Mathematics test

Paper 1

Calculator not allowed

Please read this page, but do not open your booklet until your teacher tells you to start. Write your name and the name of your school in the spaces below. If you have been given a pupil number, write that also.

First name

Last name

School

Pupil number

Remember

- The test is 1 hour long.
- You must not use a calculator for any question in this test.
- You will need: pen, pencil, rubber, ruler and a pair of compasses.
- Some formulae you might need are on page 2.
- This test starts with easier questions.
- Try to answer all the questions.
- Write all your answers and working on the test paper – do not use any rough paper.
- Check your work carefully.
- Ask your teacher if you are not sure what to do.

For marker’s use only: Total marks
Instructions

Answers
This means write down your answer or show your working and write down your answer.

Calculators
You must not use a calculator to answer any question in this test.

Formulae
You might need to use these formulae

**Trapezium**

Area = \( \frac{1}{2} (a + b)h \)

**Prism**

Volume = area of cross-section \( \times \) length
1. Work out the **number** that is halfway between $27 \times 38$ and $33 \times 38$

Show your working.

\[ \text{Number} = \text{Halfway value} \]

2 marks

2. Solve the equation $9y + 3 = 5y + 13$

Show your working.

\[ y = \text{Solution} \]

2 marks
3. This advert was in a newspaper.

It does not say how the advertisers know that 93% of people drop litter every day.

Some pupils think the percentage of people who drop litter every day is much lower than 93%.

They decide to do a survey.

(a) Jack says:

We can ask 10 people if they drop litter every day.

Give two different reasons why Jack’s method might not give very good data.

First reason:

Second reason:
(b) Lisa says:

We can go into town on Saturday morning.
We can stand outside a shop and record how many people walk past and how many of those drop litter.

Give two **different** reasons why Lisa’s method might not give very good data.

First reason:

Second reason:

4. Fill in the missing numbers in the boxes using **only negative numbers**.

\[ \square - \square = 5 \]

\[ \square - \square = -5 \]
5. You can often use algebra to show why a number puzzle works.

Fill in the missing expressions.

Example:

Think of a number

Add 4

Now add the number you were first thinking of

Divide by 2

Subtract 2

The answer is the number you were first thinking of

Algebra:

\[ n \]

\[ n + 4 \]

\[ \vdots \]
6. The diagram shows a rectangle that just touches an equilateral triangle.

(a) Find the size of the angle marked $\alpha$
Show your working.

(b) Now the rectangle just touches the equilateral triangle so that $ABC$ is a straight line.
Show that triangle $BDE$ is isosceles.
7. Three types of mice might come into our homes. Some mice are more likely to be found in homes far from woodland. Others are more likely to be found in homes close to woodland. The bar charts show the percentages of mice that are of each type.

Key
- Yellow-necked mice
- Wood mice
- House mice

Type of mouse found

<table>
<thead>
<tr>
<th></th>
<th>More than 500m</th>
<th>500m or less</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow-necked</td>
<td>40%</td>
<td>20%</td>
</tr>
<tr>
<td>Wood</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>House</td>
<td>10%</td>
<td>30%</td>
</tr>
</tbody>
</table>
Use the bar charts to answer these questions.

(a) About what percentage of mice in homes close to woodland are wood mice?

\[
\text{........... \%} \quad (1 \text{ mark})
\]

(b) About what percentage of mice in homes far from woodland are not wood mice?

\[
\text{........... \%} \quad (1 \text{ mark})
\]

(c) The black bars show the percentages for house mice. One of the black bars is taller than the other.

Does that mean there must be more house mice in homes far from woodland than in homes close to woodland?

Tick (✓) Yes or No.

\[
\text{Yes} \quad \text{No} \quad (1 \text{ mark})
\]

Explain your answer.
8. The graph shows a straight line. The equation of the line is $y = 3x$

(a) Does the point $(25, 75)$ lie on the straight line $y = 3x$?

Tick (✓) Yes or No.

Yes [ ] No [ ]

Explain how you know.
(b) Write the coordinates of the point that lies on both the straight lines $y = 4x + 1$ and $y = 6x - 4$

You **must** show your working.

\[
( , )
\]

(3 marks)

(c) Explain how you can tell there is no point that lies on both the straight lines $y = \frac{1}{2}x + 3$ and $y = \frac{1}{2}x + 5$

(1 mark)
9. \( \frac{1}{3}, \frac{1}{8}, \frac{1}{5} \) are all examples of unit fractions.

The ancient Egyptians used only unit fractions.

For \( \frac{3}{4} \), they wrote the sum \( \frac{1}{2} + \frac{1}{4} \)

(a) For what fraction did they write the sum \( \frac{1}{2} + \frac{1}{5} \)?

Show your working.

(b) They wrote \( \frac{9}{20} \) as the sum of two unit fractions.

One of them was \( \frac{1}{4} \)

What was the other?

Show your working.
(c) What is the biggest fraction you can make by adding two **different** unit fractions?

Show your working.
10. (a) The subject of the equation below is $p$

$$p = 2(e + f)$$

Rearrange the equation to make $e$ the subject.

(b) Rearrange the equation $r = \frac{1}{2}(c - d)$ to make $d$ the subject.

Show your working.
I have a bag that contains blue, red, green and yellow counters.

I am going to take out one counter at random.

The table shows the probability of each colour being taken out.

<table>
<thead>
<tr>
<th></th>
<th>Blue</th>
<th>Red</th>
<th>Green</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.05</td>
<td>0.3</td>
<td>0.45</td>
<td>0.2</td>
</tr>
</tbody>
</table>

(a) Explain why the number of **yellow** counters in the bag **cannot** be 10

(b) What is the **smallest** possible number of each colour of counter in the bag?
12. The diagram shows the locus of all points that are the **same distance** from A as from B.

The locus is one straight line.

(a) The locus of all points that are the **same distance** from (2, 2) and (−4, 2) is also one straight line.

Draw this straight line.

(b) The locus of all points that are the **same distance** from the x-axis as they are from the y-axis is **two** straight lines.

Draw both straight lines.
(c) Look at points C and D below.

Use a straight edge and compasses to draw the locus of all points that are the **same distance** from C as from D.

Leave in your construction lines.
13. Cars more than three years old must pass a test called an MOT.

The testers measure the right and left front wheel brakes, and give each brake a score out of 500

Then they use the graph.

For example: A car has $R = 300$, $L = 350$

$\left( 300, 350 \right)$ is in the white region, so the car passes this part of the test.
(a) A man takes his car to be tested.

\[ L = 200 \]

Approximately, between what values does \( R \) need to be for his car to pass this test?

\[ \ldots \ldots \text{ and } \ldots \ldots \]

1 mark

A different part of the test uses \( R + L \)

To pass, \( R + L \geq 400 \)

(b) On the graph, draw the straight line \( R + L = 400 \)

Then shade the region where the car fails, \( R + L < 400 \)

1 mark

(c) If \( L = 200 \), between what values does \( R \) need to be to pass both parts of the test?

\[ \ldots \ldots \text{ and } \ldots \ldots \]

1 mark
14. A picture has a board behind it.

The drawings show the dimensions of the rectangular picture and the rectangular board.

(a) Show that the two rectangles are not mathematically similar.

(b) Suppose you wanted to cut the board to make it mathematically similar to the picture.

Keep the width of the board as 14 cm.

What should the new height of the board be?

Show your working.
15. A robot can move N, S, E or W along the lines of a grid. It starts at the point marked ● and moves one step at a time. For each step, it is equally likely that the robot will move N, S, E or W.

(a) The robot is going to move 3 steps from the point marked ●.

What is the probability that it will move along the path shown?
Show your working.

(b) The robot is going to move 3 steps from the point marked ●.

What is the probability that it will reach the point marked X by any route?
16. The two rectangles below have the same area.

Use an algebraic method to find the value of $y$

You **must** show your working.

\[ y = \ldots \]

4 marks
17. I fill a glass with orange juice and lemonade in the ratio $1:4$

I drink $\frac{1}{4}$ of the contents of the glass, then I fill the glass using orange juice.

Now what is the ratio of orange juice to lemonade in the glass?

Show your working, and write the ratio in its simplest form.
18. This prism was made from three cuboids.

Not drawn accurately

(a) Show that the area of the cross-section of the prism is \(24x^2 + 3xy\)

(b) The volume of the prism is \(3x^2(8x + y)\)

What is the depth of the prism?

Show your working.