The National Strategies
Secondary

Assessing pupils' progress in mathematics at Key Stage 3: Standards File

Pupil R

department for
children, schools and families

QCA Qualifications and Curriculum Authority

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Pupil R – Year 8 – Secure level 6

Assessing pupils' progress in mathematics at Key Stage 3

Assessment summary

Pupil R's overall attainment across mathematics is judged to be secure level 6. Her understanding of numbers and the number system is assessed as high level 5 and low level 6 for handling data. Her attainment is secure in all the other assessment focuses.

Using the Standards Files

- The current Standards Files are based on work planned and assessed in relation to the 1999 National Curriculum programme of study. A new set of Standards Files based on the 2008 National Curriculum are currently in production, but the current set will provide useful guidance on making APP assessments against national standards in the transition period as the new programmes of study are introduced.
- The commentaries in the Standards Files are provided for guidance and reference, and are much more extensive than any teacher would be expected to make when carrying out APP assessments. It is also important to remember that APP encourages and enables a broader overview of current learning, and that there is no need to collect special portfolios of pupils' work. Evidence from pupils' written and oral work, backed up by brief teacher's notes where necessary, is all that is required.
- The evidence base presented in each Standards File is necessarily partial, as it would of course be impractical to reproduce all of each pupil's work. Examples of each pupil's work have been selected to provide evidence to support judgements against APP criteria. This evidence should be considered in conjunction with the teacher's notes, which will provide a broader context and further justification for the assessments that are made.

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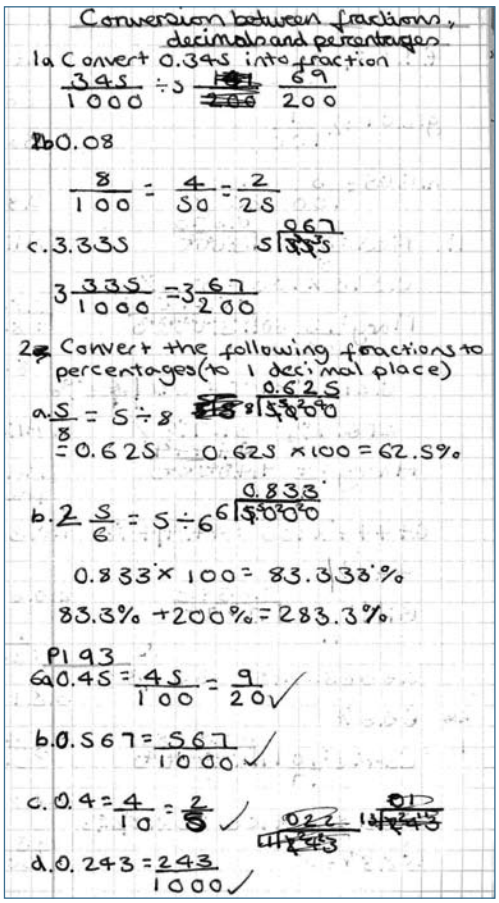
Assessment focus

Numbers and the number system; Calculating

Context

Classwork: At the beginning of a unit on proportional reasoning, pupils reviewed what they knew of equivalence between fractions, decimals and percentages.



Teacher's notes

- knows and uses procedures for the conversion of fractions, decimals and percentages
- expresses decimals with two or three decimal places as fractions with a denominator of 100 or 1000 appropriately
- reduces fractions to their simplest form by cancelling common factors
- expresses 3.335 as a mixed number and 2 5/6 as a percentage greater than 100
- divides the numerator by the denominator to convert a fraction to a decimal, using a written method of division, and then multiplies by 100 to convert the decimal to a percentage

Next steps

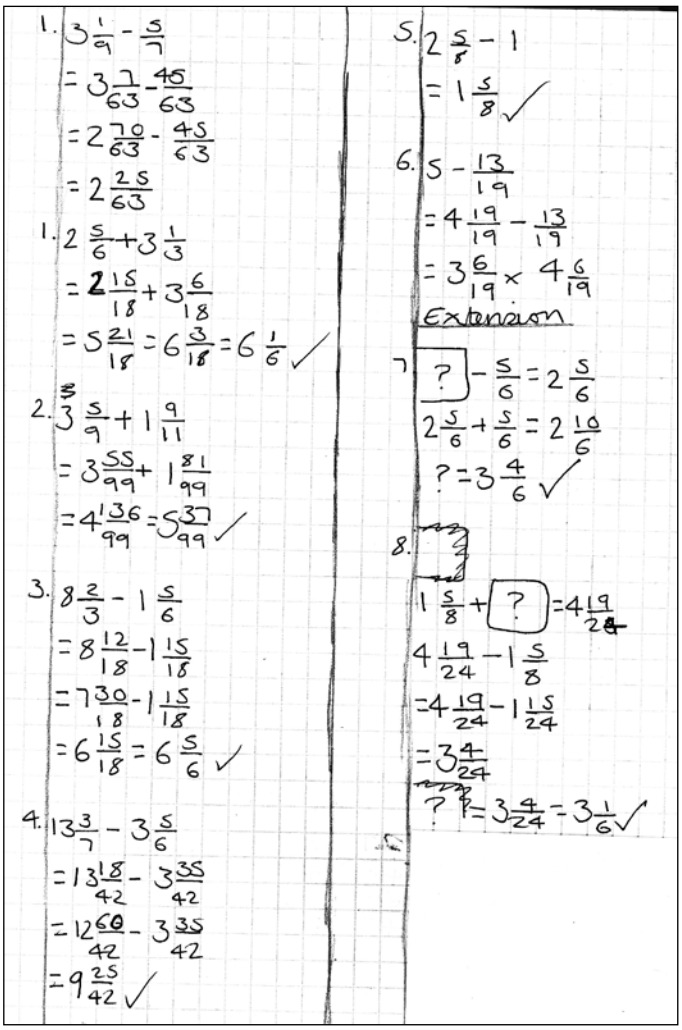
- use the conversion of fractions, decimals and percentages to solve problems such as finding the better deal when one price is given inclusive of VAT and one exclusive of VAT
- consider the types of fraction that give rise to recurring decimals
- develop more formal and correct notation (for example, to show cancelling as in question 1a)

Assessment focus

Calculating

Context

Classwork: During a unit on proportional reasoning, the class discussed mixed fractions. Pupils built on earlier work that involved adding and subtracting fractions to add and subtract mixed numbers.



Teacher's notes

- adds and subtracts mixed numbers by expressing the fractional parts with a common denominator
- solves subtraction problems that involve breaking down a unit, e.g. $3\ 1/9 - 5/7$
- uses addition and subtraction as inverse operations to find missing values
- usually reduces fractions to their simplest form

Next steps

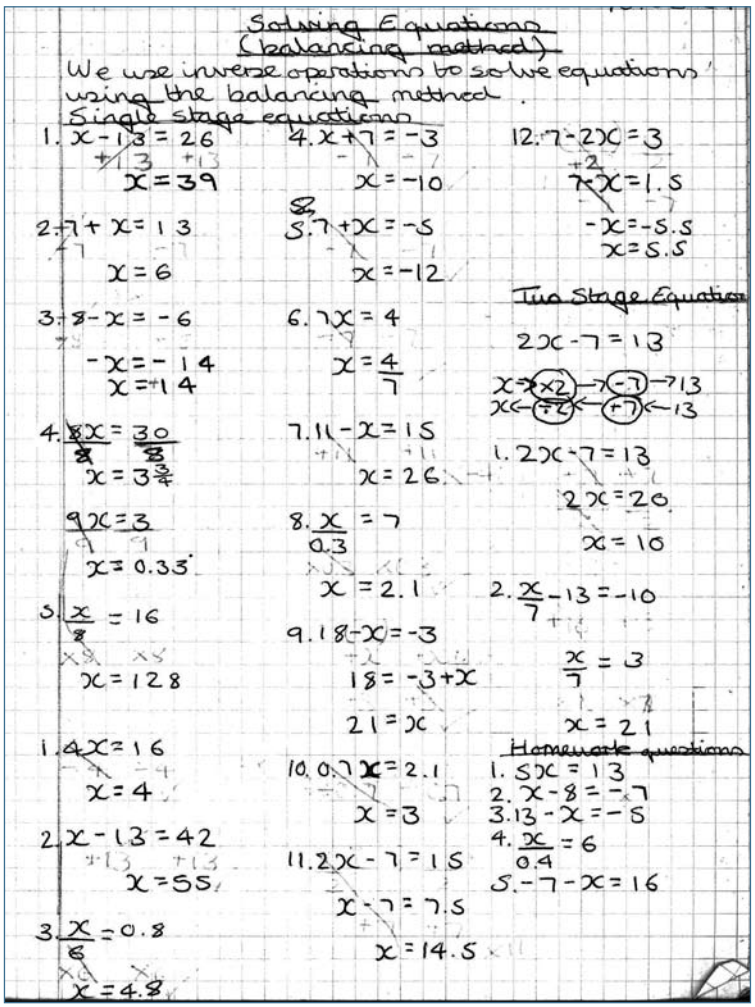
- check her work to see if she has always used the lowest common multiple as the common denominator and if resulting fractions are expressed in their simplest form
- consider the advantages and disadvantages of expressing values as a fraction, a decimal or a percentage

Assessment focus

Algebra

Context

Classwork: At the beginning of an algebra unit on solving linear equations pupils solved simple linear equations.



Teacher's notes

- solves linear equations with the unknown on one side only
- generally transforms both sides in the same way
- also uses two-step inverse operations
- calculates with decimals mentally

Next steps

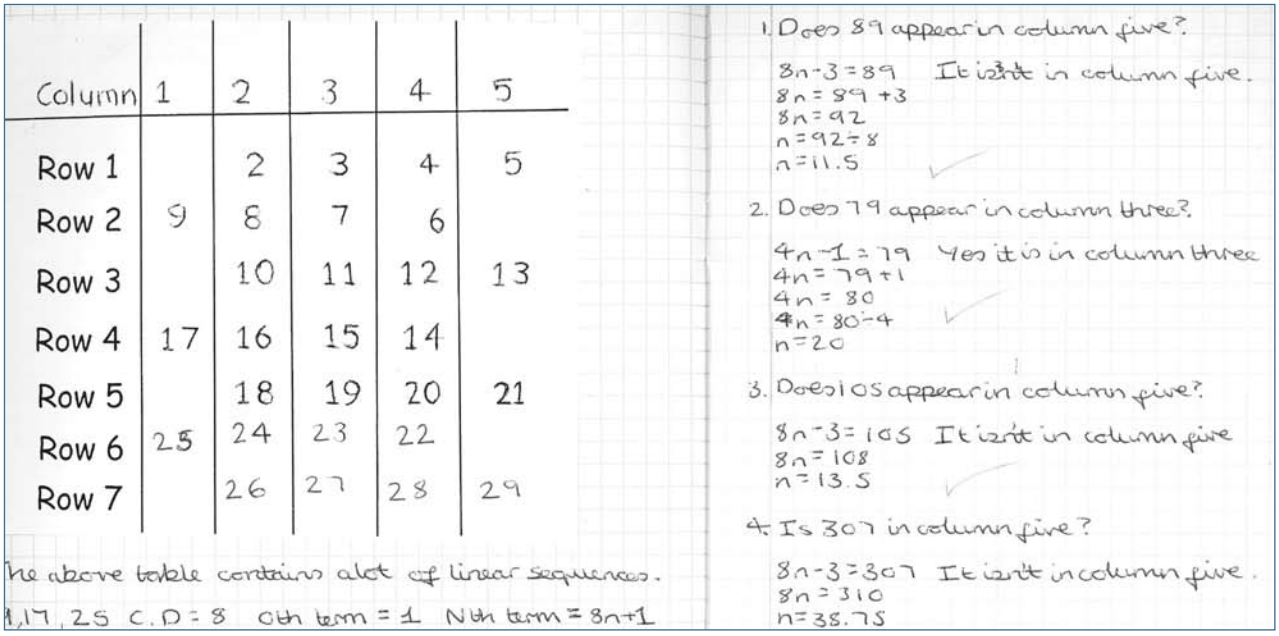
- check solutions by substituting her value for *x* into the original equation
- discuss the accuracy of giving ⅓ or $0.\dot{3}$ as the solution to $9x = 3$
- extend methods to solve equations with the unknown on both sides

Assessment focus

Algebra

Context

Classwork: In a unit on algebra, pupils reviewed what they knew about term-to-term rules for various sequences and position-to-term rules for linear sequences. They followed instructions to write the numbers from 2 to 29 into the rows. They then used the numbers in the columns to form the terms of linear sequences and found the *n*th term of these sequences. They used their formulae to check if given numbers would appear later in a sequence.



Column	1	2	3	4	5
Row 1		2	3	4	5
Row 2	9	8	7	6	
Row 3		10	11	12	13
Row 4	17	16	15	14	
Row 5		18	19	20	21
Row 6	25	24	23	22	
Row 7		26	27	28	29

The above table contains alot of linear sequences.
1, 17, 25 C.D = 8 0th term = 1 Nth term = 8n+1

1. Does 89 appear in column five?
8n-3=89 It isn't in column five.
8n=89+3
8n=92
n=92÷8
n=11.5

2. Does 79 appear in column three?
4n-1=79 Yes it is in column three
4n=79+1
4n=80
4n=80÷4
n=20

3. Does 105 appear in column five?
8n-3=105 It isn't in column five
8n=108
n=13.5

4. Is 307 in column five?
8n-3=307 It isn't in column five.
8n=310
n=38.75

Teacher's notes

- considers the sequences of numbers in each column and whether they can use a term-to-term or a position-to-term rule to describe them
- expresses the rule for the *n*th term of the sequences where the rule is linear
- sets up and solves an equation to check if a larger number is in a sequence
- recognises that the resulting value for *n* will be a positive integer if the number is in the sequence because *n* represents the position in the sequence
- explains '89 couldn't be in column 5 because 89 isn't 3 less than a number in the 8 times table… it's in between 85 and 93 and they're the eleventh and the twelfth number in the sequence'

Next steps

- consider whether particular negative numbers might be in a sequence if it is extended backwards

What the teacher knows about Pupil R's attainment in number and algebra

Pupil R generates sequences in various ways including, for example, using a spreadsheet, constructing an 'adding on' formula and filling down. Pupil R formulates the rule for the *n*th term of a given arithmetic sequence. She solves linear equations by transforming both sides in the same way and performing inverse operations. She also solves equations such as $x^2 - 1 = 48$ mentally where there are integer solutions and with the support of probing questions recognises, in this case, that there are two possible values for x.

Pupil R plots the graphs of linear functions by hand and using a graphing calculator. Plotting the graphs of $y = x + 2$ and $y = 4 - x$ and investigating where they intersect, she reasoned that the point of intersection was 'like solving $x + 2 = 4 - x$'. She solved this equation to find the value of x and related this to the x coordinate of the point of intersection. Pupil R is beginning to solve simultaneous linear equations algebraically as well as by plotting the graphs.

Pupil R uses the equivalence of fractions when she adds and subtracts mixed numbers and fractions with different denominators by expressing them with a common denominator. Pupil R investigated 'Egyptian fractions', expressing any fraction as the sum of unit fractions, for example finding that $\frac{6}{7}$ could be expressed as $\frac{1}{2} + \frac{1}{3} + \frac{1}{42}$. She converts fractions to decimals or percentages and vice versa. Pupil R compares fractions by converting them into decimals using a calculator as well as by expressing them with a common denominator. Pupil R divides quantities into two or more parts using a given ratio. She solves problems involving percentage increase and decrease using written methods. She uses her reasoning about fractions, decimals and percentages to decide, for example, whether an offer of '3 for the price of 2' is better than '30% extra free'.

Summarising Pupil R's attainment in number and algebra

Pupil R's attainment in number and algebra is best described as secure in level 6. To demonstrate further progress and evidence of working at high level 6, Pupil R needs to construct functions to represent a wider range of problems. She needs to develop her understanding of operations with fractions to include dividing by as well as multiplying by a fraction. She needs to develop her reasoning about proportional change so that she can begin to use multiplicative methods to solve problems such as finding the original cost of an article labelled '15% off… now £55.25' or problems involving ratio. She needs to develop her understanding of the equivalence of graphical, algebraic and numerical representations.

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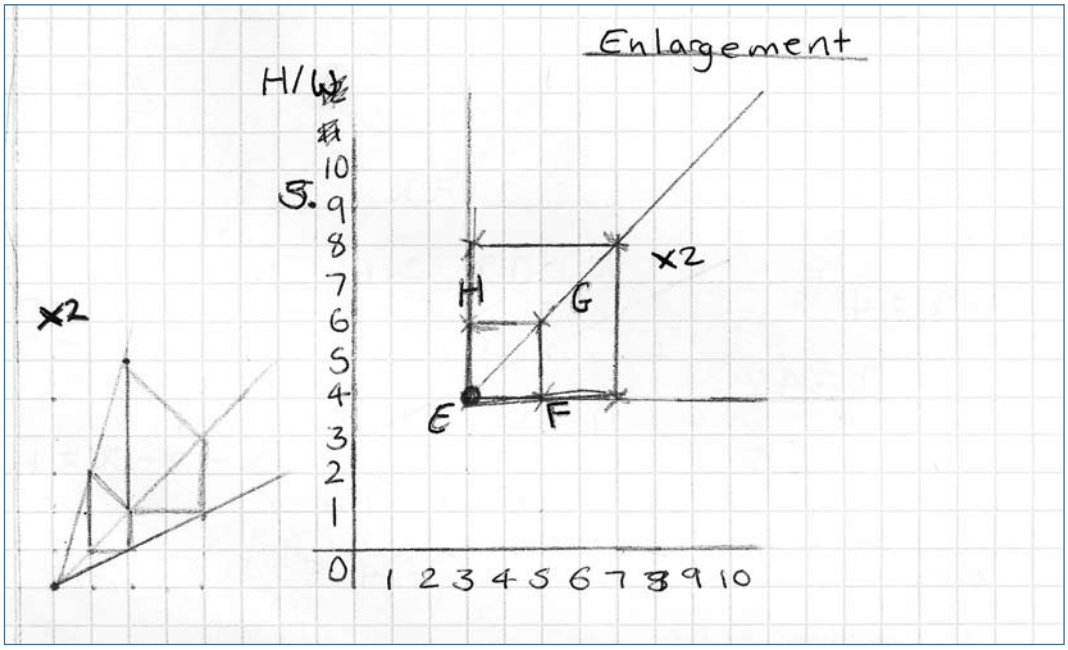
Assessment focus

Shape, space and measures

Context

Homework: At the beginning of a unit on transformations, pupils created diagrams to show their understanding of enlarging a shape by a scale factor of 2.



Teacher's notes

- shows a square enlarged with one of its vertices as the centre of enlargement
- enlarges a trapezium using a centre of enlargement that is external to the shape
- in discussion explains 'the coordinates are doubled when you enlarge by a scale factor of two and (0, 0) is the centre for the enlargement, but that doesn't work if the centre is somewhere else on the grid'

Next steps

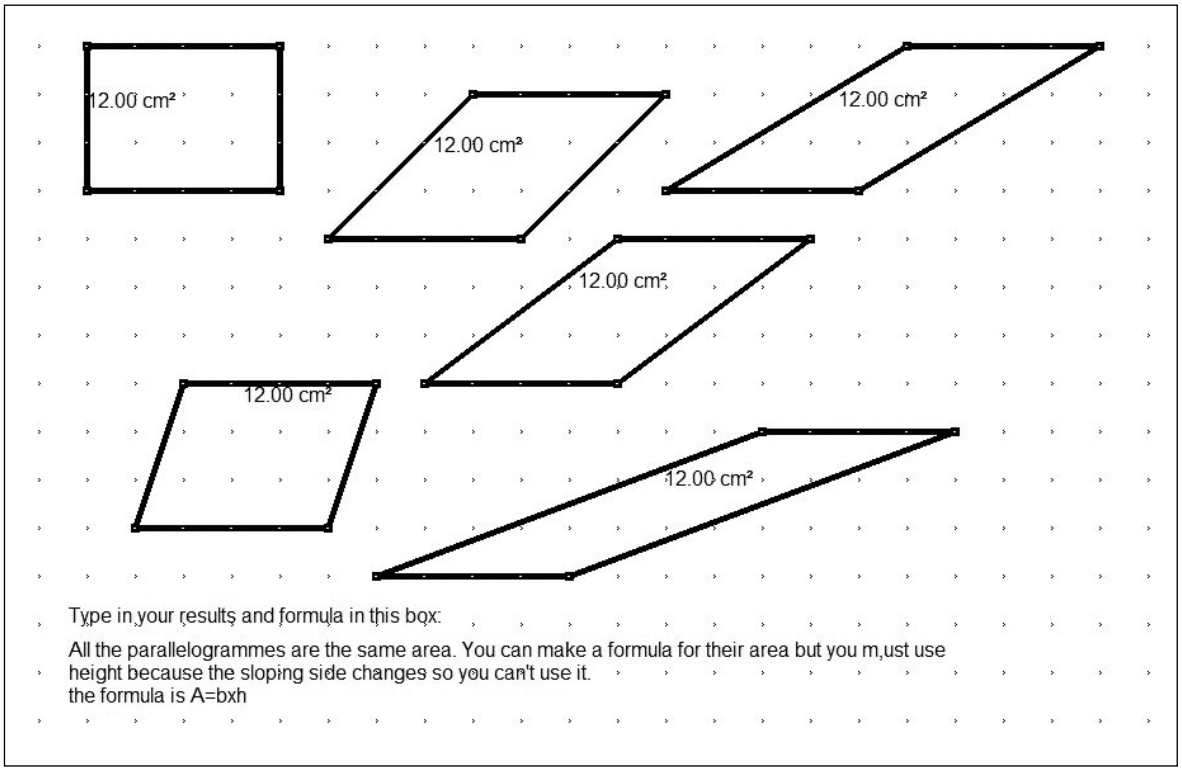
- enlarge shapes drawn in other quadrants, using (0, 0) as the centre of enlargement, and look at the effect on coordinates of vertices
- enlarge shapes using different centres of enlargement and scale factors

Assessment focus

Shape, space and measures; Using and applying mathematics

Context

Classwork: pupils used dynamic geometry software to investigate parallelograms with the same base and area as a given rectangle. They then investigated parallelograms relating to different rectangles. Their aim was to identify a formula for the area of a parallelogram.

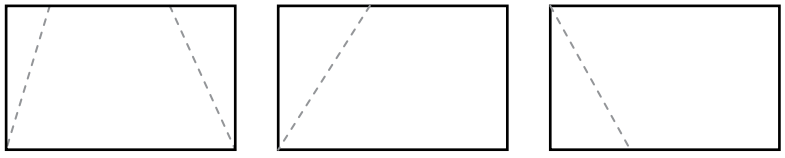


Teacher's notes

- uses trial and improvement to create parallelograms with the same base and area
- looks for relationships between the parallelograms and the rectangle other than base and area
- concludes that they all have the same (perpendicular) height even though the sloping sides are different lengths
- asserts a formula is possible 'but you must use height because the sloping side changes so you can't use it'
- gives the formula as $A = b \times h$

Next steps

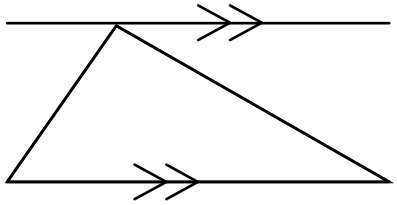
- create trapezia on a square grid by removing right-angled triangles from a rectangle of a given size, and investigate the properties of the trapezia that have the same height and area



What the teacher knows about Pupil R's attainment in shape, space and measures

Pupil R creates and interprets drawings of 3-D objects such as prisms and cuboids using isometric grids. She also represents more complex shapes. For example, she represented a shape made with five linking cubes and its reflection in a mirror line.

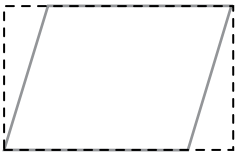
Working with quadrilaterals, Pupil R uses properties relating to pairs of parallel sides, the lengths of sides and diagonals and angles at the vertices and where diagonals intersect. She uses the angle properties of regular polygons, knows the sum of exterior angles is 360° and expresses the sum of interior angles in algebraic form. For example, Pupil R was given a diagram of a triangle with a line parallel to its base drawn though the opposite vertex:



She used the properties of parallel and intersecting lines to identify equal angles. She used the sum of angles at a point on a straight line to prove that the angle sum of a triangle is 180°.

Pupil R reflects, translates and rotates 2-D shapes on grids. She understands that when she transforms shapes in these ways, lengths and angles do not change. Pupil R also enlarges shapes using a centre of enlargement and a positive scale factor. She knows that when the centre of enlargement is the origin, (0, 0) and the image is enlarged by a scale factor of 2 then the coordinates of each vertex are doubled. She also knows that this rule does not apply to other centres of enlargement. Pupil R recognises that when enlarging shapes the side lengths are multiplied by the scale factor.

Pupil R knows and uses the formula for the area of a triangle. When she investigated parallelograms with the same base and area as a given rectangle, she found they had the same perpendicular height. She used this to derive a formula for the area of a parallelogram. She calculated the areas of other parallelograms using the base length and the perpendicular height. With the support of group discussion, Pupil R found a different way to calculate the area so that she could check her formula. She completed a rectangle around a parallelogram on her grid:



She then calculated the area of the rectangle and subtracted the areas of the triangles.

Summarising Pupil R's attainment in shape, space and measures

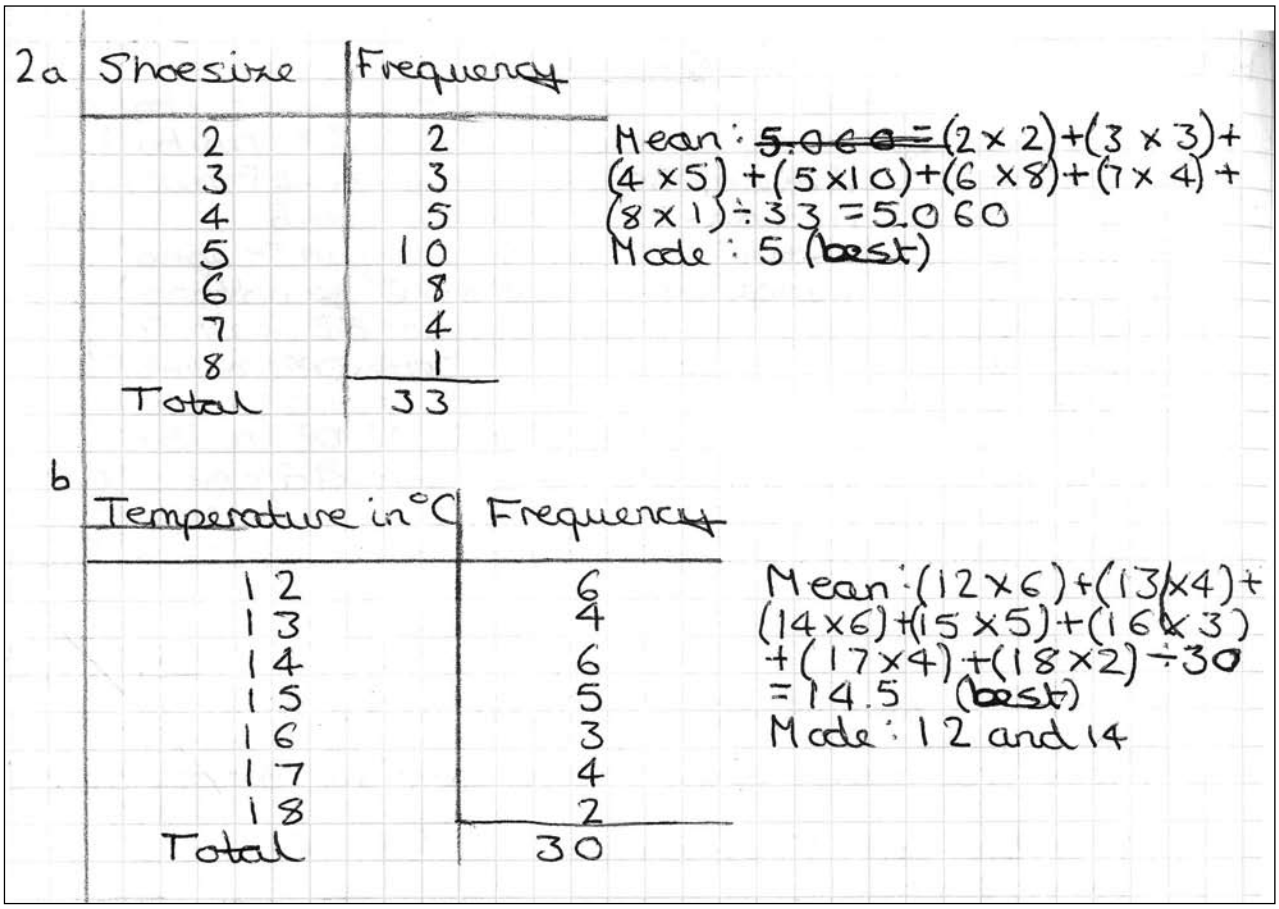
Her teacher judges Pupil R's attainment in shape, space and measures to be secure level 6. To make further progress within level 6, Pupil R needs to develop the use of a straight edge and pair of compasses for standard constructions. She needs to solve problems involving the volume and surface area of cuboids. She needs to begin to solve problems involving a compound measure such as speed.

Assessment focus

Handling data

Context

Classwork: In a unit on statistics, pupils processed and interpreted data. They calculated the mean and mode of discrete data in a frequency table. Pupils discussed the most appropriate average to use in each situation.



2a

Shoesize	Frequency
2	2
3	3
4	5
5	10
6	8
7	4
8	1
Total	33

Mean: ~~5.060~~ = (2 × 2) + (3 × 3) + (4 × 5) + (5 × 10) + (6 × 8) + (7 × 4) + (8 × 1) ÷ 33 = 5.060
Mode: 5 (best)

b

Temperature in °C	Frequency
12	6
13	4
14	6
15	5
16	3
17	4
18	2
Total	30

Mean: (12 × 6) + (13 × 4) + (14 × 6) + (15 × 5) + (16 × 3) + (17 × 4) + (18 × 2) ÷ 30 = 14.5 (best)
Mode: 12 and 14

Teacher's notes

- identifies the mode and calculates the mean of sets of data
- explains and justifies choice of average in discussion

Next steps

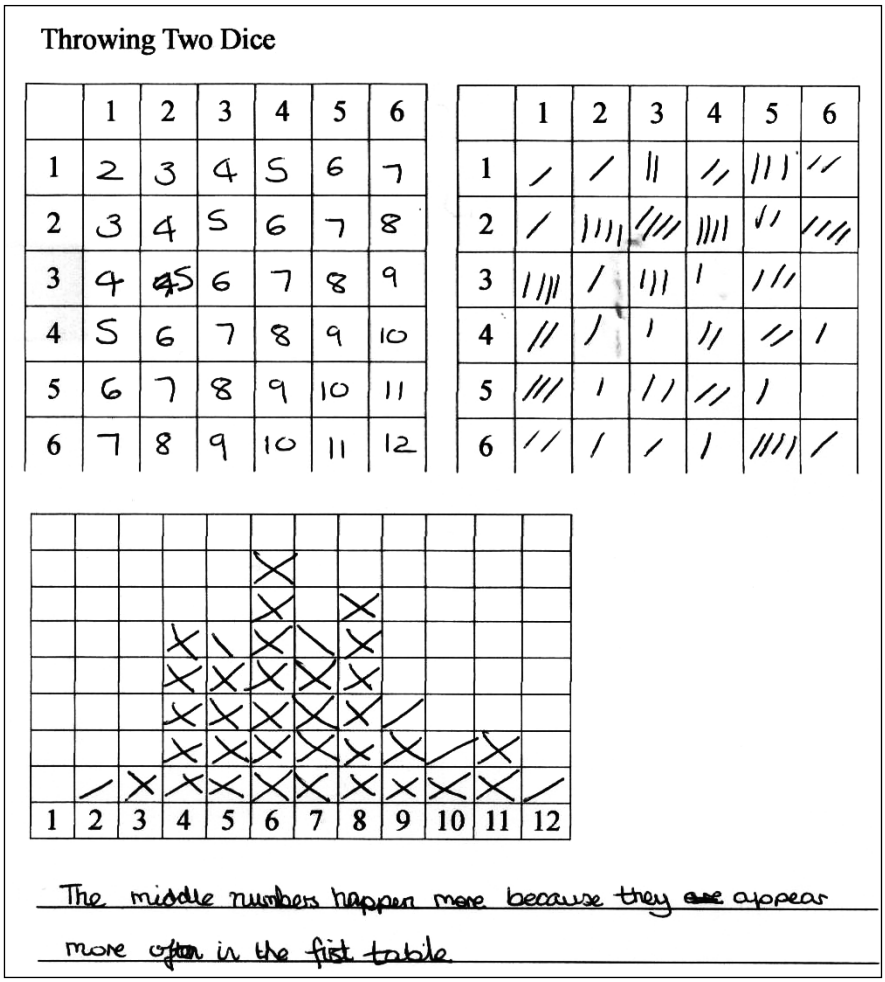
- use brackets or break calculations into stages to indicate correct order of operations when finding the mean
- use the mode and mean with range to compare sets of data
- relate measures of average and spread to graphs of frequency distributions

Assessment focus

Handling data

Context

During a unit on probability, pupils worked in pairs to explain the results of throwing two dice.



Throwing Two Dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

	1	2	3	4	5	6
1	/	/	//	///	////	/////
2	/	////	/////	////	///	////
3	////	/	///	//	///	
4	///	/	//	///	///	/
5	/////	//	///	///	//	
6	///	/	/	//	////	/

1	2	3	4	5	6	7	8	9	10	11	12

The middle numbers happen more because they ~~are~~ appear more often in the first table.

Teacher's notes

- uses a prepared table to list outcomes of throwing two dice and adding the scores
- chooses to use another copy of the table to record a tally of her results
- decides to throw the dice 70 times to have a reasonable sample size
- summarises her results using / for one event and X for two
- explains 'I expected to get more sevens than anything else and then sixes and eights but it doesn't always happen exactly'

Next steps

- use ICT and a simulation programme to collect and represent larger amounts of data on throwing two dice
- investigate whether experimental data for larger sets is closer to the theoretical probabilities

What the teacher knows about Pupil R's attainment in handling data

Pupil R identifies the mode and median and calculates the mean of a set of data. Talking about the usefulness of each measure, she concluded 'the mean is quite good for comparing the maximum daily temperatures for each month in different places…like for Seville in August the mean daily temperature is 38 °C and that's too hot for a lot of people even if some days aren't as hot as that'. She also explained 'the mode is better for shoe sizes because the mean is 5.06 and that doesn't make sense for a shoe size'. She also thought the median was a good way to compare data about shoe sizes 'because it's the shoe size for the middle person'.

When representing data sets, such as the time spent watching television over one weekend by the pupils in her year group, she reviews the listed data and chooses appropriate class intervals for a frequency diagram. Pupil R also creates bar charts and pie charts, using ICT. She understands that each sector of a pie chart represents a fraction of a population. She has used pie charts to compare the music tastes of pupils in the class with national trends. Pupil R and her group categorised a published list of the top 100 tracks in the country. They each used ICT to create a pie chart to represent the information. Pupils in the class voted for the category of music they preferred. Pupil R then created a similar pie chart for comparison. Comparing the pie charts, she found that the tastes of pupils in the class were different from tastes across the country. She concluded that the pupils had different musical tastes compared with the national results, and suggested that this may have been because the national data had involved a wider age range.

Pupil R records all possible outcomes for two successive events in an organised way. For example, she recorded in a table the possible outcomes of throwing two dice and totalling their scores. She used the table to predict the most likely scores and experimented with two dice to test her prediction. She recognised that her results were close to her prediction. She suggested that 'you can't expect it to happen exactly like that but if we did more trials I think we'd get closer'. Pupil R also expresses a probability as a fraction and understands that the sum of the probabilities of mutually exclusive outcomes will be one.

Summarising Pupil R's attainment in handling data

Pupil R's attainment in handling data is best described as low level 6. To make further progress within level 6 and towards level 7, she needs to apply her data handling skills to a more substantial investigation. For example, she might make hypotheses about GDP and adult literacy or child mortality and then decide how to collect, process and represent real data to test her ideas. She needs to use scatter diagrams and begin to understand correlation. She needs to use her charts and diagrams to illustrate and justify her conclusions.

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What the teacher knows about Pupil R's attainment in using and applying mathematics

Pupil R applies the mathematics she has learned to solve new problems. Working with arithmetic sequences she formulated an algebraic expression for the *n*th term. She understood that her algebraic expression could be used to calculate the 50th term, for example, but she reasoned that given a term she could also find its position in the sequence, using what she knew about solving linear equations. Pupil R and her partner calculated later terms for the other to work out their positions by setting up and solving equations. When challenged to say if other numbers were terms of the sequence, Pupil R used a similar approach. For example, she solved $8n - 3 = 89$ and interpreted the result 11.5 as showing that 89 is greater than the 11th term and less than the 12th term. She justified this by adding that '89 isn't three less than a multiple of 8… and the numbers on either side would be 85 and 93'.

Pupil R interprets information presented in a variety of forms. For example, she interprets and compares linear functions and explains which graph will be steeper or cross the *y*-axis at a particular point. She interprets bar charts, pie charts, frequency tables and diagrams.

She interprets geometrical diagrams and symbols that identify equal angles, equal lengths and parallel lines, for example. She reasons about lines, angles and polygons. She finds angles in diagrams of parallel and intersecting lines and understands a proof of the angle sum of a triangle. She learned to enlarge 2-D shapes using a centre of enlargement and a positive scale factor. She then reasoned how to find the centre of enlargement and the scale factor when given a shape and its enlargement.

Summarising Pupil R's attainment in using and applying mathematics

Pupil R's attainment in using and applying mathematics is best described as secure level 6. To make further progress within level 6 she needs to solve a wider range of problems, particularly problems that need to be broken down into smaller more manageable tasks. She needs to develop her understanding of appropriate degrees of accuracy, for example, by considering the appropriateness of rounding a decimal at different stages in reaching a solution. She is developing reasoned arguments in group discussion and needs to incorporate explanatory text when she records solutions.

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Pupil name……R……**Class/group**……**Date**……

	Using and applying mathematics	Numbers and the number system	Calculating	Algebra	Shape, space and measure	Handling data
Level 6	• solve problems and carry through substantial tasks by breaking them into smaller, more manageable tasks, using a range of efficient techniques, methods and resources, including ICT; give solutions to an appropriate degree of accuracy • interpret, discuss and synthesise information presented in a variety of mathematical forms • present a concise, reasoned argument, using symbols, diagrams, graphs and related explanatory texts • use logical argument to establish the truth of a statement ☑	• use the equivalence of fractions, decimals and percentages to compare proportions ☐	• calculate percentages and find the outcome of a given percentage increase or decrease • divide a quantity into two or more parts in a given ratio and solve problems involving ratio and direct proportion • use proportional reasoning to solve a problem, choosing the correct numbers to take as 100%, or as a whole • add and subtract fractions by writing them with a common denominator, calculate fractions of quantities (fraction answers), multiply and divide an integer by a fraction ☑	• use systematic trial and improvement methods and ICT tools to find approximate solutions to equations such as $x^3 + x = 20$ • construct and solve linear equations with integer coefficients, using an appropriate method • generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT; write an expression to describe the nth term of an arithmetic sequence. • plot the graphs of linear functions, where y is given explicitly in terms of x; recognise that equations of the form $y = mx + c$ correspond to straight-line graphs • construct functions arising from real-life problems and plot their corresponding graphs; • interpret graphs arising from real situations ☑	• classify quadrilaterals by their geometric properties • solve geometrical problems using properties of angles, of parallel and intersecting lines, and of triangles and other polygons • identify alternate and corresponding angles; understand a proof that the sum of the angles of a triangle is 180° and of a quadrilateral is 360° • devise instructions for a computer to generate and transform shapes and paths • visualise and use 2-D representations of 3-D objects • enlarge 2-D shapes, given a centre of enlargement and a positive whole-number scale factor • know that translations, rotations and reflections preserve length and angle and map objects onto congruent images • use straight edge and compasses to do standard constructions • deduce and use formulae for the area of a triangle and parallelogram, and the volume of a cuboid; calculate volumes and surface areas of cuboidsknow and use the formulae for the circumference and area of a circle ☑	• design a survey or experiment to capture the necessary data from one or more sources; design, trial and, if necessary, refine data collection sheets; construct tables for large discrete and continuous sets of raw data, choosing suitable class intervals; design and use two-way tables • select, construct and modify, on paper and using ICT: – pie charts for categorical data – bar charts and frequency diagrams for discrete and continuous data – simple time graphs for time series – scatter graphs and identify which are most useful in the context of the problem • find and record all possible mutually exclusive outcomes for single events and two successive events in a systematic way • know that the sum of probabilities of all mutually exclusive outcomes is 1 and use this when solving problems • communicate interpretations and results of a statistical survey using selected tables, graphs and diagrams in support ☑
Level 5	• identify and obtain necessary information to carry through a task and solve mathematical problems • check results, considering whether these are reasonable • solve word problems and investigations from a range of contexts • show understanding of situations by describing them mathematically using symbols, words and diagrams • draw simple conclusions of their own and give an explanation of their reasoning ☐	• use understanding of place value to multiply and divide whole numbers and decimals by 10, 100 and 1000 and explain the effect • round decimals to the nearest decimal place and order negative numbers in context • recognise and use number patterns and relationships • use equivalence between fractions and order fractions and decimals • reduce a fraction to its simplest form by cancelling common factors • understand simple ratio ☑	• use known facts, place value, knowledge of operations and brackets to calculate including using all four operations with decimals to two places • use a calculator where appropriate to calculate fractions/percentages of quantities/measurements • understand and use an appropriate non-calculator method for solving problems that involve multiplying and dividing any three-digit number by any two digit number • solve simple problems involving ordering, adding, subtracting negative numbers in context • solve simple problems involving ratio and direct proportion • apply inverse operations and approximate to check answers to problems are of the correct magnitude ☐	• construct, express in symbolic form, and use simple formulae involving one or two operations • use and interpret coordinates in all four quadrants ☐	• use a wider range of properties of 2-D and 3-D shapes and identify all the symmetries of 2-D shapes • use language associated with angle and know and use the angle sum of a triangle and that of angles at a point • reason about position and movement and transform shapes • measure and draw angles to the nearest degree, when constructing models and drawing or using shapes • read and interpret scales on a range of measuring instruments, explaining what each labelled division represents • solve problems involving the conversion of units and make sensible estimates of a range of measures in relation to everyday situations • understand and use the formula for the area of a rectangle and distinguish area from perimeter ☐	• ask questions, plan how to answer them and collect the data required • in probability, select methods based on equally likely outcomes and experimental evidence, as appropriate • understand and use the probability scale from 0 to 1 • understand and use the mean of discrete data and compare two simple distributions, using the range and one of mode, median or mean • understand that different outcomes may result from repeating an experiment • interpret graphs and diagrams, including pie charts, and draw conclusions • create and interpret line graphs where the intermediate values have meaning ☐
BL	☐	☐	☐	☐	☐	☐
IE	☐	☐	☐	☐	☐	☐

Key: BL-Below Level IE-Insufficient Evidence

Overall assessment (tick one box only) Low 5 ☐ Secure 5 ☐ High 5 ☐ Low 6 ☐ Secure 6 ☑ High 6 ☐

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