Weaker factor: not anticipating or spotting misconceptions

The starter activity for this lesson was a revision quiz on number patterns. This Year 11 bottom set contained 11 pupils, nine of whom had special educational needs. The teacher was not a mathematics specialist.

After the pupils had worked for several minutes on the quiz questions, the teacher asked individual pupils for their answers. One question was 'What is the first number in the sequence 3n+1?' A pupil correctly answered '4' and the teacher praised him and moved on to the next question. However, this answer could be derived incorrectly by ignoring the n in the expression 3n+1 and simply adding 3+1. The teacher did not check that the pupil knew to substitute 1 for n. Later, the inspector reminded the pupil about the question and his correct answer of 4 for the first number, and then asked what the second number in the sequence was. The pupil replied '5', adding 'then 6, and 7'.

How might it be improved?

The teacher did not know the common difficulties that pupils have with expressing sequences algebraically. When asked to find the value of an expression such as 3n+1, pupils who do not understand algebra often ignore the variable n and just calculate using the numbers, 3+1 in this case. This means that the correctly calculated answer for the first term is indistinguishable from the incorrect, as will always be the case for the first term of such a sequence. Asking the pupil how he worked out the answer, and going on to check the value of the next few terms would ascertain whether the pupil understood fully.

Moreover, pupils often write incorrect expressions like n+3 for a linear sequence that 'goes up in 3s', for example, 5, 8, 11, 14 … instead of the correct nth term, 3n+2. They struggle to move from spotting the term-to-term pattern to writing a general expression for any term in the sequence.

Assessing attainment: one piece of work at a time?

85. Many of the schools had prioritised the improvement of assessment, supported by materials from the National Strategies on assessment for learning. Typically, this involved whole-school training and changes to the assessment policy. For instance, some schools insisted that at regular intervals a piece of marked work should be assigned a National Curriculum level. However, too few schools recognised that the policy needed to be tailored to particular subjects. In mathematics, the nature of the subject content is hierarchical with many individual topics assigned to discrete levels and each level comprising a range of topics. Therefore, because levels are largely governed by the topic being covered, assigning a level to one piece of work is not representative, or even possible, especially if much of the work is incorrect. For example, work on circumference and area of circles is generally considered to be a Level 6 topic.

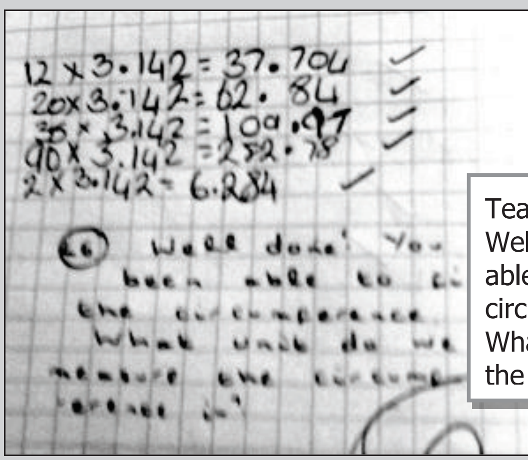
Incorrect or misunderstood work cannot be called Level 5. Similarly, to improve a piece of work to achieve a higher level is not always possible in mathematics; for instance the closest topic to work on circles might be volumes and surface area of cylinders, but this represents new learning rather than how to improve existing work. To get around this problem, schools typically set a test and give a level based on the mark awarded. However, this does not necessarily lead to an accurate or diagnostic assessment of what pupils can do well or need to improve.

86. The following extracts from pupils' work illustrate how levels had been assigned without sufficient (a) depth of coverage of the whole topic and (b) mastery of the topic.

Weaker factor: thin assessment evidence

(a) The pupil has calculated the circumference of five circles of different diameters.

The answer to each calculation is presented as it would be shown on the calculator when the approximation of 3.142 is used for π (pi). The pupil has not rounded the answer or given the units of measurement, as noted by the teacher, who adds a prompt about this.

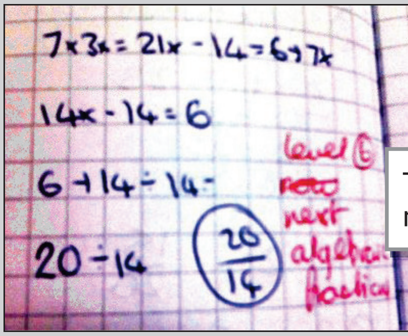


Teacher's comment: **L6**
Well done! You have been able to find the circumference.
What unit do we measure the circumference in?

This is a narrow set of questions: the pupil has not found the circumference when the radius rather than the diameter is given, or solved a variety of problems involving the circumference, diameter and radius of a circle. The teacher has assessed this work as Level 6 (written as L6), but the work does not provide convincing evidence of adequate depth of knowledge and understanding of the topic.

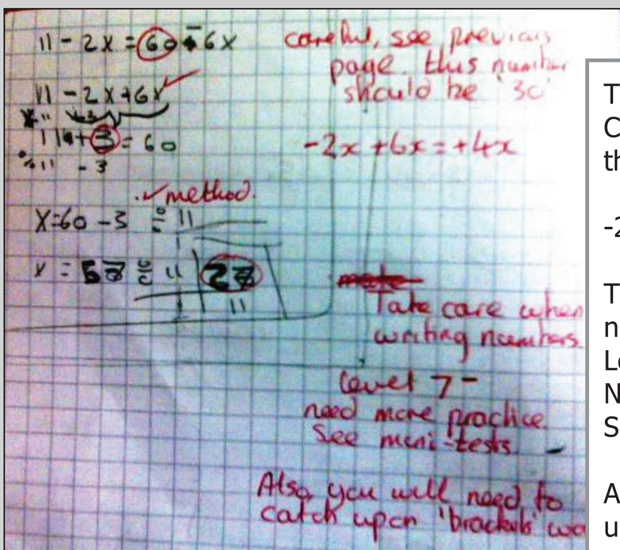
(b) Neither of the two illustrative extracts below show a sufficiently good grasp of linear equations to be assessed at Level 6 in the first example and a low Level 7 in the second. In both cases, the pupils had tackled several linear equations, with these being the last and most difficult question.

The solution below to 7(3x – 2) = 6 + 7x was correct but was poorly set out and the answer presented as an unsimplified vulgar fraction, rather than $x = 1^3/_7$. The teacher's 'next step' comment, 'algebraic fractions' appears to point to the next topic rather than the next step for the pupil.



Teacher's comment: Level 6
next algebraic fractions

The solution below to 11 – 2x = 6(5 – x) had more errors than the teacher had identified and the method was not correct. However, the teacher's comments are constructive.



Teacher's comments:
Careful, see previous page.
this number should be '30'.

-2x + 6x = +4x

Take care when writing numbers.
Level 7⁻
Need more practice.
See mini-tests.

Also you will need to catch up on 'brackets' work.

How might it be improved?

To use only one piece of assessed work every six weeks or half a term is not a reliable way to track a pupil's attainment and progress. A broader range of evidence, such as homework and classwork, tests and longer problem solving and investigative work would give a richer assessment. Some schools make good use of 'Assessing Pupils' Progress' sheets to capture a more holistic view of what pupils can do and their progress within each aspect of mathematics.[8]

[8] Assessing Pupils' Progress is a structured approach to assessment against National Curriculum levels.

87. Almost all the schools gathered such assessment information on a regular basis, typically termly or half-termly, and used it systematically to track pupils' progress and identify underachievement. However, if the assessment is not accurate, the system becomes flawed, and support may not be provided for those pupils who most need it.

Marking: the importance of getting it right

88. Inconsistency in the quality, frequency and usefulness of teachers' marking is a perennial concern. The best marking noted during the survey gave pupils insight into their errors, distinguishing between slips and misunderstanding, and pupils took notice of and learnt from the feedback. Where work was all correct, a further question or challenge was occasionally presented and, in the best examples, this developed into a dialogue between teacher and pupil.

89. More commonly, comments written in pupils' books by teachers related either to the quantity of work completed or its presentation. Too little marking indicated the way forward or provided useful pointers for improvement. The weakest practice was generally in secondary schools where cursory ticks on most pages showed that the work had been seen by the teacher. This was occasionally in line with a department's marking policy, but it implied that work was correct when that was not always the case. In some instances, pupils' classwork was never marked or checked by the teacher. As a result, pupils can develop very bad habits of presentation and be unclear about which work is correct.

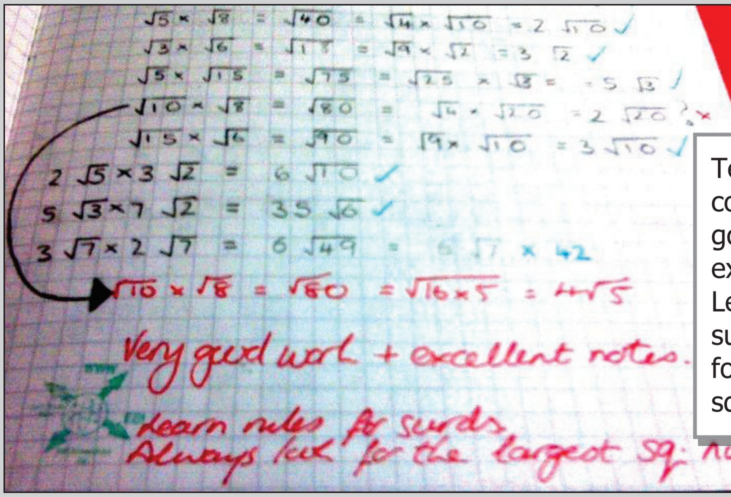
90. A similar concern emerged around the frequent use of online software which requires pupils to input answers only. Although teachers were able to keep track of classwork and homework completed and had information about stronger and weaker areas of pupils' work, no attention was given to how well the work was set out, or whether correct methods and notation were used.

91. Teachers may have 30 or more sets of homework to mark, so looking at the detail and writing helpful comments or pointers for the way forward is time consuming. However, the most valuable marking enables pupils to overcome errors or difficulties, and deepen their understanding. The following two examples illustrate this point.

Prime practice: helpful marking

Simplifying numbers written as square roots (known as 'surd form') is one of the more demanding higher tier GCSE topics. In the first illustration below, the pupil did not simplify $\sqrt{80}$ fully. The teacher demonstrates the simplification, and the helpful comment 'always look for the largest square number' relates specifically to where the pupil's solution had fallen down.

Note also that this teacher is checking work marked already by the pupil and comments on the quality of the notes made in class by the pupil.

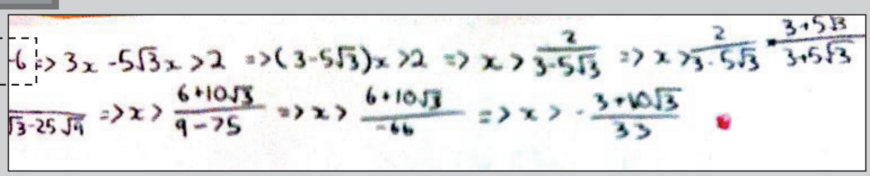


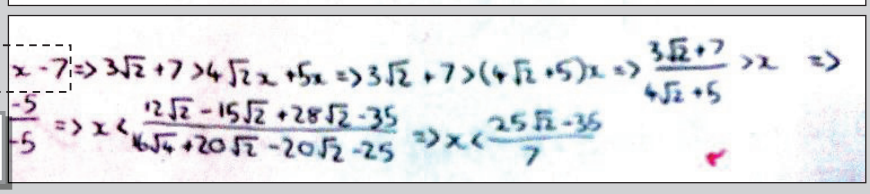
Teacher's comments: Very good work and excellent notes. Learn rules for surds. Always look for the largest sq. no.

Weaker factor: unhelpful marking

The teacher's marking and comment, 'more work is needed on this', did not help the pupil to understand the errors that arose when solving the following two challenging inequalities.

$3x - 8 > 5\sqrt{3}x - 6$

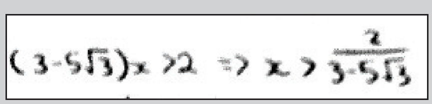




$3\sqrt{2} - 5x > 4\sqrt{2}x - 7$

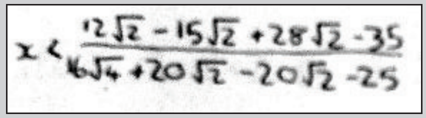
Neither of the pupil's errors was pinpointed in the marking.

In the first question, an important error occurred when the pupil divided by $(3 - 5\sqrt{3})$.



Because $(3 - 5\sqrt{3})$ is negative, x should be less than, not greater than, $2/(3 - 5\sqrt{3})$.

The pupil's error in the second question is likely to have been a simple slip. The first number in the numerator, 12√2, comes from multiplication of 3√2 by 4√2. It should be 24. Otherwise, the method was correct.

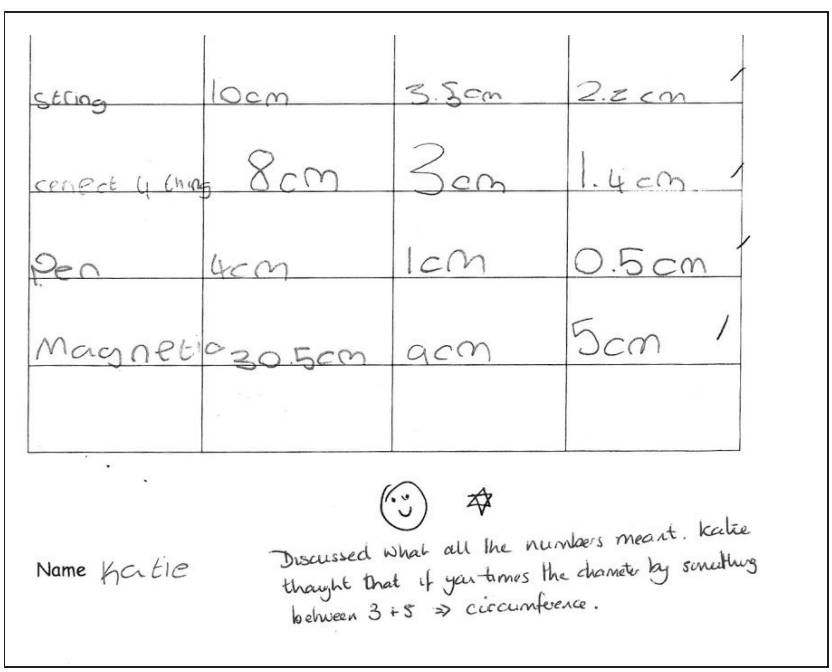


How might it be improved?

It is not clear whether the teacher had identified the nature of these two errors. He/she could consider making model solutions available to pupils so that they could see for themselves where they went wrong. This would also encourage their independence and focus subsequent discussion on any remaining areas of difficulty.

The teacher might have anticipated the first error. The first step in rearranging the inequality 3x – 6 > 5√3x – 4 was likely to lead to the negative expression, 3x – 5√3x, on the left hand side of the inequality. If other pupils had made the same error, it would make a good teaching point for the next lesson.

92. Examples of good annotation of pupils' work were observed in some primary schools, sometimes accompanied by photographs. The example below shows part of a Year 4 pupil's table of measurements of the circumference, diameter and radius of several objects. The teacher has recorded the pupil's generalisation from her results, 'Discussed what all the numbers meant. Katie thought that if you times the diameter by something between 3 and 5 ⇒ circumference.'



string	10cm	3.5cm	2.2cm
conect 4 thing	8cm	3cm	1.4cm
Pen	4cm	1cm	0.5cm
Magnetic	30.5cm	9cm	5cm

Name Katie

Discussed what all the numbers meant. Katie thought that if you times the diameter by something between 3 + 5 ⇒ circumference.

93. The following examples represent brief but helpful advice or questions provided by teachers in marking pupils' work.

- 'Check all reflections with tracing paper especially ones in the diagonal line y=x.'
- 'Practise adding and subtracting recurring fractions – try the A/A* booster on [online software].'
- 'Look up bearings as you've missed this work. What are the three key points to remember when using bearings?'
- 'Try this question again. What type of triangle is it?' (The pupil had previously not spotted that the unusually orientated triangle was isosceles, but redid the question correctly following the teacher's hint.)
- 'What is 20cm in mm?' asked a teacher after praising a pupil's correct work on shape. The pupil had written 200 in response. This Year 5 teacher regularly asked pupils short questions at the end of their work and they responded. The pupils explained to the inspector that they checked for the teachers' comments and questions in their mathematics and English books during register time at the start of morning school. Such good practice is not typical.

94. Some marking did not distinguish between types of errors and, occasionally, correct work was marked as wrong. For example, in marking homework on probabilities of single events, a teacher marked unsimplified fractions as wrong in exactly the same way as incorrect answers that showed misunderstanding. For instance, the probability of selecting a heart from a pack of playing cards is ¼ but a pupil's unsimplified but correct answer of $^{13}/_{52}$ was marked wrong, instead of indicating that this fraction was correct but cancels down to ¼. The same pupil's incorrect answer of ¼ for the probability of selecting a king from a pack was similarly marked wrong without any comment. The pupil might have reasoned, incorrectly, that the probability of selecting one king, knowing that a pack has four kings, is ¼ but this misunderstanding was not addressed through the marking. (The correct answer is $^{4}/_{52}$ which simplifies to $^{1}/_{13}$.)

95. At other times, teachers gave insufficient attention to correcting pupils' mathematical presentation, for instance, when 6 ÷ 54 was written incorrectly instead of 54 ÷ 6, or the incorrect use of the equals sign in the solution of an equation.

96. Most marking by pupils of their own work was done when the teacher read out the answers to exercises or took answers from other members of the class. Sometimes, pupils were expected to check their answers against those in the back of the text book. In each of these circumstances, attention was rarely paid to the source of any errors, for example when a pupil made a sign error while expanding brackets and another omitted to write down the '0' place holder in a long multiplication calculation. When classwork was not marked by the teacher or pupil, mistakes were unnoticed. In the example below, the pupil made a

common, but important, error by not drawing the quotient line correctly in the formula for solving a quadratic equation. Because the work was not marked, the error was not corrected and the pupil could continue to make the same mistake repeatedly in the future.

$x^2 - 4x + 2 = 0$

$x = 4 \pm \frac{\sqrt{(16 - 4 \times 1 \times 2)}}{2 \times 1}$

$x = 4 \pm \frac{\sqrt{8}}{2}$

x = 5.4 or 2.6, which are incorrect solutions

The correct first step should have been $x = \frac{4 \pm \sqrt{(16 - 4 \times 1 \times 2)}}{2 \times 1}$

97. The involvement of pupils in self-assessment was a strong feature of the most effective assessment practice. For instance, in one school, Year 4 pupils completed their self-assessments using 'I can …' statements and selected their own curricular targets such as 'add and subtract two-digit numbers mentally' and 'solve 1 and 2 step problems'. Subsequent work provided opportunities for pupils to work on these aspects.

98. An unhelpful reliance on self-assessment of learning by pupils was prevalent in some of the schools. In plenary sessions at the end of lessons, teachers typically revisited the learning objectives, and asked pupils to assess their own understanding, often through 'thumbs', 'smiley faces' or traffic lights. However, such assessment was often superficial and may be unreliable, as illustrated in the following case study.

Weaker factor: unreliable self-assessment of understanding

A low-attaining Year 8 class had worked on multiplying and dividing by 10, 100 and 1,000. In the work on multiplying, the pupils had completed a page of simple multiplications, mostly multiplying whole numbers by powers of 10, but also including some where they had to multiply a decimal number. One pupil who had ticked her answers right explained to the inspector, 'to multiply by 100, you add two zeros'. She went on to say that when it was a decimal you had to move the decimal point. She knew to move the point the same number of places as the number of zeros in the multiplier, so twice when multiplying by 100. However, she was unsure in which direction the point moved. In fact, she gave the answer to 4.6 x 10 as 0.46 but, because she believed that 0.46 was the same as 46, she marked her answer as correct. She thought that she understood the topic but her understanding was very shaky.

How might it be improved?

When assessing pupils' understanding at the end of such a lesson, the teacher might have asked pupils to display their answers on miniwhiteboards so that errors would be picked up immediately. Alternatively, a quick matching task that involved questions and possible wrong and right answers (say 46, 4.60, 0.46, … for 4.6 x 10) would allow misconceptions to be revealed.

99. Rather than asking pupils at the end of the lesson to indicate how well they had met learning objectives, some effective teachers set a problem which would confirm pupils' learning if solved correctly or pick up any remaining lack of understanding. One teacher, having discussed briefly what had been learnt with the class, gave each pupil a couple of questions on pre-prepared cards. She took the cards in as the pupils left the room and used their answers to inform the next day's lesson planning. Very occasionally, a teacher used the plenary imaginatively to set a challenging problem with the intention that pupils should think about it ready for the start of new learning in the next lesson.

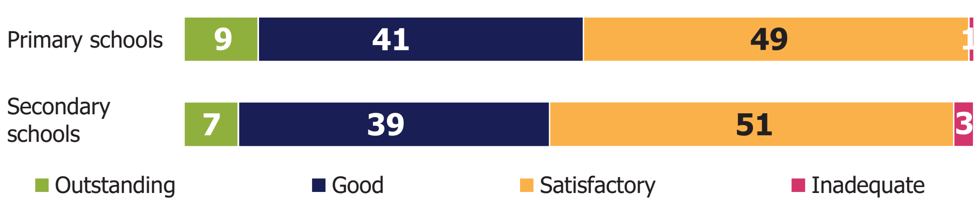
Curriculum

100. Important weaknesses in the curriculum remain. Inspectors continue to be concerned about the lack of emphasis on 'using and applying mathematics'. In addition, curriculum plans in many schools did not offer the necessary support to help teachers' tailor lessons to meet the needs of different ability groups. This section draws attention to the negative impact of early entry to GCSE on many of our pupils.

The curriculum – is it good enough?

101. Since the last survey, the secondary National Curriculum, GCSE and AS/A-level specifications, and the National Strategy frameworks have all been revised. Most schools have amended their schemes of work, some on more than one occasion. Despite this, or perhaps because of it, the mathematics curriculum was judged good or outstanding in less than half of the schools inspected (47). While it was inadequate in only seven schools, it often had important shortcomings that impeded pupils' better progress.

Figure 4: Quality of the curriculum in mathematics in the schools surveyed (percentages of schools)



Percentages are rounded and do not always add exactly to 100.

102. Day-to-day management of the curriculum is generally stronger than strategic leadership. Action to bring improvement tends to be reactive rather than proactive. Lack of robust evaluation of curricular developments and initiatives, including intervention programmes, and insufficient interrogation of information from monitoring activities and question-level analyses too often mean searching