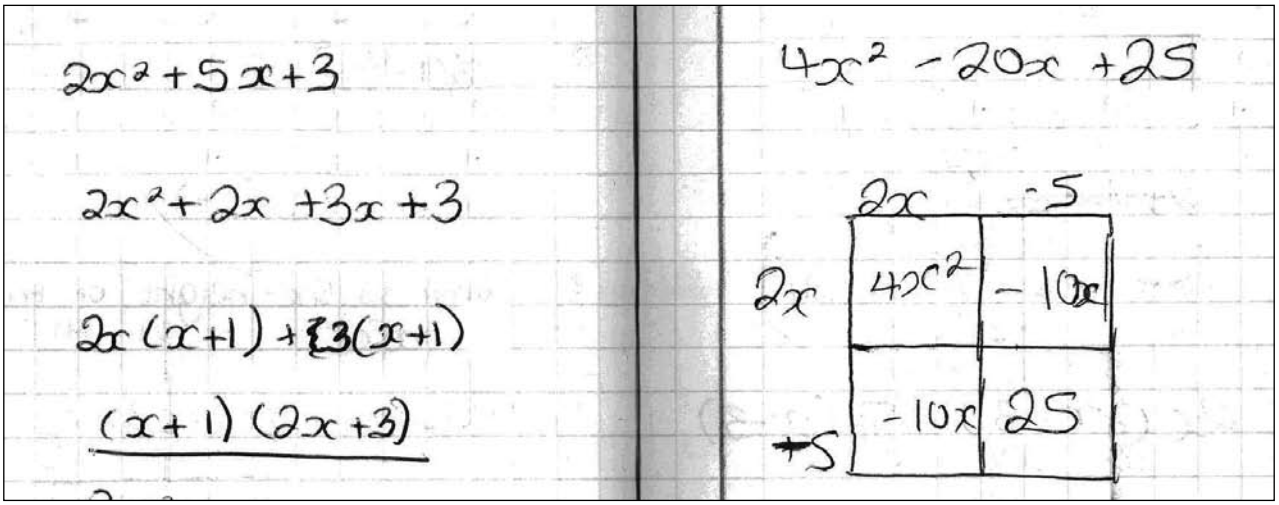
- extend understanding of surds from squares and square roots to other powers and roots
- solve problems using surds
- make connections between surds and index form

Assessment focus

Algebra

Context

Classwork: Pupils factorised quadratic expressions in a lesson developing skills for manipulating algebraic expressions and equations.



Teacher's notes

- factorises quadratic expressions with a positive coefficient of *x*-squared
- partitions the *x*-term to obtain common factors
- uses a grid method to find factors

Next steps

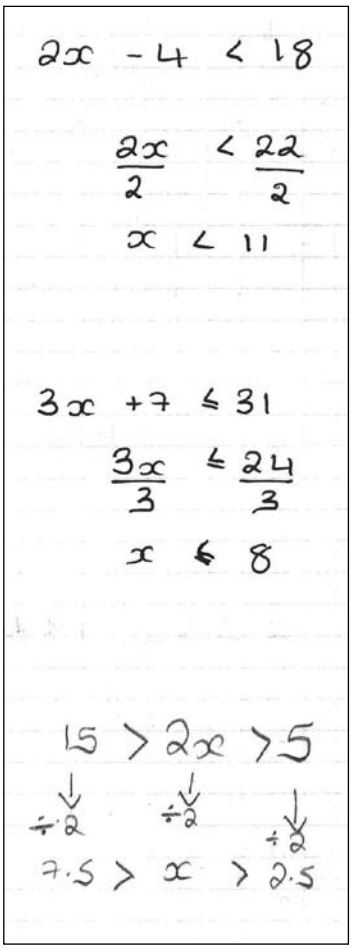
- justify her method
- develop more efficient methods for factorising quadratic expressions
- extend to situations with negative and rational coefficients
- use factorisation to solve quadratic equations
- link algebraic and graphical representations of quadratic expressions

Assessment focus

Algebra

Context

Homework: Pupils solved simple inequalities early in a series of lessons on using and solving inequalities.



Teacher's notes

- solves simple inequalities
- records methods and solutions
- combines several steps into a single line of working

Next steps

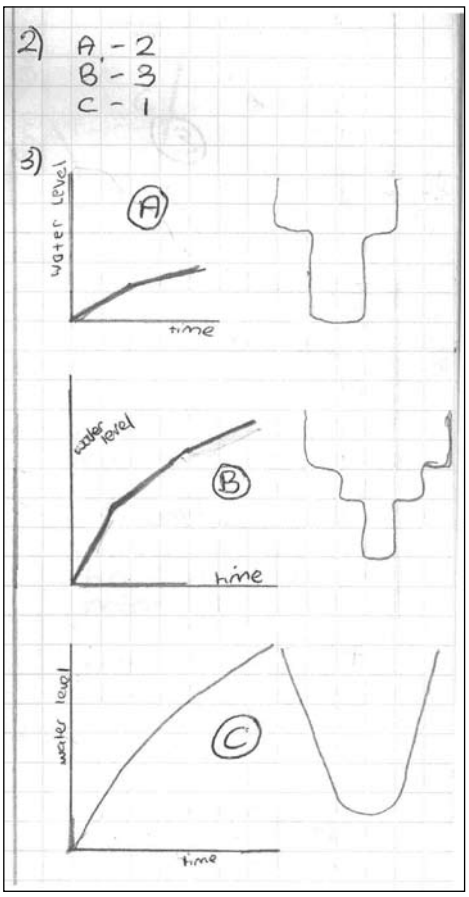
- develop more formal algebraic presentation
- use a number line to represent solution sets
- use inequalities to represent real contexts
- begin to solve inequalities in two variables
- represent inequalities graphically

Assessment focus

Algebra; Using and applying mathematics

Context

Classwork: In a unit on using mathematics to model and represent situations, pupils undertook practical activities filling different-shaped containers, and drawing graphs of water level against volume of water. As a follow-up activity pupils sketched graphs and discussed possible shapes of container to show the water level when the volume was increasing at a constant rate.



Teacher's notes

- sketches graphs to represent height of water against volume of water in the container (denoted by time as it was assumed that the volume was increasing at a constant rate)
- justifies shape of the graph
- uses mathematical language to discuss results

Next steps

- consider a range of other situations, particularly distance–time graphs
- identify dependent and independent variables
- use ICT to capture data and develop mathematical models

What the teacher knows about Pupil N's attainment in number and algebra

Pupil N understands different ways of representing rational and irrational numbers. She used an informal method of changing recurring decimals into fractions by using denominators of 9, 99 and 999. She can demonstrate that some rational numbers are equivalent to terminating decimals and some have recurring decimal expansions. Pupil N also uses an algebraic method to convert between different representations.

Pupil N uses a range of mental and written methods to solve numerical problems in all areas of mathematics. She uses fractions and decimals when solving problems involving proportional change. She looks for efficient approaches. For instance, working on surds she developed a strategy of looking for factor pairs that contained a square number. She also calculates using indices and standard form, choosing to use a calculator where necessary. Pupil N checks answers using mental methods and a calculator.

Pupil N manipulates algebraic expressions and equations. She factorises quadratic expressions and solves simultaneous equations arising from different contexts by substitution and by elimination. Pupil N showed creativity by making connections between a method for finding factor pairs by decomposition of the linear term and an informal method based on a grid method for multiplication. When pupils used graph plotting software to explore the properties of functions, she commented on the links between the graphical and algebraic representations of functions and the relationship between intercepts on the graph, roots of equations and factors.

Pupil N creates functions and graphs to model real situations. She has attended an external event where she explored using functions to fit curves and straight lines to selected features on pictures of buildings. She understands the connection between the gradient of a curve and rates of change. In the exercise on filling containers she recognised that vertical sides resulted in a straight-line graph. She described the relationship between the gradient and the width of the vessel and explained why the rate of filling was greater when the container was narrower.

Pupil N explores the effect of positive and negative values of a and b on the graphs of $y=ax^2 + b$ and $y=ax^3 + b$. For example, she described the properties and key features of the curves using appropriate vocabulary and related the transformations to her understanding of translation and reflection in geometry.

Summarising Pupil N's attainment in number and algebra

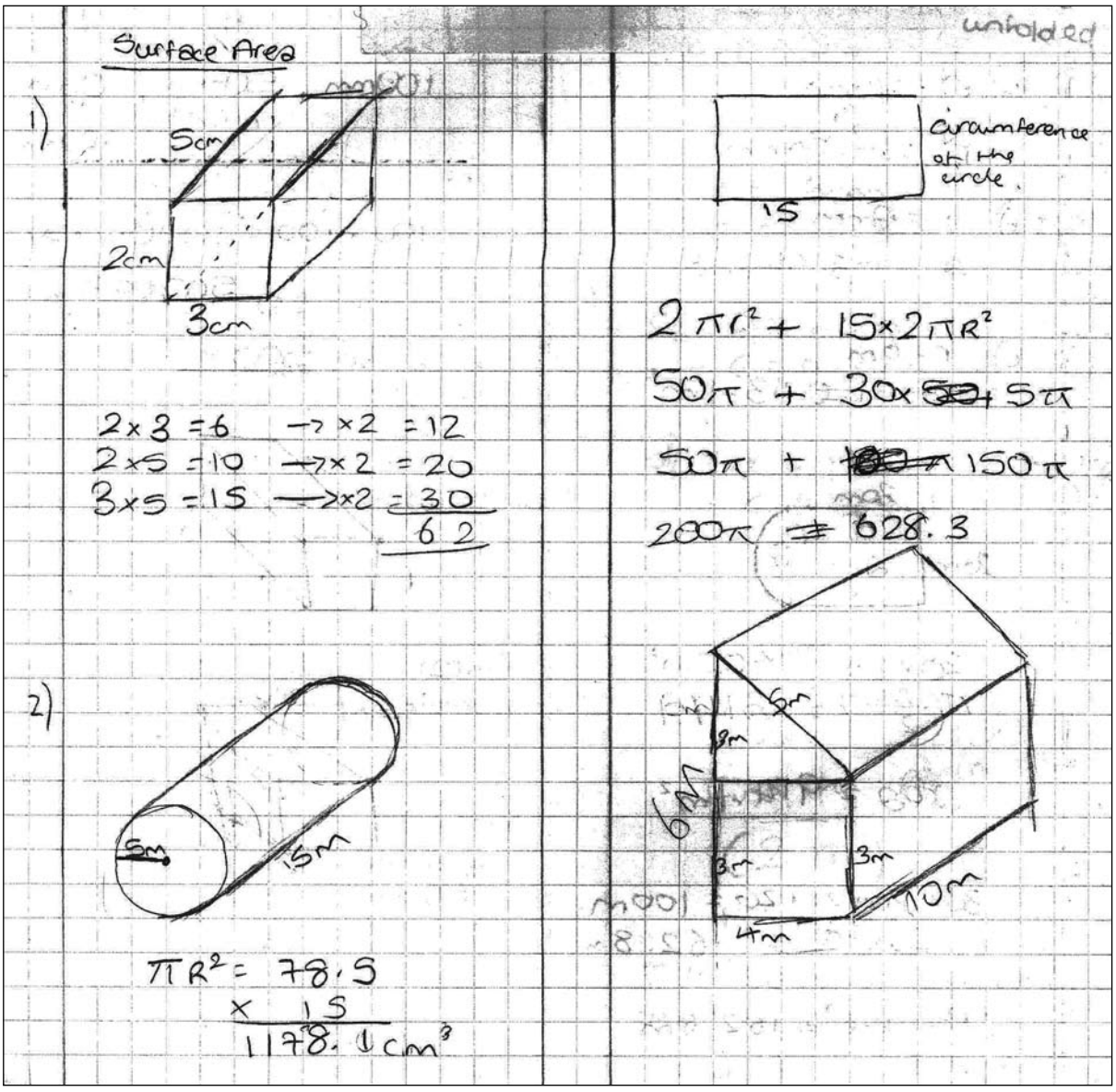
Pupil N's attainment in number and algebra is best described as low level 8. She should extend understanding of inequalities in two variables using algebraic and graphical methods. She needs to apply numerical, graphical and algebraic methods in a range of contexts to help her make further connections between different areas of mathematics.

Assessment focus

Shape, space and measures

Context

Classwork: Pupils calculated the surface area of 3-D shapes.



Teacher's notes

- uses standard formulae for mensuration
- calculates volumes and surface areas of cylinders and right prisms

Next steps

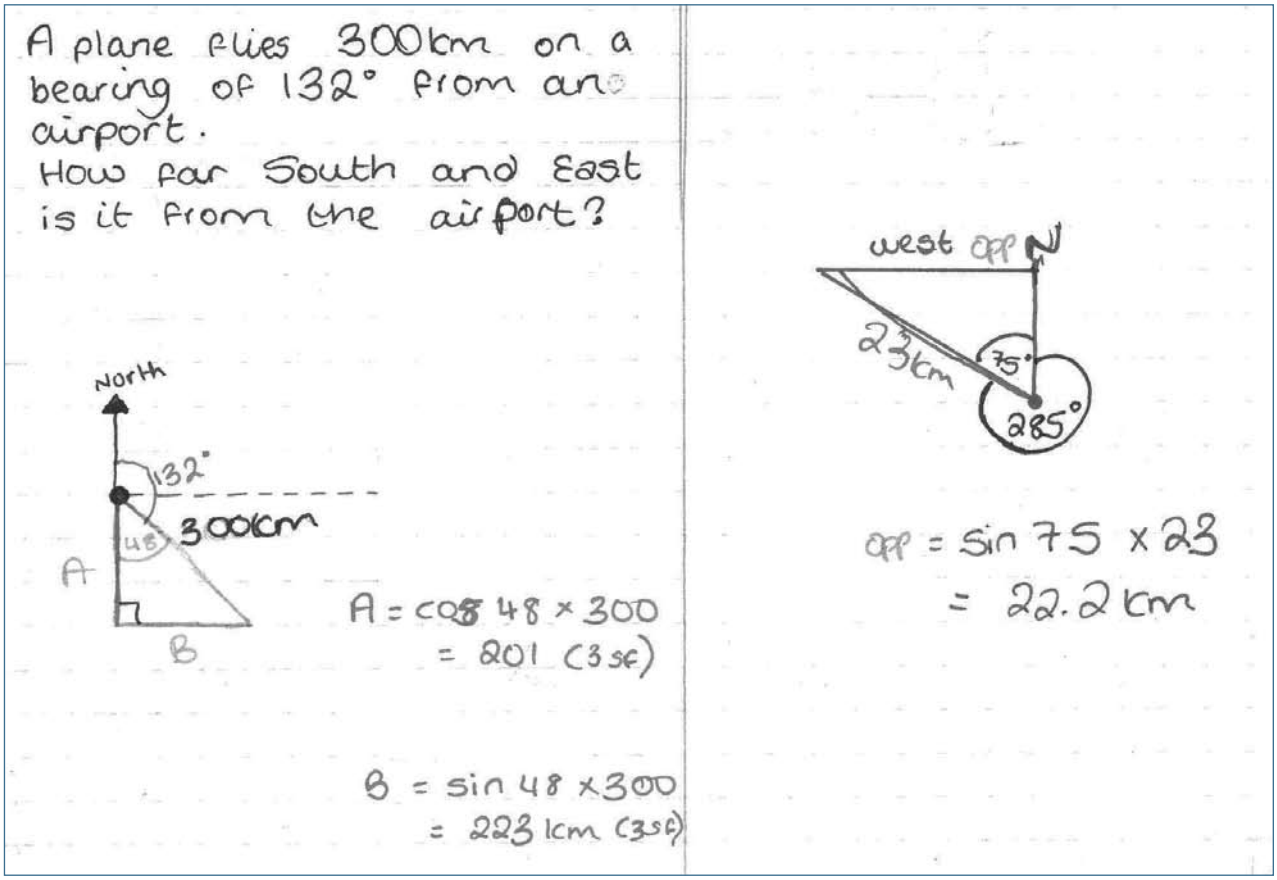
- use units consistently and accurately
- lay out calculations clearly and correctly and record interim values accurately
- analyse more complex situations

Assessment focus

Shape, space and measures

Context

Classwork: At the end of a unit on geometrical reasoning, pupils worked on a variety of problems requiring them to select appropriate mathematical tools.



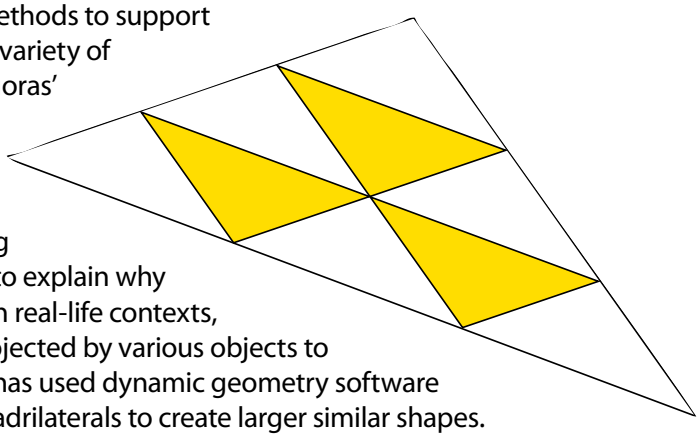
Teacher's notes

- represents the situation using diagrams
- selects an appropriate method
- calculates with trigonometric ratios
- rounds to three significant figures

Next steps

- develop accurate and more conventional notation, for example, capital letters for vertices, lower case for lengths, specifying the units in sin 48°
- extend use of trigonometric ratios and Pythagoras' theorem to a wider range of problems

What the teacher knows about Pupil N's attainment in shape, space and measures



Pupil N uses a wide range of techniques and methods to support geometric reasoning. She solves problems in a variety of contexts using trigonometric ratios and Pythagoras' theorem. She makes links with other areas of mathematics, leaving numerical answers in surd form where appropriate.

Pupil N recognises similarity and uses reasoning about angles in polygons and on parallel lines to explain why shapes are similar. She has explored similarity in real-life contexts, for instance using photographs of shadows projected by various objects to estimate unknown distances and lengths. She has used dynamic geometry software to translate, reflect and rotate triangles and quadrilaterals to create larger similar shapes.

Pupil N solves mensuration problems competently, calculating surface areas and volumes of cuboids, prisms, cylinders and compound solids. She worked with a small group to investigate optimisation problems. They used a spreadsheet to search for values that would minimise the amount of material needed to make a cylinder to contain one litre.

Summarising Pupil N's attainment in geometry and measures (shape, space and measures)

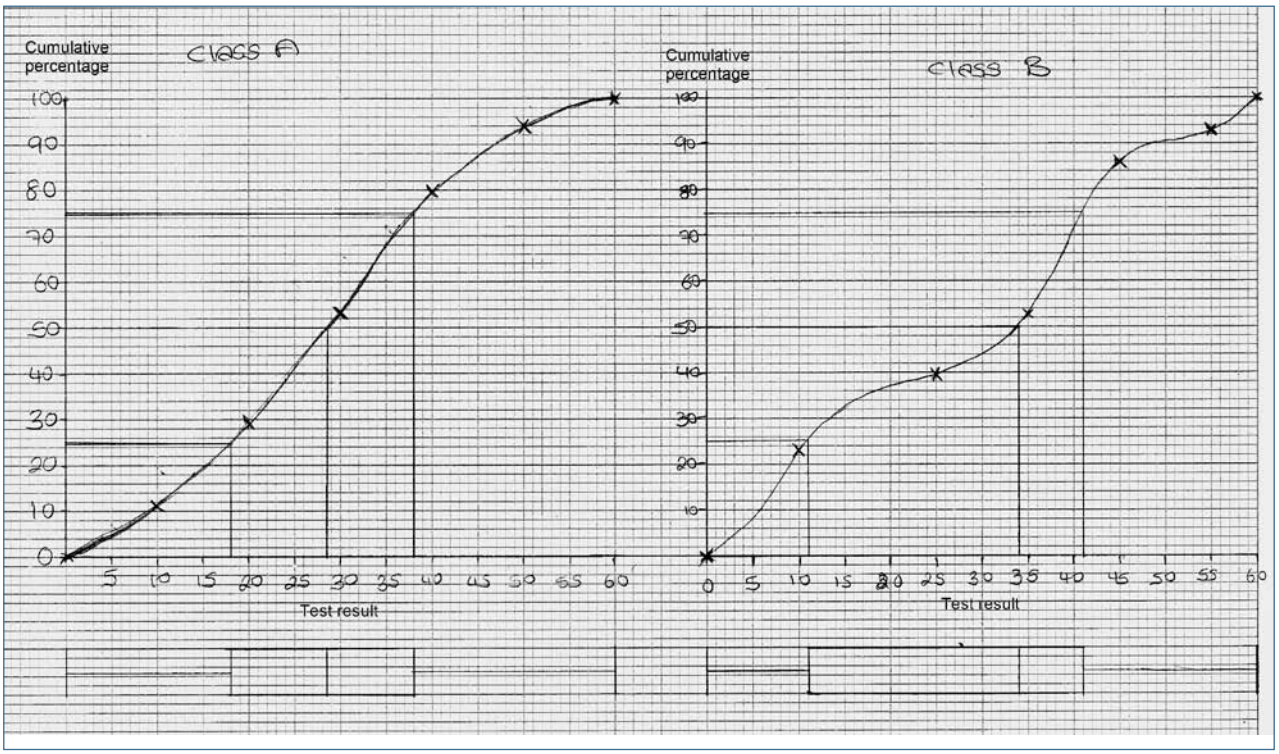
Pupil N's attainment in geometry and measures is best described as low level 8. She has procedural fluency with a range of techniques and applies them appropriately. To move further in the level she needs to extend understanding of mathematical similarity and congruence to the results of transformations and combining transformations. She needs to generalise her understanding of the relationships between dimensions and formulae, for example, to explain why πr^2 cannot be the formula for the volume of a solid.

Assessment focus

Handling data

Context

Classwork: As an introduction to a project using the handling data cycle to explore a hypothesis, pupils compared data sets using different graphical representations.



Teacher's notes

- constructs cumulative frequency curves
- constructs box plots from cumulative frequency curves

Next steps

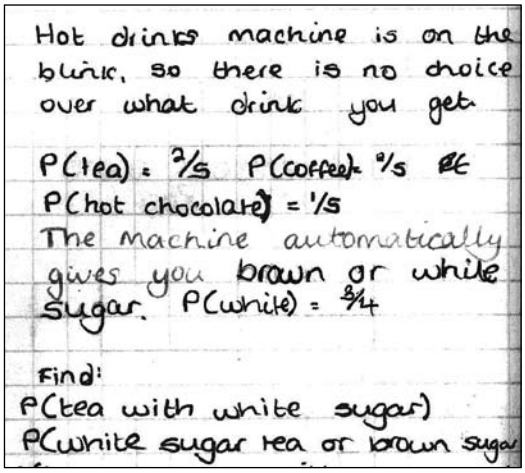
- investigate the effect of grouping data, for example, compare the estimated statistics with those obtained using the raw data
- use ICT to analyse large data sets
- apply statistical techniques to a substantial problem

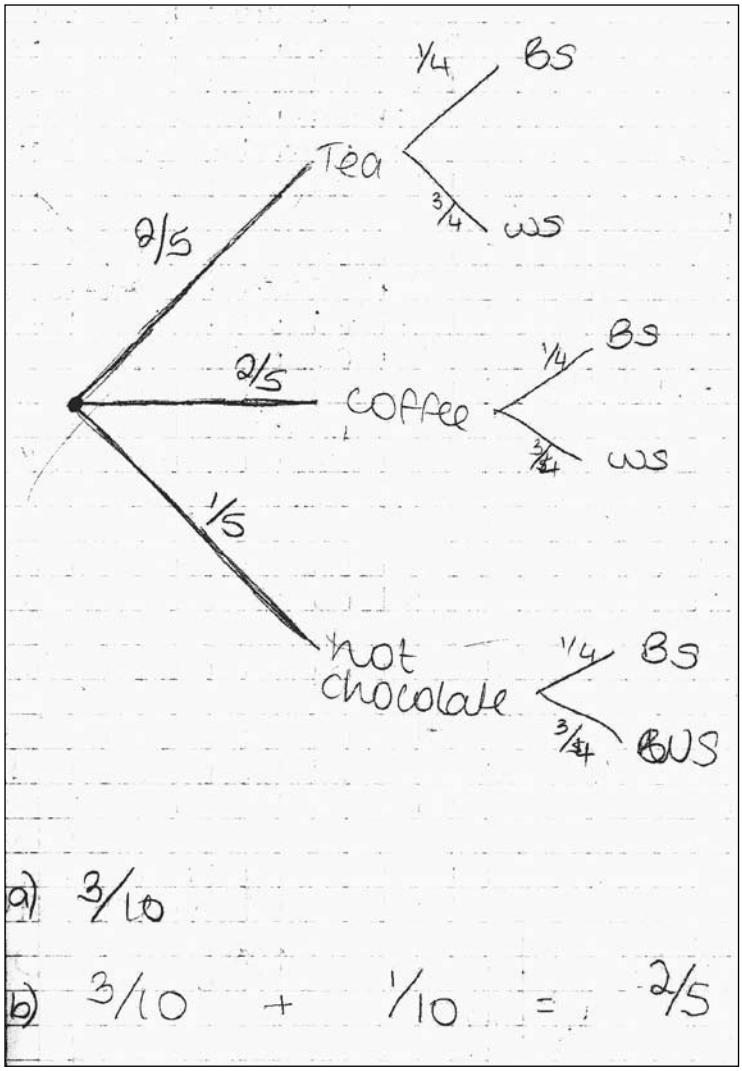
Assessment focus

Handling data

Context

Classwork: Pupils constructed tree diagrams to represent conditional probability.





Teacher's notes

- draws tree diagrams to represent outcomes
- calculates compound probabilities

Next steps

- generalise rules for calculating probabilities of independent events
- work on a wider range of contexts including simple instances of conditional probability
- consider the validity of statistical independence in different contexts

What the teacher knows about Pupil N's attainment in handling data

Pupil N takes steps to avoid bias when she has identified possible causes. For example, when comparing the heights of pupils in the UK with South Africa using the Census at School database, she ensured that the age distributions were as wide as possible and were similar for both countries.

Pupil N calculates descriptive statistics accurately. She constructs suitable statistical diagrams including cumulative frequency diagrams, box plots and histograms with equal class intervals. She estimates the median and quartiles from diagrams. She describes distributions by noting the shape of a diagram and the value of its mean and range where appropriate.

Pupil N uses tree diagrams and other representations to solve probability problems and knows how to combine probabilities appropriately. She checks her answers by ensuring that the probabilities of the outcomes in the sample space sum to one. In earlier work she has conducted experiments that compare estimated probability (from relative frequency) with theoretical probability and understands that more trials usually lead to a more accurate estimate.

Summarising Pupil N's attainment in handling data

Pupil N's attainment in statistics is best described as high level 7. She calculates a range of sample statistics accurately and uses these to describe the shape of a distribution. To consolidate level 7 and progress into level 8 she needs to apply this knowledge to problems within the context of the whole handling data cycle. She needs to pose questions to explore and develop her ability to interpret and evaluate the results of data collection, make inferences and relate results to a hypothesis. She needs to extend her work with grouped data to working with unequal class intervals and investigating the effect of grouping data on estimated statistics. She needs to calculate probabilities in a wide range of situations to consolidate her understanding of sample spaces and approaches to calculating probabilities of combinations of events.

What the teacher knows about Pupil N's attainment in using and applying mathematics

Pupil N poses questions and solves problems. She applies previously learnt material well and draws on mathematics from a range of contexts. For example, she suggested modifying the volume optimisation problem to find the largest closed cylinder volume that could be made from an A4 piece of card. She generated formulae and used a spreadsheet to find values for radius and length although she didn't fully appreciate all the constraints that were introduced until she tried to explain the results.

Pupil N selects appropriate mathematics to apply to a situation and communicates effectively. She gives reasons for her choices in response to probing questions. In the work on surds and indices she understood that when surds were multiplied simplification was often possible. She chooses ICT effectively to support calculations, to analyse large data sets and to explore geometric and graphical situations.

Pupil N uses short chains of deductive reasoning to solve problems. With prompting, she appreciates the difference between mathematical explanation and experimental evidence. For example, she initially used the angle measuring function to show that the triangles she created with dynamic geometry were similar. When asked if there was a different way she could show this, she used congruence properties ('the small triangles are the same') and the sum of angles at a point on a straight line to give a better explanation.

Summarising Pupil N's attainment in using and applying mathematics

Pupil N's attainment in using and applying mathematics is best described as high level 7. To progress further to level 8 she needs to comment on and justify results more independently. She also needs to compare approaches based on experimental evidence and those based on inductive and deductive proof. For example, she might add her own commentary to given diagrams that demonstrate a proof or arrange the steps of a proof given on cards.

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Pupil name……N……Class/group……Date……

	Using and applying mathematics	Numbers and the number system	Calculating	Algebra	Shape, space and measure	Handling data
Level 8	• develop and follow alternative methods and approaches • reflect on lines of enquiry when exploring mathematical tasks • select and combine known facts and problem-solving strategies to solve problems of increasing complexity • convey mathematical meaning through precise and consistent use of symbols • examine generalisations or solutions reached in an activity, commenting constructively on the reasoning and logic or the process employed, or the results obtained • distinguish between practical demonstration and proof; know underlying assumptions, recognising their importance and limitations, and the effect of varying them ☐	• understand the equivalence between recurring decimals and fractions ☑	• use fractions or percentages to solve problems involving repeated proportional changes or the calculation of the original quantity given the result of a proportional change • solve problems involving calculating with powers, roots and numbers expressed in standard form, checking for correct order of magnitude and using a calculator as appropriate ☐	• factorise quadratic expressions including the difference of two squares, e.g. $x^2 - 9 = (x + 3)(x - 3)$ • manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions • derive and use more complex formulae and change the subject of a formula • evaluate algebraic formulae, substituting fractions, decimals and negative numbers • solve inequalities in two variables and find the solution set • sketch, interpret and identify graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations • understand the effect on a graph of addition of (or multiplication by) a constant ☑	• understand and use congruence and mathematical similarity • understand and use trigonometrical relationships in right-angled triangles, and use these to solve problems, including those involving bearings • understand the difference between formulae for perimeter, area and volume in simple contexts by considering dimensions ☑	• estimate and find the median, quartiles and interquartile range for large data sets, including using a cumulative frequency diagram • compare two or more distributions and make inferences, using the shape of the distributions and measures of average and spread including median and quartiles • know when to add or multiply two probabilities • use tree diagrams to calculate probabilities of combinations of independent events ☐
Level 7	• solve increasingly demanding problems and evaluate solutions; explore connections in mathematics across a range of contexts: number, algebra, shape, space and measures, and handling data; refine or extend the mathematics used to generate fuller solutions • give reasons for choice of presentation, explaining selected features and showing insight into the problems structure • justify generalisations, arguments or solutions • appreciate the difference between mathematical explanation and experimental evidence ☑	• understand and use proportionality ☐	• calculate the result of any proportional change using multiplicative methods • understand the effects of multiplying and dividing by numbers between 0 and 1 • add, subtract, multiply and divide fractions • make and justify estimates and approximations of calculations; estimate calculations by rounding numbers to one significant figure and multiplying and dividing mentally • use a calculator efficiently and appropriately to perform complex calculations with numbers of any size, knowing not to round during intermediate steps of a calculation ☑	• square a linear expression, and expand and simplify the product of two linear expressions of the form $(x \pm n)$ and simplify the corresponding quadratic expression • use algebraic and graphical methods to solve simultaneous linear equations in two variables • solve inequalities in one variable and represent the solution set on a number line • use formulae from mathematics and other subjects; substitute numbers into expressions and formulae; derive a formula and, in simple cases, change its subject • find the next term and nth term of quadratic sequences and functions and explore their properties • plot graphs of simple quadratic and cubic functions, e.g. $y = x^2$, $y = 3x^2 + 4$, $y = x^3$ ☐	• understand and apply Pythagoras' theorem when solving problems in 2-D • calculate lengths, areas and volumes in plane shapes and right prisms • enlarge 2-D shapes, given a centre of enlargement and a fractional scale factor, on paper and using ICT; recognise the similarity of the resulting shapes • find the locus of a point that moves according to a given rule, both by reasoning and using ICT • recognise that measurements given to the nearest whole unit may be inaccurate by up to one-half of the unit in either direction • understand and use measures of speed (and other compound measures such as density or pressure) to solve problems ☐	• suggest a problem to explore using statistical methods, frame questions and raise conjectures; identify possible sources of bias and plan how to minimise it • select, construct and modify, on paper and using ICT, suitable graphical representation to progress an enquiry including frequency polygons and lines of best fit on scatter graphs • estimate the mean, median and range of a set of grouped data and determine the modal class, selecting the statistic most appropriate to the line of enquiry • compare two or more distributions and make inferences, using the shape of the distributions and measures of average and range • understand relative frequency as an estimate of probability and use this to compare outcomes of an experiment • examine critically the results of a statistical enquiry, and justify the choice of statistical representation in written presentation ☑
BL	☐	☐	☐	☐	☐	☐
IE	☐	☐	☐	☐	☐	☐

Key: BL-Below Level IE-Insufficient Evidence

Overall assessment (tick one box only) Low 7 ☐ Secure 7 ☐ High 7 ☐ Low 8 ☑ Secure 8 ☐ High 7 ☐

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